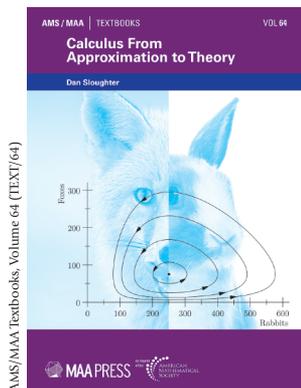


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## Calculus from Approximation to Theory

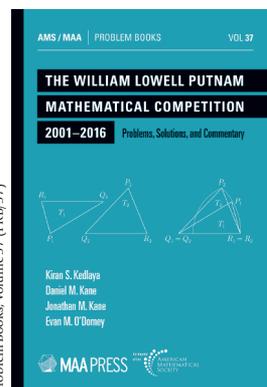
by Dan Sloughter

How would you describe what calculus is about if asked at a cocktail party? You might say something like, “The central idea of calculus is that the way to measure something curved is to break it up into many, many small pieces, approximate those pieces with flat replacements, and add up the measures of the flat things. As you take more and more, smaller and smaller, flat pieces, your approximation gets better and better.” If pressed about the mismatch between “measure” and “approximation,” you’d probably mutter something about technicalities and try to change the subject. But, if you wanted to take your interrogator seriously, you’d try to explain something about limiting processes and you’d probably draw a circle and approximating regular inscribed polygon and talk about the sequence of approximations as the number of sides increases.

I would be thrilled to hear one of my calculus students explain the subject this way, but I’m sure it would never happen. I’d have to redesign my entire calculus course. I imagine that I would define limits in terms of limits of sequences. I would want to spend a fair bit of time talking about approximation, and error in approximation, and order of an approximation. I’d make free and copious use of computation. The derivative would be defined as the best affine approximation to a curve in a natural way. The integral would be obvious once my students learned to think this way. Power series would be the natural generalization of approximating polynomials. I’d probably throw in a concluding chapter on differential equations just to pre-emptively answer the cocktail party follow-up question: “Why would anyone care?”

The AMS Bookshelf is prepared bimonthly by AMS Acquisitions Specialist for MAA Press titles Stephen Kennedy. His email address is [skennedy@amsbooks.org](mailto:skennedy@amsbooks.org).

This is exactly what Dan Sloughter has done in *Calculus from Approximation to Theory*. And, it feels right. The exposition is clear and compelling, the problems are varied and meaningful, and everything is rigorous. This would make a great alternative to the standard Calculus-as-a-collection-of-tools approach. It feels unified and whole and very satisfying.



## The William Lowell Putnam Mathematical Competition 2001–2016: Problems, Solutions, and Commentary

by Kiran S. Kedlaya, Daniel M. Kane, Jonathan M. Kane, and Evan M. O’Dorney

Possibly the most difficult Putnam problem ever was B6 on the 2011 exam: Let  $p$  be an odd prime. Show that for at least  $(p+1)/2$  values of  $n$  in  $\{0, 1, 2, \dots, p\}$ ,  $\sum_{k=0}^{p-1} k!n^k$  is not divisible by  $p$ . No competitor received a positive score. Do you see what to do?

This is the fourth collection of Putnam problems published by MAA and it covers the years 2001–2016. It contains full reports of all individual and team results and a variety of statistics about the scores. It also contains, of course, the problems. But the meat of the volume is the collection of solutions. Often more than one solution, especially if there exists an interesting alternative solution. The four authors, collectively, are ten-time Putnam Fellows. Naturally, their perspective is enlightening. They include a short section on strategy for success at the Putnam. Also, and even more interesting because it reveals something of how they approach and think about the problems, there is a section of very brief hints sandwiched between the problem statements and solutions. About this the authors say, “The hints may often be more mystifying than enlightening. Nonetheless, we hope they encourage readers to spend more time wrestling with a problem before turning to the solution section.” Still puzzling over 2011 B6? The hint from these authors suggests you count the roots of  $\sum_{k=0}^{p-1} x^k/k!$ .