exams, the pedigree of schools, and status of letter writers. While these are not irrelevant data points, in order to prevent bias, it is important to start by asking what kinds of things we are looking for evidence of. We may be looking for evidence of strong background knowledge, of ability to do creative research, and/or intellectual independence and maturity. We can then consider the various pieces of evidence within the application towards these ends. For example, we can ask: what does a particular GRE score tell us about one's ability to do creative research? Or about their general background knowledge? In fact, evidence seems to show that it only gives very limited information about these things, and does it in a way which is very much skewed along gender and racial lines. In using this for evidence, one therefore needs to take these factors into account, or, as many are doing, ignore these scores entirely. Such criteria can also be very helpful in reading letters of recommendation. Letter writers often exhibit a range of biases themselves, and as we read such letters, we have to constantly consider how their narrative helps or doesn't help us evaluate a candidate according to our criteria in order to protect ourselves against their biases.

How do we know these things? While I am compelled to point out that I am speaking beyond my own expertise, nevertheless, the questions I've sought to address here, on how to address bias in various aspects of academic life, are not beyond our ability to answer. These are questions of social science, which various people have endeavored to study and provide some answers. Below I'll point out just a bit of the literature that I've looked at which relates to some of the assertions I've made here.

There are many studies of *unconscious bias in hiring*, which suggest the adoption of explicit criteria (evidence-based approaches to hiring). While not specifically about academia, this article summarizes various studies and points to explicit and transparent evaluation criteria as a method to reduce implicit bias in hiring and promotion: `https://doi.org/10.1111/0022-4537.00234`.

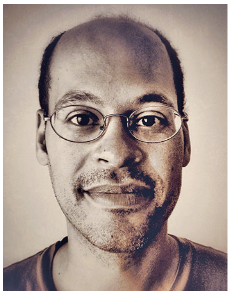
It does seem to be true that awareness of bias lessens its effects, as was discussed here: `https://doi.org/10.1038/s41562-019-0686-3`. However, we should not imagine implementing formal training for all of us to "remove bias" will necessarily solve the problem. To this effect I'd point out this nice review: `https://doi.org/10.1146/annurev.psych.60.110707.163607`, which highlights a real lack of evidence of efficacy of such programs.

The resistance of people to implement procedures to reduce bias has also been documented: `https://doi.org/10.1108/02683941111138985`.

Gender bias in letters of recommendation is something which needs to be taken into account. This has been discussed in a number of places in the literature. I found these articles informative: `https://doi.org/10.1037/a0016539` and `https://doi.org/10.1038/ngeo2819`.

As we look for unbiased criteria for graduate students, *GRE subject scores seem problematic*. While I haven't found data on the math subject score as a predictor of success, the usefulness of subject tests has been examined in general as well as in physics in particular. While moderate positive correlations seem to exist between GRE subject scores and grades during the first year of graduate school (`https://doi.org/10.1037/0033-2909.127.1.162`), it didn't seem correlated to other measures of success for physics graduate students (`https://doi.org/10.1126/science.274.5288.710`). Besides this, evidence that GRE subject scores in physics suffer from gender bias is discussed in various places, for example: `https://www.aps.org/publications/apsnews/199607/gender.cfm`.

These studies really just scratch the surface of a literature in which I am still largely uninitiated; however, I believe that they do serve to illustrate that these are ideas whose validity can be reasonably explored, tested, and refined as we try over time to make the moral arc of our profession bend towards inclusion.



Danny Krashen

Credits

Photo of Danny Krashen is courtesy of Max S. Gerber.

Rethinking the Teaching and Learning of Mathematics in Light of COVID-19

Della Dumbaugh and William McCallum

Viewing the Pandemic as an Opportunity

During the spring of 2020, COVID-19 prompted the nonprofit Bill McCallum cofounded, Illustrative Mathematics (IM), to put considerable energy into developing resources

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for K–12 teachers using the IM curriculum in all the different combinations of in-person, virtual, synchronous, and asynchronous that schools have implemented. It began to dawn on Bill and his colleagues that amid all the chaos and anxiety there were real opportunities to rethink the teaching and learning of mathematics. We wondered about this at the collegiate level. For this article, Bill interviewed colleagues at the large public institution where he formerly served on the faculty, and Della Dumbaugh drew from her work at a small private institution, particularly her success with oral exams, to explore the lessons learned for mathematics teaching at the collegiate level.

Large Public Institution: University of Arizona

Bill interviewed two of his former colleagues at the University of Arizona, Deb Hughes Hallett and Rob Indik, about their experiences with online learning.

I won't dwell on all the logistical problems they faced, which I am sure are familiar to any readers who had to teach online in the spring of 2020. Rather I'll highlight a few things that seem worth developing or that point to changes in teaching practice even when things return to normal.

Deb found that the structure imposed by screen sharing led to some good practices. She found that "for people teaching proofs or computations, where students have to follow an argument with several lines, online can be very good because of the animation features of PowerPoint or Beamer. In theory you can stop and ask 'what comes next' at the end of each line of writing on the board, but usually instructors are so occupied with writing that they don't. If you use animation, it's very easy to do synchronously online. This I would definitely take back into the classroom with me."

It works the other way as well, when students share their screens with instructors. Deb found that the necessity of students sharing their screens and writing about their problem enabled her to get a much clearer picture of what they were thinking.

Both Deb and Rob were teaching a class that had students working in groups at tables of four, which were replaced by Zoom breakout rooms. This was not a perfect solution because it is difficult to know how much discussion is going on in these rooms simultaneously. It is possible for instructors to drop into the Zoom rooms, but in practice this takes more time than wandering around a classroom eavesdropping. If Zoom made it possible to switch quickly between observing different breakout rooms, it would be a big improvement.

Because students were not required to have their cameras on, it was difficult to know who was really listening. In one of Rob's classes almost nobody turned on their cameras. To mitigate this they instituted multiple-choice questions in the form of polls for the full-class sessions, which meant that each student in the class had to commit to an answer. Of course, instructors have been doing something similar for years, with concept tests and various response systems. However, the power of multiple-choice questions to ask deep questions and reveal gaps was made vivid to Rob when the students were invisible and the responses were the only thing he could see. He said that one thing he'll do differently when he goes back to in-person teaching is use a lot more of these sorts of multiple-choice questions. In this case, it's not so much that the pandemic has led to a newly developed teaching practice as that it is likely to make an existing practice more widespread. We will see another example of this in the next section.

According to Deb, the big challenge for everyone was exams. It is difficult to stop students getting help, although they did come up with a way of detecting that: students were required to take the exams simultaneously on a Zoom call with their video on and they were scanned constantly. Questions were presented in randomized order, and for multiple-choice questions the options were randomized as well. They caught one student talking to someone else not present, and were able to listen in. Nonetheless, they both felt that they probably missed some cheating. Rob said, "Of course, I strongly suspect there is cheating for our in-person exams that we don't catch as well. My inclination is to avoid focusing on cheating and cheating prevention." Maybe this will make us rethink the way we assess students.

Learning from her experience in the spring, Deb asked everybody in a summer class to turn on their cameras during class (unless they were having bandwidth problems). And, learning from another colleague's experience in the spring, she started cold calling on the students, beginning with those she thought unlikely to answer on their own. As a result she got to know everybody and they all got to know each other.

Small Private Institution: University of Richmond

For Della at the University of Richmond, COVID-19 led to a revival of the oral exam.

The University of Richmond is a liberal arts institution with about 3200 undergraduates. Our mathematics classes have enrollments of about 22–28 students. Although I had taken oral exams in graduate school (live, in-person oral exams at the chalkboard in some professor's office), the oral exam was not a tool I used in my undergraduate classroom before March 2020. Teaching mathematics online during COVID-19, however, inspired me to infuse the seemingly antiquated tactic of oral exams with fresh purpose.

With solutions and other resources readily available online, I noticed students gained a false sense of security about their understanding of mathematical concepts and I had a difficult time assessing what students actually knew themselves. Oral exams provided a way to address these issues simultaneously. In the process, reviving and updating oral exams for the online classroom helped students improve their communication, conquer anxiety, solve problems quickly, and think creatively. They also gave students an opportunity to articulate layers of sophisticated thinking in an organized fashion, a skill they could take with them to any workplace.

Secrets for Success with Oral Exams

1. Set clear guidelines about what the exam will cover and how long it will last. Less is more. By way of an example, for Calculus, I might limit the oral exam to questions about what the derivative tells us about a function. For abstract algebra, I might focus on questions related to Lagrange's theorem or permutation groups. Announce the amount of time for the oral exam in advance and stick to it. I set up a google doc for students to schedule their oral quizzes and exams. Listing appointments for exams at 11:00, 11:10, 11:20, etc., reinforces that these opportunities for assessment last a prescribed amount of time. It also helps students understand that they must attend their oral exam on time.
2. Harness the power of peers watching peers do mathematics. To prepare your students for this type of assessment, record sample oral exams and post them to the class electronic learning site (Blackboard in my case). This simple technique is the single most effective tool to ensure success with oral exams. These students have grown up with a video camera nearby their entire lives; they are comfortable making videos. I convinced a couple of students to take their oral exam early and allow me to record and post it to the class. Naturally, these "sample" students worked and reworked problems so they would demonstrate mathematical finesse in their video. Quite unexpectedly, these videos not only served as sample oral exams, but they also underscored the process involved in thinking about problems in mathematics. It was one thing for me to work problems in my instructional videos, it was quite another for students to watch their 19-year-old classmate articulate each step of a problem. "You want me to find where this function has a horizontal tangent line," one student began her sample oral quiz. "This means I need to take the first derivative and set it equal to zero. This function is a quotient so I will need to use the quotient rule…." For whatever reason, students absolutely loved to hear their peers speak like mathematicians. That motivated them to learn the same language. And here is the funny thing. After two weeks of my repeatedly asking for volunteers that I could record taking a sample oral exam, students became eager to do so.

What Students Gain from an Oral Exam

1. Communication skills. The students were responsible for communicating sophisticated mathematics in a timed setting. As one senior put it, "I wish I would have done this last year. This would have really helped me prepare for my tech interviews." One first-year student wrote on her Calculus evaluation that "at first I was afraid of the oral quizzes. After I took one, though, I realized how much I liked to talk about math. I started meeting with a friend after that so we could talk about problems together." The oral exams required students to articulate ideas about sophisticated mathematics in an organized manner. This skill will be useful in any workplace.
2. They learned to press through and conquer anxiety. One 6′7″ talented basketball player would appear in his rectangle visibly nervous at the start of his oral quiz. After I gave him his problem, he would repeat it back to me and then take a deep breath. Those 17 seconds seemed to help him gain his focus and direction. Then he would begin, "this is a problem about a derivative…." That he can talk about mathematics with finesse and successfully aim a 9.5 inch ball into a net will both serve him well in his future.
3. Fast-paced problem solving. At times, students had 10 minutes to solve three problems. They used their mastery of the material to answer three different types of questions in quick succession. They had to identify the essence of the problem, weigh various approaches, and act. These are skills that will serve them well in many professions. Thus oral exams foster skills for a student who aims to pursue a career as a Wall Street Stock Exchange trader. In that role, she will have to organize fast-changing information, determine a best course of action, and take steps to put decisions in place.
4. Creativity. Students created elaborate testing environments at home that they could capture with their computer screen. One student taped paper to the freezer portion of her refrigerator and presented her work like a teacher. Later, I gave a telephone reference for this student. I described my work with her in four mathematics classes and then relayed this story about the freezer as a sort of lighthearted addition to the conversation. The employer went crazy. "What a great problem solver," he gushed, "that is just the kind of spirit we are looking for."
5. They dressed for success. These students who regularly attended our remote class in sweats and t-shirts took showers, brushed their hair, and put on button down shirts. I did not include dressing professionally as part of the instructions. They recognized this was important and they rose to the occasion.
6. Students learned how to talk about mathematics, and in particular, they learned how to talk about mathematics with each other, to *do* mathematics together. They met over Zoom and practiced with each other.
7. Real-time feedback on their work. Usually, students take some sort of assessment on paper in class and submit it. Even if faculty grade these assessments promptly, students will still have to wait several days for feedback. With an oral exam, you can see exactly what a student

understands and point them to particular areas for further study. One student worked a derivative problem beautifully but could not evaluate $\sin(\pi)$ at the end. I looked at the list of oral quizzes, which ended at 5:30. I responded along the lines of "you just did beautiful, sophisticated work that showed a solid understanding of how a derivative works and what it tells us about the function. Your last calculation hinges on a solid understanding of an important trig function, $f(x) = \sin x$. I'm going to schedule you for a follow-up discussion at 5:30. I'd like for you to have learned the graph of $f(x) = \sin x$ by then and use it to evaluate some basic x-values." At 5:30, the student reappeared on Zoom with a solid understanding of the $\sin x$.

Harnessing the Power of Oral Exams Elsewhere in the Classroom

The success of the sample oral quizzes and exams led to the creation of an entirely new approach to studying for the final exam. I assigned individual students one important problem to prepare for the final exam. I met with them over Zoom and recorded each student working their problem. I collected these individual videos in a folder on our Blackboard site titled "Video Final Exam Review." I encouraged students to utilize even three minutes of free time to view one of the videos and work through a problem that would help prepare them for the final. Students absolutely loved this collection of problems and reported that, indeed, in the moments before dinner, or right before bed, or in an afternoon lull, they would go the folder, click on a video, and work a problem. "Sometimes I watched the same video a few times just so I could get down all the steps," one student commented.

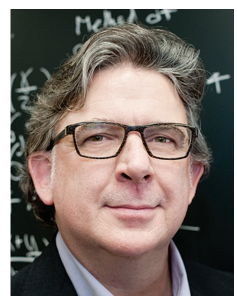
What Faculty Gain from Oral Exams

1. First and foremost, faculty gain an accurate assessment of what your student knows (and does not know) about the material. It only takes five or ten minutes to gain insight into a student's understanding of mathematics. Students cannot hide in an oral exam.
2. More efficient grading. You can grade on the spot. (No electronic grading!) I purchased a large stack of legal pads. I wrote out the problems on the paper and used the margin to keep track of assessment. I used a similar sort of grading rubric as I did for written exams. For example, for a Calculus problem worth 10 points that asks where a function has a horizontal tangent line, I would allocate three points for translating the problem into a question about derivatives, four points for doing the calculation, and three points for determining where the derivative equals zero. This allowed students to earn some or all of the possible points, as with a written exam. I generally did not interrupt students, even if they were moderately off track. If they were well into a problem and completely headed in an unfruitful direction, I might suggest they begin the problem again with a different approach and then subtract some points for this "tip." I made all of my oral quizzes worth 10 points. For tests with five problems, I made each problem worth 10 points and doubled the score.
3. **Joy.** I had struggled to find the same delight in the remote setting of a classroom consisting of 1 × 2 inch rectangles on my computer screen as I did when teaching in a "normal" semester. An oral exam with each student reminded me of why I got into this business in the first place—to build and strengthen lives through mathematics.

Realistic Outcomes for the Future

Taken together, these experiences suggest that the restrictions of the pandemic present an opportunity to give thought to new and, perhaps, improved ways to teach mathematics. To be sure, there is confusion and anxiety in the current moment. Maybe now is not the best time to completely revamp teaching styles. But now may be the best time to reconsider seemingly tried and true techniques from a fresh perspective. The lessons we gather now could inform meaningful changes for the future.


Della Dumbaugh


William McCallum

Credits

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Photo of William McCallum is courtesy of the author.

Building Equity-minded Online Programs

Justin Lanier and Marissa Kawehi Loving

So you want to start a program to help build and serve the mathematical community. And you want your program

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