The Ruth Lyttle Satter Prize recognizes an outstanding contribution to mathematics research by a woman in the previous six years.

Kaisa Matomäki, University of Turku, Finland, received the 2021 Ruth Lyttle Satter Prize for her work (much of it joint with Maksym Radziwiłł) in the field of multiplicative functions.

Previous Ruth Lyttle Satter Prize winners include:

2019 Maryna Viazovska, for her groundbreaking work in discrete geometry and her spectacular solution to the sphere-packing problem in dimension eight.

2017 Laura DeMarco, for her fundamental contributions to complex dynamics, potential theory, and the emerging field of arithmetic dynamics.

The Joan and Joseph Birman Fellowship for Women Scholars awards exceptionally talented women extra research support during their mid-career years.

Karin Melnick, University of Maryland, College Park, received the 2020/2021 Birman Fellowship for her work on differential-geometric aspects of rigidity.

Previous Joan and Joseph Birman Fellows:

2019–2020 Lilian Pierce, for her research in analytic number theory and harmonic analysis.

2018–2019 Margaret Beck, for her exceptional research on stability problems in partial differential equations and spatially extended dynamical systems.

The Ruth Lyttle Satter Prize in Mathematics was established in 1990 by Joan S. Birman in memory of her sister, botanist Ruth Lyttle Satter. The Joan and Joseph Birman Fellowship for Women Scholars was established in 2017 by Joan and her husband, Joseph. The AMS is grateful to the Birmans to their commitment to advancing women in mathematics.

Thank You

To learn more and make a gift, visit www.ams.org/giving
A WORD FROM...

Catherine A. Roberts, Executive Director of the AMS

The opinions expressed here are not necessarily those of the Notices or the AMS.

March 2021

February 1st each year is significant on the AMS calendar. On this day, we welcome dozens (and dozens) of new volunteers who have agreed to serve our society across nearly a hundred committees. Their willingness to contribute to advancing research and building our profession is what makes the AMS, well, the AMS.

This is essential and central work. Our volunteers serve as journal editors and leaders in governance. They serve on award selection committees and policy committees addressing publications, meetings, education, science policy, and the profession. These committee volunteers are joined by thousands of others who serve as journal reviewers, blog editors, MathSciNet® reviewers, authors, donors, and supporters. The AMS exists because of the collective contributions of all of its members. I am especially grateful on February 1st when I am reminded of the scale of the AMS operation. And, before I forget, please consider nominating yourself or a colleague to serve on AMS committees! We would love for you to join us.

Given that this is Women’s History Month, today I want to give special thanks to three remarkable women whose AMS terms ended on February 1, 2021.

Carla D. Savage

On February 1st, Carla completed eight years as AMS secretary. In the years since our society was formed in 1888, Carla was only our tenth secretary and the first woman.

A few words about this position, since it’s hard to tell from the title “secretary” what this is all about. The Office of the Secretary is the primary point of contact between the AMS and its members. The secretary implements scientific policies of the society, oversees the scientific program at AMS conferences, manages our nearly 100 committees, oversees all AMS prizes and awards, and coordinates our AMS elections. Carla serves as the institutional memory for the AMS, which is such an important role. She was instrumental in onboarding me, and has certainly played the same role for countless others.

There are so many things I could thank Carla for. Given that this issue is celebrating women’s history, let’s focus there. We all recognize that we have further to go before women reach parity in mathematics. Behind-the-scenes efforts by Carla have had a significant impact on this shared goal. Since 1988, the Notices has been publishing an annual summary showing the relative numbers of men and women in categories such as invited hour addresses at AMS meetings, speakers at Special Sessions at AMS meetings, and the number of female members of editorial boards of AMS journals. Along with serious and thoughtful efforts by many members of the mathematics community serving the AMS, Carla has helped make the AMS more inclusive. We have seen broader participation of women in many aspects of the society’s governance and leadership during her term.

More recently, the Council’s new Prize Oversight Committee is at work to develop and implement an approach to recruit larger, more diverse pools of nominees. I am optimistic that the efforts Carla nudged along in the background to improve the participation of women at the AMS will contribute to this work. I look forward to helping the AMS welcome increased participation from all underrepresented mathematicians. I hope our efforts to include

Catherine A. Roberts is executive director of the AMS. Her email address is exdir@ams.org.

1 https://www.ams.org/gov-committees
2 https://www.ams.org/committee-nominate
more racially diverse mathematicians, as well as more early-career mathematicians, career teaching faculty, and those working in business, industry, and government will become something to celebrate.

**Jane M. Hawkins**

Jane had served a full decade as AMS treasurer when she stepped aside February 1st. She was also the first woman to serve in this role. The treasurer is responsible for long-term financial management and auditing oversight of our society. She ensures that AMS assets, which all belong to our members, are invested in a manner consistent with our mission.

In her role, Jane worked very closely with the chief financial officer overseeing our budget, finances, annual audit, and investments. She served as chair of our Investment Committee, during which time the AMS endowment grew substantially. Her expert and careful stewardship means our society can continue to advance its mission, even during the current uncertain economic times.

Jane also has been deeply involved with the AMS Office of Government Relations, supporting the work of its director and thinking carefully about how we can best support our mathematics research community through policy advocacy. Jane and I share an appreciation for how our community’s voice is strengthened when more mathematicians become members of the AMS. She has been a true advocate for all mathematicians.

**Jill C. Pipher**

On February 1st, Jill became AMS past-president. Jill was the third woman to be elected AMS president (following Julia Robinson and Cathleen Synge Morawetz). I see Jill’s fingerprints everywhere, and here are just a few examples.

Recently, our AMS Council added a new policy committee on Diversity, Equity, and Inclusion. This would not have happened without the goodwill and energy of dozens of people, but Jill was the catalyst. She asked what it would take to establish a permanent and high-level diversity committee at the AMS. She led the effort and navigated the conversations to make it happen.

During a 2020 Council conversation about how the AMS should engage with the racial reckoning happening in the United States, Jill helped establish the Council’s Task Force on Understanding and Documenting the Historical Role of the AMS in Racial Discrimination. This is one part of an Action Plan designed by the AMS in summer 2020. She understands the value of learning about our history as we seek real change for our mathematics community.

In her position as AMS president, Jill made hundreds of appointments to committees and helped set ballots for elected positions, in consultation with our nominating committees. Even though I was fortunate to be in the rooms when much of this progress happened, I still do not know how Jill managed to accomplish so much in two years. She donated hours of time and talent to the AMS, and on behalf of all of us, I extend my gratitude. Her impact on our profession will be felt for a long, long time.

In closing, I have highlighted three remarkable women whom I have had the special privilege of working with at the AMS. I want to thank them for their incredible service. The AMS is stronger because of their dedication to our mathematics community.

---

Catherine A. Roberts

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3https://www.ams.org/coedi-home
4https://www.ams.org/understanding-ams-history
5https://www.ams.org/messageofsupport
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LETTERS TO THE EDITOR

Mathematicians Call on their Colleagues and on Mathematical Societies to Support the Egyptian Mathematician Laila Soueif and her Children

The case of Laila Soueif: https://lailasoueif.org/call/
Cliquez ici pour la version française: https://lailasoueif.org/appel/

The Egyptian mathematician and activist Laila Soueif gets no respite. She is a Professor of Mathematics at Cairo University and a founder of the March 9 Movement for University Autonomy in Egypt, and she and her family know all too well Egypt’s political repression. In the 1980s, as their daughter Mona was born, her husband Ahmed Seif El-Islam, who had become known as an eminent lawyer and defender of human rights, was in prison for his left-wing political activity. In November 2013 following the coup d’état of Marshal Abdel Fattah al-Sissi, Soueif saw her son Alaa, who had been the icon of the Egyptian Revolution of 2011, arrested once more. A few months later in June 2014, it was the turn of her daughter Sanaa to be arrested in a demonstration calling for freeing political prisoners including Alaa. Hardly two months later, Soueif’s husband Seif died from complications following heart surgery. Laila and her three children, two of them in prison, undertook a hunger strike which went on more than two months, protesting the imprisonment of Alaa and Sanaa.

In September 2019, Alaa, soon after being released from prison, was arrested anew, and Laila again worked for his release, along with her teaching at the University. After organizing a small demonstration calling for prisoners to be freed during the coronavirus epidemic, she was arrested in turn, but let go the next day. Prison visits being suspended and communication with families cut off, Laila Soueif began a sit-in in front of the prison demanding that she be allowed to receive a letter from Alaa. She was physically attacked there, along with her daughters who had come to join her. When they went to the authorities to report the attack, Laila saw Sanaa taken away by out-of-uniform police, and held in prison.

Laila and Mona are being smeared constantly in state-owned newspapers which routinely publish articles claiming they are foreign agents, terrorist sympathizers, etc…

When the press is severely muzzled, supporters of human rights and democracy urgently need international support. Currently Laila Soueif continues to struggle for the release of her two children. We call on our colleagues and on mathematical societies to support her and her children by writing, in your respective countries, to

• your Ministry of Foreign Affairs;
• the Egyptian Embassy.

We invite you also to send messages to the Egyptian authorities, namely to

• the Public Prosecutor’s Office;
• the Ministry of Justice;
• the Ministry of Higher Education;
• the Presidency.

A list of Egyptian authorities’ addresses is available here: https://lailasoueif.org/take-action/

Signatories:
Ahmed Abbes, mathematician, Director of Research at CNRS, Paris.
Sir John Ball, FRS, Professor of Mathematics at Heriot-Watt University, former President of the International Mathematical Union.

*We invite readers to submit letters to the editor at notices-letters@ams.org.

https://mesana.org/awards/awardee/mesa-academic-freedom-award/laila-soueif
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https://www.newsclick.in/alaa-abd-fattah-hungers-strike-freedom
https://www.democracynow.org/2020/7/6/egyptian_activist_laila_soueif_on_the
Ivar Ekeland, FRSC, Professor Emeritus of Mathematics and former President, University of Paris-Dauphine.

Dusa McDuff, Kimmel Professor of Mathematics at Barnard College, Columbia University.

John W. Milnor, Laureate of the Abel Prize (2011), Distinguished Professor of Mathematics at Stony Brook University.


Bào Châu Ngô, Professor of Mathematics at the University of Chicago, Laureate of the Fields Medal (2010).

Graeme Segal, FRS, Professor of Mathematics Emeritus, All Souls College, Oxford, former President of the London Mathematical Society.


Cédric Villani, Member of the French National Assembly, Laureate of the Fields Medal (2010).

There are 26 signatories in all.

Letter to the Editor

Just before Christmas 2019 two secretaries and the librarian at the Mittag-Leffler Institute at Stockholm were given notice of the immediate termination of their appointments. These measures were taken by the management of the Institute, together with the chairman of the Board, under the auspices of the Royal Swedish Academy of Science which "owns" the Institute. The Board itself was never consulted, the excuse being that this was an internal affair concerning personnel and none of the business of the Board, besides which they probably would not have approved. In fact, the Board did not and several members, especially the non-Swedish, voiced great concern. Strong pressure was applied on the chairman to convene an extra board-meeting, which was subsequently done, during which the matter was heatedly discussed, but no generally acceptable explanation was forthcoming beyond references to conflicts of personality.

One may argue that the handling of the staff is wholly within the jurisdiction of the appointed manager, who is responsible solely to the Board, and that the staff exists to serve an Institute (in this case Institut Mittag-Leffler) and not the other way around. But to fully appreciate the matter one needs to understand the special nature of the Institute on which I unfortunately do not have space to elaborate. Suffice it to say that the Institute is intimate in size and the devotion of the staff during its modern phase as an International Institute has been essential for its success.

The two secretaries who were responsible for contacts with visitors counted for almost half the staff, their departure necessitating a reorganization, the extent of which is yet to be known and whose consequences, due to the interruption by Covid-19, are yet to be evaluated. Former managers of the Institute were disturbed by these events, and were in particular worried that the control of the Institute would no longer lie with the mathematicians but be taken over by the Academy, within which mathematicians play but a minor role.

The pride of the Institute is its library, at the time of Mittag-Leffler’s donation the most distinguished private mathematical library in the world with many remarkable items in its holdings. The present management has decided that no new books should be acquired and that in view of digitalization invited guests should rely on their home institutions to access new literature or possibly use the library of the mathematics department of Stockholm University six miles away. Consequently the office of the librarian is now considered redundant. Admittedly this seems to be a world-wide trend, whose long-term consequences are yet to be seen. But the management seems not to have understood that the real treasure of the Institute lies in its historical archives (in particular the Weierstraß-Nachlass) which are now for all intents and purposes abandoned, to the dismay and frustration of the community of historians of mathematics who can no longer consult it. The former librarian Mikael Rågstedt, who has served the Institute faithfully for almost forty years, knows it better than anyone else in the world. Personally indignant as I may be of the cruel treatment levied against him I need nevertheless to put the interests of the Institute and mathematics above that of the individual. The Institute has an obligation to mathematics to avail itself of his unique expertise, just as he has an obligation to assist in conveying it to his successors. As it is, he is now prevented from doing his duty.

What is at stake is the continued integrity of a cultural treasure well deserving a mathematical form of a UNESCO protection.

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Letters to the Editor

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Guest Professor at Uppsala University
Chair, Board of Institut Mittag-Leffler 1997–2003
Uppsala, Sweden

Arild Stubhaug
Author of biographies of Abel, Lie, and Mittag-Leffler
Oslo, Norway

Time Zone Shift Improves Access to JMM
I want to thank the JMM organizers for holding the meeting on Mountain Standard Time.

As a resident of the Pacific time zone, this made it possible for me to attend the full day of events each day, even if it meant starting early. My colleagues in Alaska and Hawai‘i, who are currently one and three hours earlier than us, respectively, were especially grateful. In the US alone, there are six time zones, and in January they cover seven hours, so scheduling the conference in the middle was inclusive, and I applaud your consideration.

If we had remained on Eastern time, the Invited Paper Session I organized would have begun at 4:40 a.m. my time. While 6:40 is still early, it was a reasonable compromise.

Even this schedule might not have worked for some participants—the earliest events may still have been too early for people in the Western time zones and the latest events might have been late for people in the Eastern time zones. But having the conference on Mountain time at least tried to balance the extremes.

I am also so impressed with how well the meetings went. Like everyone, I am burnt out on Zoom and was not sure how a multi-day virtual conference would go. It was fantastic! Don’t get me wrong—I cannot wait to do this in person again. But given our constraints...

I can’t imagine how much work it was to put together such a large and complex virtual meeting. I know how much work the in-person conference is, and this must have been so much harder. Thank you to everyone who worked on this meeting for such a great success.

—Nancy Ann Neudauer
Professor of Mathematics
Pacific University

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The Inimitable F. N. David: A Renaissance Statistician

Amanda L. Golbeck and Craig A. Molgaard

1. Highlights

Florence Nightingale David (1909–1993) was known to readers of her scholarly publications as “F. N. David” and to her colleagues as “David” or “FND.” David has been recognized as the leading, most accomplished and most memorable British woman statistician of the mid-20th century ([11], [14]). She was a professor at University College London (UCL), and then at the University of California (UC). When we were graduate students in the early 1980s at UC-Berkeley, where David had an affiliation before and after her retirement from UC-Riverside, she was already a legend in statistics.

"Enormous energy" and “prolific output.” These are words that statistician D. E. Barton wrote in an obituary to describe his close UCL colleague. Barton added that these qualities were part of what made David “an exciting colleague to work with” [1]. These qualities are also what make it difficult to pigeonhole David’s illustrious 60-year-long career, which was packed with probability and statistical ideas. The numbers alone make clear the extent of her immense energy and output: She wrote nine books, over one-hundred published articles, over fifteen secret (classified) war reports, and various forest service white papers. She was working on a tenth book and other articles and papers when she died.

David’s life as a statistician began at age 22, when she walked into Karl Pearson’s office at UCL in 1931 and asked him for a job. Pearson (1857–1936) was in his mid 70s and an enormous intellectual figure, a protégé of Sir Francis Galton (1822–1911). Pearson had built the world’s first university statistics department at UCL twenty years earlier, in 1911 [24]. He was acknowledged to be a Renaissance scientist, that is, a broad thinker (see [20]). He did, after all, write The Grammar of Science in 1892. When he retired, it took three people to fill his job.

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David was Pearson’s last statistics protégé. It is no wonder David took after him and became a Renaissance statistician. David worked in the department Pearson founded for most of her career. This was 29 years in all, between 1931–1939 and 1946–1967. She rose up the academic ranks from research assistant to full professor. David’s many accomplishments at UCL include the following:

1. David worked with renowned statisticians Egon S. Pearson (1895–1980, the son of Karl Pearson), Jerzy
Neyman (1894–1981, founder of the UC-Berkeley statistics department), and William Sealy Gosset (1876–1937, publishing under the pseudonym “Student" because he worked in industry) on “a thorough consideration of the fundamentals of statistics” [1].

2. She collaborated with statisticians Egon Pearson and Norman Johnson (1917–2004) to further the excellence of the undergraduate program in statistics and to develop the statistics department into both a research center and a center for post-graduate studies.

3. She became the first faculty member in the statistics department to be titled in both probability and statistics.

4. She was the first woman in the statistics department to be promoted to the rank of professor. David’s academic career at UCL was interrupted by World War II. The British government put her to work doing critical military research for six long years (1939–1945). Among David’s accomplishments for the war effort are the following:

5. David conducted vital civil defense research with Frank Garwood on plans for evacuating London during the German Blitz, as well as the civilian populations of other British cities.

6. She conducted critical research with Sir Austin Bradford Hill (1897–1991) and others on mass bombing casualties in Britain, known as the Bombing Census.

7. She worked on the placement of anti-tank mines in the African desert in the campaign against Rommel. The UC recruited David in 1967. She emigrated to the United States and finished her career at UC-Riverside and UC-Berkeley, while doing significant contract work for the US Forest Service. Among David’s accomplishments are the following:

8. David was the founding chair of the department of statistics at UC-Riverside.

9. She was one of the first women to be a research university chairperson in the sciences in the United States.

10. She was an amazing storyteller, a skill that won her a distinguished teaching award at UC-Riverside.

11. She was the first winner of the prestigious Committee of Presidents of Statistical Societies (COPSS) Elizabeth L. Scott Award.

12. In 2000, COPSS established an award named after her, the COPSS Florence Nightingale David Award.

13. In 2018, COPSS named a lecture after her, the COPSS Florence Nightingale David Lecture.

The first David award was presented to Nan Laird in 2001. The first David Lecture was given by Susan S. Ellenberg in 2019. This was the first lecture named after a woman that was ever given at the Joint Statistical Meetings (JSM) [7].

In this paper, we aim to introduce readers to F. N. David and her contributions to statistics. In order to tell more of David’s story, we accessed archival materials and conducted site visits on two continents, researched published literature and web articles, and conducted interviews with people who knew her. We stumbled upon benefactors like Emma Styles, Funding and Development Officer of the Colyton Grammar School Academy Trust in the United Kingdom. Our research is ongoing, for example, we are still searching for evidence of a rumored connection beyond the David family and the Florence Nightingale (1820–1910) family that extends beyond the namesake. Here we will feature some elements of the David story that piqued our interest, especially drawing from our own research, a “conversation article” [12]. David’s newly unclassified war reports, several interviews with statistician Jim Baldwin in 2019, and a presentation by statistician Roxy Peck who participated with us in an invited session that we organized at the 2019 JSM. Baldwin and Peck both knew David when they were doctoral students at UC-Riverside. Baldwin served as David’s head teaching assistant; David recruited him to the US Forest Service, where he spent most of his career.

David was a prodigy, warrior, professor, writer, leader, and celebrity. She was an inimitable statistician.

2. Prodigy

David spent eight years (1931–1939) at UCL before World War II. She was a West England girl, growing up within sight of Offa’s Dyke, the ancient boundary between England and Wales. In fact, her father was of Welsh descent. Both of her parents were school teachers; her father was a headmaster. David was a new college graduate when she had the temerity to knock on Karl Pearson’s door at UCL.

Pearson saw promise in David: he gave her a start as a statistician by renewing her scholarship so that she could pursue graduate studies in statistics. As his research assistant, she ably helped him carry out his projects and edit the journal Biometrika. Reflecting back on this time in her life, David told Baldwin that Pearson would look at her and just shake his head at her, because she was a woman. She told Laird that he was the only person who truly scared her, but he was also very kind.

In her first year on the job, David received a published acknowledgement. William Sealy Gosset (aka “Student”)
published an article in *Biometrika* on the $z$-test [25]. Karl Pearson wrote a review that appeared with the article in which he mentioned David. Student had proposed that the $z$-test was suitable for use in small samples with highly correlated individuals. Pearson doubted that this would be good practice. David produced random samples from a population so that Pearson could conduct a series of experiments to lend evidence to his argument.

In her second year on the job, David became a published author. She was only 23 years old. She worked on a Bessel function problem together with Pearson and sociologist Samuel Stouffer (1900–1960) that was published in *Biometrika* [19]. The purpose of the paper was to discuss an alternative to the ratio method for dealing with certain coefficients.

Thus, David quickly established herself as a statistics prodigy. David’s first acknowledgement was by Pearson. Her first publication—which was a hefty 58 pages long—was with Pearson. At the time, Pearson was viewed in the statistics community as the god of the discipline.

In the early 1930s, the professoriate at UCL and other research universities was almost exclusively male. UCL appointed its first woman to the rank of professor in 1949 [10, p. 213]. David nonetheless thrived as a young woman academic. There were several enabling factors.

One is that UCL had a liberal philosophy and global outlook. The university was built upon the great social reformer Jeremy Bentham’s philosophy that education should be made available to all, not just the wealthy. The auto-icon of Bentham (1748–1832) that still greets you at the university’s philosophist tradition. Today UCL boasts that it was the first university in England to accept students of any race, class, or religion. It also asserts that it was the first to accept women and men on equal terms. Yet the whole time David was at UCL, the staff common rooms were segregated by gender. Thus, even though David found an opening within the Bentham-style liberal philosophies, she did not find complete relief.

A second likely reason that David did well at UCL is that Karl Pearson was a progressive and broad thinker with a long interest in gender issues, and he created a work culture that included women. When David approached him for a job, Pearson already had employed and promoted a number of brilliant women. We will mention two here. Alice Lee (1858–1939) was one of the first women to earn a PhD at UCL; she became a research lecturer in applied mathematics. Ethel Elderton (1878–1954) worked her way up to the position of reader at UCL. Lee and Elderton didn’t rise to the position of professor, but it could be argued that they helped to pave the way for David.

A third likely reason that David did well at UCL is that she arrived there with a stellar educational background and love for learning. At age five, her parents arranged for the rural church parson to teach her algebra, Greek, and Latin. At age 11, David won a scholarship to a top school, Colyton Grammar School. David was Head Girl there four years in a row. This was a significant position, being akin to a student council president in the United States. David won many academic honors and prizes at the school, especially in mathematics and English. At age 19, she won a major merit scholarship that she used to attend Bedford College for Women and then UCL. She graduated from Bedford with a BSc in pure mathematics. David was well prepared to pursue graduate studies at UCL.

David’s first eight years at UCL (in the 1930s) coincided with a seminal period in statistics. The biggest names in statistics were present. Karl Pearson’s son Egon succeeded him as professor and head of the UCL statistics department. Jerzy Neyman was a reader in the department for four years (1934–1938). Neyman and Pearson are known for having developed, among other things, the modern theory of hypothesis testing that included the concepts of type I and type II errors, power, and simple versus composite hypotheses [13]. Ronald A. Fisher (1890–1962) was a professor and developed, among other things, the analysis of variance, maximum likelihood, and foundations of the design of experiments [2]. The statistics department was a magnet for people like William Sealy Gosset (aka “Student”) who is best remembered for developing the $t$-test [18]. David sized it up by saying that she “saw all of the protagonists from a worm’s eye point of view” [12].

In the prewar years, David had a seat at the table with these major actors. She achieved many milestones during this period: She was promoted to lecturer in 1935, published an enduring book on correlation in 1938 (see section 5), earned a PhD in 1938, and wrote her first of a series of papers on Neyman’s smooth test of goodness of fit. David was a rising star among stars. She was a bridge between first generation (like Karl Pearson) and second generation (like Jerzy Neyman) modern statisticians. See, for example: [https://en.wikipedia.org/wiki/Guy_Medal](https://en.wikipedia.org/wiki/Guy_Medal).

Then World War II happened.

3. Warrior

David engaged in work related to World War II during the period 1939–1945. She transitioned back to UCL during the period 1945–1947, when statistics department staff reconstructed the department and completed their war work.

David was a highly effective and ubiquitous warrior during World War II. She was a crucial part of the statistical brains of the Royal Air Force located at Princes Risborough. She helped to conduct the Bombing Census of Great Britain with Sir Bradford Hill. She carried out detailed statistical analyses of the casualties and damage to English cities. These analyses led to the plans for bombing operations against Nazi-controlled Europe that helped end the war and bring peace.

These actions began for David in 1938 in what was known as “the Czech Crisis.” Great Britain and Germany
butted heads over the border between Germany and Czechoslovakia, amid mutual recriminations and threats. All assumed war was imminent, which would bring on strategic bombing of population centers. David served on a planning committee for the evacuation of London’s population to safer rural areas, such as Wales, during this period. She also created statistical models to predict damage from potential German bombing to high-density areas and populations in London and other English cities. These included possible numbers living and dead, reactions to fires and damaged buildings, damage to communications and utilities, etc.

By September 1939, the war began for real. On the very day that Hitler invaded Poland (September 1, 1939) the British government drafted five statisticians to work at UCL on the war effort. The five were David, Egon Pearson, Norman L. Johnson, B. L. Welch, and D. J. Bishop. They were transferred from UCL to the Ordinance Board at the Ministry of Supply.

It was known at the time, but is not well remembered, that Winston Churchill (1874–1965) recognized the value of quantitative insights. He began his interest in quantitative research with studies of comparative attrition from the Somme Battle in 1916 in World War I among the British and German armies involved. He disputed the English War Office statistics on the Somme Battle, and used German and British archival material to prove that attrition occurred on both sides during that battle, but was far worse for the English. This analysis was a major part of Churchill’s book, The World Crisis. His primary point there was that General Haig was wearing out the English army faster than he was wearing out the German army. It was a splendid example of a military vital statistics analysis ([15]; https://winstonchurchill.org/publications/finest-hour/finest-hour-172/battle-of-the-somme-2/).

Churchill was largely responsible for bringing “the quantifiers” from Oxford, Cambridge, and UCL into the British World War II effort. Churchill’s vision from 1939 as First Lord of the Admiralty and from 1940 as Prime Minister was that new statistical services and organizations were needed to manage quantitative information as it increased during the war. This led to the creation of the S-Branch (later known as the Prime Minister’s Statistical Section) under the direction of Frederick Lindeman (1886–1957), an Oxford physics professor. This organization was tremendously active during the war, among other things working on quantitative problems of national economics, defense equipment, machine tools, shipping, rationing, and manpower reports. The last included the Beveridge Report on social insurance, which led to the creation of—and still influences—the British health care system.

David partnered with Edward van Rest, Bradford Hill, and Egon Pearson on multiple statistical projects in the first years of the war. This included development of the Bombing Census (with van Rest and Bradford Hill, beginning in November 1940) and problems of anti-aircraft gunnery (with Egon Pearson). This research was mainly supported by the Royal Air Force (RAF).

The initial goal of the Bombing Census was to assess the bomb distribution and effects over the entire country (from 1940 to 1945 around 61,000 British citizens were killed in these raids). For example, what was the frequency and nature of injuries and causes of death when domestic homes and shelters were bombed? The resulting bomb plots and supporting information for each and every German raid were passed on to the Ministry of Home Security Research and Experiments Branch for analysis. Data were collected on a form designed by Bradford Hill. Bradford Hill was the senior statistician on this project, and David was the deputy statistician.

The statisticians carried out most of the analysis at the data center at Special Section No. 8 at Princes Risborough, Buckinghamshire. This was where David lived and worked for most of the war (1942–1945), in very austere circumstances in shared quarters with two other women. Princes Risborough, because of its analytic strengths, was known as “the Brains of the RAF” [21, p. 194].

The commanding officer of this Special Section was Squadron Leader Dewdney, an expert on oil production and transportation. Dewdney eventually surmised that the English bombing raids on continental Europe oil plants were heavily overestimating effects, describing them as “delusional.” This led to the Butts Report of 1941, which heavily criticized RAF bombing raids that commonly missed their targets by ten miles.

The Bombing Census findings were used to project damage from British bombing raids to German industry and housing throughout the war. David was deeply involved in this reverse engineering analysis, especially on German raid data from London, Birmingham, Coventry, and Liverpool.

What did the statisticians learn? They learned that concentrated use of incendiary bombs, followed by high explosive bombs to cause drafts, was the most effective technique of air assault against large industrial centers. One author noted that “Damage was heaviest in the congested working-class districts, which suggested that these were optimal targets” [16, p. 56]. The goal then became attacking the homes and lives of industrial workers living near factories to limit their productivity.

To summarize, statisticians such as David were called in to study the effects of the bombings by the Germans on England (defensive damage assessment: 1939–1941). As the war turned in favor of the allies, data from the initial damage assessments were used to mount effective offensive operations against the Axis powers (1942–1944). The British returned to conducting research on defensive planning and techniques in the second half of 1944 and 1945 as a result of the German introduction of V-1 and V-2 robot bombs. David was deeply involved in finding the robot bomb launching sites in France, Belgium, and the Netherlands to target allied air power on the sites to destroy them.
As the war wound to a close, David was also heavily engaged in research on clearing mines from beaches as allied beach assaults began in 1944. Earlier she had helped train British troops to use random number tables when laying mines to protect tank parks in the western desert of Africa when fighting Rommel and the Afrika Corps. David also trained American statisticians such as Neyman at Princes Risborough in British bombing strategy and tactics to be used in finishing the war with Japan. Neyman stayed at Risborough with David for six weeks in late 1944 studying these British approaches.

It is our opinion that no other British statistician carried out such a wide range of applications for the war effort, with the possible exception of Frank Yates (1902–1994) at Rothamsted Experimental Station. This included production of twenty classified statistical papers from 1942 to 1945 for the Ordinance Board and the Research and Experiments Department R.E. 8. Her work also took her outside of England as when she flew to Philadelphia in 1944 for a site visit of the first electronic general purpose super computer (the ENIAC—Electronic Numerical Integrator and Computer), whose first program was a study of the feasibility of thermonuclear war. Following German surrender in 1945, she was asked to join the American Manhattan Project in planning the atomic attacks on Japan.

David turned the Manhattan Project invitation down. She had had enough of war, and instead returned to UCL in 1946 and began helping to restore the libraries there, as the campus had been badly damaged by incendiary bombing on two different occasions. David began to organize and transport library books from the Welsh salt mines, where they had been stored for safety from the wartime blitz, to the UCL campus.

4. Professor

After the war, David returned to UCL in 1946. She picked up where she left off in her faculty role. In all, she remained at UCL until 1967. During this time, David researched and published articles on distribution of chromosomes, mapping of karyographs (a method of displaying characteristics of chromosomes), diversity in ecology, spread of forest fires, symmetric functions, k-statistics, experimental design, history of probability, combinatorics, correlation, least squares, goodness of fit, and sampling theory. David’s work on space-time interaction in epidemics, with its emphasis on person-to-person infection, surely can inform current day coronavirus modeling. David wrote 10 papers with Norman Johnson on statistical aspects of experimental design, and about 30 on diverse combinatorial problems and their statistical applications [1].

During this period, David earned a DSc in 1952. In the United Kingdom, the DSc is a higher degree than the PhD. The many reviewers of David’s work described it as rigorous, sound, careful, meticulous, substantial, sturdy, useful, and notable.

There is an adage in higher education that those who publish early will publish often. This certainly applied to David. She was a prolific scientific author with considerable breadth and depth. She authored over 100 journal publications in all, not counting her classified war papers or her white papers for the US Forest Service. She had a continual output of publications, many of which are still cited today. Barton wrote in her obituary: “Her contribution to statistics was wide-ranging and substantial.” One of David’s doctoral students, Colin Mallows (of Mallow’s \( C_p \)), which assesses model fit in ordinary least squares regression), said this about David: “I admire her continual output of research papers. I think this ‘rubbed off’ on me since I have been trying all my life to complete the next paper and get it published” (Mallows, 2018, personal communication).

In support of her many successes at UCL, David had the backing of a number of champions. We will mention three.

Egon Pearson. After becoming professor and head of the statistics department, Egon Pearson promoted David multiple times in recognition of her outstanding work. Eventually he promoted her to the position of reader. In the British system, the only higher faculty position was professor. Egon Pearson was a champion of David, but his championship had its limits. Many thought that she was qualified to succeed Egon Pearson as professor and head of the statistics department. But when he retired, Egon Pearson made sure that David did not succeed him by arguing to his provost that she did not have the temperament for the position. According to Lehmann [14, p. 117], “...prejudice against women prevented her appointment....”

Maurice Bartlett. His support of David was unqualified. It was Bartlett (1910–2002) who succeeded Egon Pearson as the professor and head of the statistics department. In the British system, the norm was to have one professor who was also the chair. Bartlett could have enjoyed his position as the sole professor in the department. But David’s work was so exceptional that he decided to make things right for her, at least as right as he could. He promoted her to the rank of professor, making her only the fourth person (after Karl Pearson, Egon Pearson, and Bartlett) to hold the rank of professor in the department. It was almost unheard of to be a professor without the department head responsibilities. It speaks volumes about both Bartlett as a person and David as a prolific scholar.

Jerzy Neyman. He was without a doubt David’s biggest supporter. David met Neyman in 1934 when he came to work in the department of statistics at UCL. She was his first graduate student there. He is credited with insisting she finish her PhD. The two taught probability together, and she wrote her second book on this course material (see section 5). She became the first reader in the statistics department to be titled in both statistics and probability. David and Neyman became lifelong colleagues and friends.

When Neyman died in 1981, there was a memorial session for him at the European Meeting of Statisticians.
David was a speaker. She said this about him: “I knew him over 50 years. He could be quite impossible and we quarreled strongly every six months or so. But I loved him” [3]. David had a strong personality. She would, without hesitation, tell you what was on her mind. But this didn’t get in the way of her having very close professional and personal relationships.

5. Writer

David loved books. She wrote, cowrote, or edited nine of them on a variety of subjects:


These books helped to propel the use of statistics in science, to set a standard for the teaching of statistics, and to promote the understanding of statistics and statisticians in historical contexts.

One collection of David’s books greatly facilitated the use of statistics in scientific research at a time before the field transitioned to the use of computational algorithms to compute probabilities. David’s three books of statistical tables of probability integrals were definitive, and she produced them with great care. Her first book of tables, written when she was only 29 years old, was a monumental contribution to the correlation coefficient (David, 1938; reprinted as recently as 2009). Using three mechanical Brunsviga calculating machines in series, she computed exact probability integrals of the sample correlation coefficient with 20-figure accuracy. A second book of tables was a major contribution to combinatorics (David, Kendall, and Barton, 1966, as a companion to the theoretical book David and Barton, 1962). In this book, David and her co-authors summarized “all of the techniques which we have found useful and paved the way for further research work” on symmetric functions (p. x). A third book of tables was an important contribution to the analysis of ordinal or very non-normal data (David et al., 1968). It was primarily concerned with normal centroids, normal medians, and normal scores. All three books contained tables that had a high degree of accuracy, with almost no errors. Importantly, they also contained detailed, theoretical introductions to the subjects that gave readers the foundations they needed to use the tables with skill. These sections of the books could have easily stood alone as important monographs.

Another collection of David’s books helped to set a firm foundation for the teaching of statistics and development of the statistics workforce. These books were original, rigorous, and authoritative. One was a book on probability theory for statistical methods, based on some of David’s lectures at UCL (which she conducted with Neyman and continued after he left UCL for UC-Berkeley) to second-year mathematics students who wanted to learn some statistics (David, 1949). Here David described the mathematical theory of probability as “a bridge, however inadequate it may seem, between the sharply defined but artificial country of mathematical logic and the nebulous shadowy country of what is often termed the real world.” Another book was a first course in statistics, based on some of her lectures to life scientists across nearly all science departments at UCL (David, 1953, later updated and republished as David, 1971). David wrote that it was for scientists who were “able to digest simple statistical ideas and such mathematical symbols as are necessary to formulate them.” A third book which was written with Egon Pearson was on elementary
In general, David cared about explaining difficult statistical concepts in a simple, straightforward, and engaging manner; illustrating these concepts with a wide variety of meaningful examples; promoting student practice of the manipulation of real-world data; and making her enthusiasm for the subjects known and infectious.

Standing proudly alone in a category was David’s classic book on the history of probability, titled *Games, Gods and Gambling: A History of Probability and Statistical Ideas* (David, 1962). This was the first book on the history of probability theory to be published in 100 years. It is still read today after having seen multiple reprints by multiple publishers. The book contains both prehistory and history, beginning with ancient Egypt and ending with the death of de Moivre in the middle of the 18th century. It is filled with “interesting facts and fresh ideas.” This is no wonder, as David used many kinds of historical sources in the book (as she did in her teaching), and also poetry, classical literature, and archaeology. It was David’s philosophy that: “the man creates the mathematical theorem, but the events of a man’s life create the man, and the three are indissoluble.” David’s book blends biographies and social environments with assessments of discoveries, making the book very readable. A reviewer wrote: David’s “keen interest in the personalities makes itself felt on almost every page, but it is equaled by her profound knowledge of the subject matter” [23].

Also of special mention is the book *Research Papers in Statistics* (David, 1966), which David edited as a Festschrift for her close friend and colleague Neyman, to celebrate his 70th birthday. A reviewer wrote: “Jerzy Neyman has put all statisticians in his debt by his numerous contributions … The present volume…is an expression of the affectionate esteem in which colleagues the world over hold Jerzy Neyman…I can think of no more fitting tribute [than David’s book] to [Neyman’s] assured place in the annals of statistical theory” [5]. The book was a collection of several dozen papers, on a wide range of topics, and from a who’s who of theoretical and applied statisticians: Egon Pearson, Maurice Bartlett, Joe Berkson, David Cox, Harold Cramer, David Kendall, Lucien LeCam, Paul Levy, George Polya, Herman Wald, and others. When you start to develop an edited book, you always wonder if people will write for you. All of these major players in the field of statistics were willing to write insightful chapters for David.

In addition to her nine books, David was a consulting editor or book editor for several journals. She wrote many of the book reviews herself. For example, a 1961 issue of a journal contained 31 book reviews, with David authoring 10 of them. David also had many doctoral students at UCL and was a highly appreciated research mentor. This was evidenced by the thanks given to her in the bound copies of the dissertations that we reviewed in the UCL statistics library. David’s love for books propelled her to devotedly help to rebuild the UCL library system after significant World War II bombing damage.

### 6. Leader

In 1967 at age 58, David packed her bags and left UCL to emigrate to the United States. Neyman, her longtime colleague and friend, had earlier reached a barrier to advancement at UCL and emigrated to the United States to become the founding chair of the statistics department at UC-Berkeley. Now David’s circumstances were similar, and she became the founding chair of the statistics department at UC-Riverside. As mentioned, David was one of the few women researchers in the sciences to break into academic administration [22]. This was an accomplishment that followed Gertrude Cox (1900–1978) who became a chair in 1941 at North Carolina State University, and that David shared contemporaneously with Elizabeth L. Scott (1917–1988) who became a chair in 1968 at UC-Berkeley [6].

Right before David emigrated, Bartlett resigned his position as head of the statistics department at UCL. Dennis Lindley (1923–2013) was hired to be his replacement as head. If Neyman was a *draw* for David to go to UC-Riverside, Lindley was a *push* for her to go. Lindley was a leading advocate of Bayesian statistics. David was a frequentist to the core. Statistician Roxy Peck recalled that, “at a conference [David] took some [UC-Riverside] graduate students to, if somebody came up to give a talk on Bayesian statistics, she took her cane and shook it and left the room knocking over her chair as she left” [17]. Perhaps it is no surprise, given David’s intellectual bond with Neyman, that she had no use for Bayesian statistics.

![Figure 3. F.N. David visiting Berkeley from UCL in 1960. With Berkeley statisticians Elizabeth L. Scott, David Blackwell, and Evelyn Fix.](image-url)
This story illustrates that David used a cane, and she used it to do more than assist with walking. As a young woman, she had a motorbike accident that caused her to often carry a cane. In Riverside, she used her cane to help clear a pathway for others. In Berkeley, she liked to wrap her fist around the middle of the cane and wave it at drivers to get them to stop so that she could cross the street. Her cane was a prop. It was part of her persona and act.

David was a clever and savvy administrator at UC-Riverside. She was creative at getting students and skillful at procuring resources, to the point where the mathematics department was jealous of her and wanted her investigated. They failed. Baldwin describes the years of problems with the math department as the “war years”; they were “very tense,” and she was fighting with “all men.”

When there were departmental meetings at UC-Riverside, David invited staff as well as faculty so that all could contribute. All were welcome. This was an example of inclusive leadership, long before the term ever existed.

David was a beloved and award-winning teacher. This was true both at UCL and UC-Riverside, but she is particularly remembered for her teaching skill while at Riverside. There she mainly taught at the introductory level. For example, she taught a general education class, using her book *Games, Gods and Gambling*. Her excitement for the subject was palpable.

David also taught a doctoral seminar on topics in the statistics literature. Students gave presentations, and these were heavily critiqued by both David and the other students in the class. The seminar was not for the faint of heart, but it thoroughly prepared students for competing in the job market and presenting papers at professional conferences. David was very supportive of the statistics graduate students. Baldwin remembers her attitude about working with applied scientists: “You wanted to learn something about the applied field but not too much so that you thought like them. That was really important to her. You needed to speak the language of whatever applied field it was, but you didn’t want to know too much about it, is what she kept telling us” (Baldwin, 2019, personal communication). David understood how statisticians can advance science by looking at applied problems with fresh and independent eyes.

In California, David explained her accent at the beginning of each course, telling her students that she spoke the pure King’s—or Queen’s—English. Her voice was loud and crystal clear. She was a great storyteller. As a teacher, she painted a picture of statistics with meaningful examples drawn from a wide variety of real applications. She would walk around the room talking, which was way ahead of her time in terms of teaching pedagogy. Her classes were packed, often with as many as 200 students in a class. Eventually her class became too popular—her class size increased to 450–500 people—it was so large that it became unmanageable, so it was discontinued.

David had a strong sense of determination. As a girl, she wanted to be an actuary, but the profession at the time was largely closed to women. She reportedly was turned down for an actuarial position when it was discovered that she was a woman. Her father told her she should not cry and give up, but instead she should get on with her work. This she did. And she passed along this message and attitude to her students. Peck remarked: “She was a role model [for her students], but she was not a typical role model. She could be a bit cantankerous, feisty, smoked cigars. None of us actually pictured our future to be like her. But we also knew that we weren’t part of her league in terms of statistical abilities. But the thing about David is that she allowed you to see the possibilities of what you could accomplish if you worked hard. And what she valued was people who would work hard for what they wanted” [17]. As a result, David had tremendous respect from her students.

David was tough: her stances were like laser beams, and those exposed felt like they could be deadly. But she joked around a lot, too, mainly in a sarcastic way. She also occasionally practiced physical humor, where she could have a pretty powerful punch. She was beloved by colleagues and students alike, for both her fiery temper and her wonderful story-telling ability. She was a role model in the mathematical sciences, but in no ways was she typical.

David retired from her position at UC-Riverside in 1977. Baldwin recalls that she “just got tired of it.” She had gotten the Riverside department going, and it was running well, so “she probably figured it was time…. “ David had strong relationships at UC-Berkeley with Neyman, Scott, Evelyn Fix (1904–1965), and others in the Bay Area statistics community. She had been commuting between Riverside and Berkeley every week for almost ten years, which is a long time to do a seven-hour commute in each direction.

After becoming Emeritus Professor, David continued to teach at UC-Berkeley and consult for the US Forest Service. She recruited Baldwin, one of her former UC-Riverside statistics graduate students, to work at the Forest Service as well. He recalled that she was interested in the measurement of natural populations, and she “described some simple-minded approaches for counting things and testing where there were trends in bird populations…Birds was a big thing for her when I was in the Forest Service. She had her two-bang theory for counting birds. It was somewhat sarcastic on her part, or maybe a whole lot sarcastic. It went like this: if birds were in a tree, you’d take a shotgun and fire to get the birds out of the tree, and then you’d use the shotgun again to shoot the birds and count them once they were on the ground. The two-bang method” (Baldwin, 2019, personal communication). It is an example of David’s wry sense of humor.

In 1993 in the Bay Area, David transitioned to the afterlife. Being British to the core, she had her body shipped back to the United Kingdom. It is laid to rest in the lovely English village of Graftham where she kept her vacation home. Sadly, her grave is unmarked. There is no stone marking the final resting place of this very strong woman and leader in statistics.
7. Celebrity

COPSS established the Elizabeth L. Scott Award. The first award was given in 1992, and it is given every other year. David was the inaugural recipient: for her efforts in opening the door to women in statistics; for contributions to the profession over many years; for contributions to education, science, and public service; for research contributions to combinatorics, statistical methods, applications, and understanding history; and her spirit as a lecturer and as a role model. It is interesting that David and Scott were personal friends. Their careers ran in parallel on both sides of the Atlantic.

About ten years later, COPSS partnered with the Caucus for Women in Statistics to establish another award named after a woman that would alternate with the Scott award. They chose to name it after David. The first award was given in 2001. It recognizes an individual who exemplifies David’s excellence in research, leadership of multidimensional teams, statistics education, and service to the professional societies. A history of the David Award is given in [8].

In 2018, COPSS unanimously decided to accept a proposal to elevate the Elizabeth L. Scott Award to the Elizabeth L. Scott Award and Lecture, and to elevate the Florence Nightingale David Award to the Florence Nightingale David Award and Lecture. These lectures are delivered at the JSM. They have the same format and standards as the former Fisher Lecture. The first Florence Nightingale David Lecture was given in 2019, and the first Elizabeth L. Scott Lecture was given in 2020. The David lecture was the first lecture named after a woman that was ever given at the JSM.

Golbeck articulated the significance of these new lectures at a meeting of the Caucus for Women in Statistics in 2018. As she put it,

The JSM has been held every year since 1840. This means that over the past 178 years, men and women who have been attending the JSM have not seen one single lecture named after a woman. The JSM is the largest gathering of statisticians in North America, and one of the largest in the world. Each year there are over 6,000 participants from over 50 countries. These huge numbers of men and women have been attending the JSM without seeing one single lecture named after a woman…ALL of us—men and women, of all ages—in our profession need to see strong human, female faces: faces like Elizabeth L. Scott and F. N. David ([17], address to the Caucus for Women in Statistics).

These lectures serve as more than lectures. The people who the lectures are named after, and the people who give the lectures, serve as role models who can instruct, inspire, and motivate statisticians to achieve greater successes.

When David started out, there were few women statisticians. Today there are many more: women account for 43% of the doctoral recipients in doctoral-granting statistics and biostatistics departments in the United States [9] and 35% of the members of the American Statistical Association (https://magazine.amstat.org/blog/2016/02/01/genderupdate16). It is fitting that there are now lectures named after two women who were brilliant trailblazers in statistics.

8. Inimitable Statistician

David’s British parents gave her a big name. Her namesake, Florence Nightingale, who died only two years before David was born, was the first woman named to the prestigious Order of Merit. Parents like David’s who named their children after Florence Nightingale were perhaps hoping that the name could instill confidence, if not mystique. David certainly had both qualities. She was instrumental in developing the field of statistics on two continents.

David saw her job as one of asking questions and trying to find the answers. She was one of the broadest statisticians of her time. Another Renaissance person and statistician, J. B. S. Haldane (1892–1964), was very impressed with David and wrote about her: “...a number of people with fellowships of the Royal Society and the like are narrow specialists compared with Dr. David…." Florence Nightingale David was a woman of letters, a Renaissance statistician.

Peck collected words that have been used to describe David. These included “feisty, unconventional, persistent, uncompromising, independent, prolific, generous, and charming” [17]. David liked to smoke Turkish cigarettes. She apparently started smoking cigars in the war because they were easier to get than Turkish cigarettes. Her colleagues at the US Forest Service in California used to collect her cigar boxes. Baldwin described her as “loud and round, and inspirational” (Baldwin, 2019, personal communication).

Both authors of this paper knew David. When Golbeck was a graduate student in statistics and biostatistics at UC-Berkeley, David interviewed her for a consulting position at the US Forest Service. Golbeck received the position. Molgaard took a class in international health while a graduate student in public health at UC-Berkeley, David was a guest lecturer. She was, on a personal level, marvelously eccentric as well as brilliant.

At Berkeley in the 1980s, David relaxed and favored wearing cowboy boots, a cowboy shirt, and blue jeans. When teaching she liked to talk at length about her namesake. Florence Nightingale was known as the lady of the lamp. It is fitting that David ended her academic career at UC-Berkeley, where the university motto is “let there be light.”

No one who met David ever forgot her.

9. Conclusion

Why should you know about F. N. David? She was a major player among the most major players in the building of the field of statistics in Great Britain and the United States.
Because she was so prolific and her work was so strong, the other major players had enormous respect for her, and she prospered against all odds as a woman within the profession. She succeeded at UCL up to the highest glass ceiling, and then she was recruited to UC-Riverside where she broke that ceiling.

What do we gain by knowing the F. N. David story? As epidemiologists put it, we gain a sense of person, place, and time in our contextual understanding of the field of statistics between 1930 and 1967. The importance of David to the development of the field is that she witnessed all of the great statistical theoretical developments of her time and contributed to an impressive number of them. She brought enormous energy to her work, was willing to work under very difficult conditions, and dedicated herself to do whatever it took to solve a problem.

David was a remarkable statistician and leader. Her prolific writing covered a broad range of topics and interests. Even though she lost seven years of academic productivity doing service work during the war, she rose up higher on the academic ladder in Britain than almost any other woman in science of her time, and she was a rare woman academic department chair in the research sciences in the United States. On the basis of our research, David was a “ceiling-cracker” for women in the statistical sciences profession. Further evidence of this is the establishment of the COPSS F. N. David Lecture, given for the first time at the 2019 Joint Statistical Meetings.

References
Susan Friedlander’s Contributions in Mathematical Fluid Dynamics

Alexey Cheskidov, Nathan Glatt-Holtz, Natasa Pavlovic, Roman Shvydkoy, and Vlad Vicol

Susan Friedlander received her undergraduate degree in mathematics from University College London in 1967. Having been awarded one of the prestigious Kennedy Scholarships to study at the Massachusetts Institute of Technology, Friedlander moved to the US and earned her MS degree at MIT in 1970. She subsequently started her

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Opening photo is of Susan Friedlander lecturing at the Fields Institute in 2008.
PhD studies at Princeton University, completing her doctorate thesis under the title “Spin Down in a Rotating Stratified Fluid” in 1972 with the supervision of the fluid dynamicist Louis Norberg Howard. After being a visiting member at New York University’s Courant Institute of Mathematical Sciences, Friedlander moved to the University of Illinois at Chicago, where she worked as a Professor until 2008. Since then, Friedlander is a Professor and the Director of the Center for Applied Mathematical Sciences at the University of Southern California.

Throughout her career, Friedlander has focused on the mathematical analysis of partial differential equations (PDEs) arising in fluid dynamics. While the fundamental models are several centuries old, to date fluid dynamics remains the source of some of the most fascinating and challenging problems at the intersection of mathematics and physics. Without a doubt, the phenomenon of “turbulence” is chief among them. A unifying theme in Friedlander’s research is an emphasis on problems of clear physical interest and importance.

Friedlander’s impact on the field of mathematical fluid dynamics, and on the mathematical community as a whole, extends far beyond her research contributions. Prior to 1989, she opened bridges to the fluids communities behind the iron curtain. Since the early 1990s she has served in several leadership positions at the American Mathematical Society, including as Associate Secretary. For the past 15 years Friedlander has been the Editor in Chief of the Bulletin of the AMS, and more recently Friedlander was one of the key figures in the founding of the Mathematical Council of the Americas.

Susan is an exceptional mentor. Since the early stages of our careers, the authors of this paper were fortunate enough to collaborate with Susan, benefiting from her guidance, academic generosity, and perspective on mathematics as a whole. Susan has helped shape both our careers and our views of mathematics, and we are truly thankful for her inspiration, thoughtful guidance, and limitless positive energy.

In this review celebrating Friedlander’s contributions we will focus on her work on hydrodynamic instability as it relates to the transition from laminar to turbulent flow, on dyadic models in fluid dynamics and Onsager’s conjecture analyzing the transfer of energy in turbulent flows, and on magnetohydrodynamics as it relates to large-scale motions in Earth’s fluid core.

1. Equations of Fluid Dynamics
The fundamental partial differential equations that describe the macroscopic properties of the motion of an incompressible, inviscid fluid with constant density are the Euler equations:
\[
\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p = 0, \quad \nabla \cdot u = 0,
\]
with the initial condition
\[
u(x, 0) = u_0(x),
\]
for the unknown velocity vector field \(u = u(x, t) \in \mathbb{R}^d\) and the pressure \(p = p(x, t) \in \mathbb{R}\), where \(x \in \mathbb{R}^d\), \(t \in [0, \infty)\), and \(d = 2, 3\). Despite the fact that Leonhard Euler introduced them in 1757, many basic questions concerning Euler equations in \(d = 3\) are still unresolved. For example, it is an outstanding problem to find out if solutions of the 3D Euler equations form singularities in finite time, from smooth initial data.

The equations modeling the macroscopic properties of viscous, incompressible, homogeneous fluids were formulated by Claude-Louis Navier (1822) and Sir George Stokes (1845). The Navier-Stokes equations that they derived are written as
\[
\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p = \nu \Delta u, \quad \nabla \cdot u = 0,
\]
with the initial condition
\[
u(x, 0) = u_0(x),
\]
and appropriate boundary conditions. As with the Euler equations the theory of the Navier-Stokes equations in three dimensions is far from being complete. One of the major open problems is global existence and uniqueness of smooth solutions to the Navier-Stokes equations in 3D. This is one of the Millennium Problems of the Clay Mathematics Institute.

Beyond the Navier-Stokes and Euler equations many other models animate modern research in mathematical fluid dynamics. Of particular interest in Friedlander’s work are the magnetohydrodynamics equations (cf. (34)–(37) below) and other equations arising in geophysics.

2. Instabilities
The late 1970s and early 1980s were marked by the discovery of a new type of instability in incompressible fluids—the so-called shortwave or broad-band instability. Such instabilities occur when a fluid rushes through a pipe leading to the formation of elliptical vortices near the walls. Although such vortices themselves are two dimensional, and in fact stable under two-dimensional perturbations, they are manifestly unstable when perturbed in the direction of their axes of rotation. Moreover, the frequency \(\kappa\) of unstable modes corresponding to the same exponential rate \(\lambda\),
\[
\mathcal{L}v \sim \lambda v, \quad v = e^{-i\kappa \cdot x} \phi(x),
\]
corresponds to a range of values \( \kappa > \kappa_0 \) instead of being uniquely determined by a dispersive relation \( \kappa = \kappa(\lambda) \). Hence, the term “broad-band.” Grounded in numerous physical works by Orszag, Patera, Bayly, Pierrehumbert, Craik, Criminale, and others, these novel instabilities lacked a rigorous foundation presenting a unique challenge for the mathematical community in the early 1980s.

2.1. The fast dynamo problem. A similar type of instability appears in the kinematic dynamo problem. This problem seeks to describe persistent growth of a magnetic field \( H(x, t) \) transported by a given velocity field of electrically conducting fluid \( u(x, t) \) in the limit of vanishing magnetic resistivity. Specifically, for \( H \) satisfying the system

\[
\frac{\partial H}{\partial t} = -u \cdot \nabla H + H \cdot \nabla u + \varepsilon \Delta H, \quad \nabla \cdot H = 0, \tag{7}
\]

the dynamo is called fast if one has

\[
\limsup_{\varepsilon \to 0} \omega_\varepsilon > 0, \tag{9}
\]

where \( \omega_\varepsilon \) is the exponential type of the \( C_0 \)-semigroup \( G^\varepsilon_t \) generated by (7)–(8). Similar to the fluid problem such instabilities are expected to be of highly oscillatory nature as the corresponding spectral problem \( \mathcal{L}_\varepsilon H = \lambda \varepsilon H \) would require an increasing range of frequencies as \( \varepsilon \to 0 \).

The groundbreaking works [FV91, FV92, VF93] laid the foundation to what is now called the geometric optics approach to shortwave instabilities, a particular case of which is the elliptic instability we mentioned in the beginning of this section.

To describe the method in more detail let us consider the example of the classical incompressible Euler system linearized around a given steady state \( u_0 \):

\[
\frac{\partial v}{\partial t} = -u_0 \cdot \nabla v - v \cdot \nabla u_0 - \nabla p, \quad \nabla \cdot v = 0, \tag{12}
\]

where \( v \) is the linear perturbation of \( u_0 \) and \( p \) is the perturbed pressure, which plays the role of projecting the right side of (12) onto the space of divergent-free fields. We consider (12) with periodic boundary conditions, \( x \in \mathbb{T}^n \), or the whole space \( x \in \mathbb{R}^n, n \geq 2 \). The method seeks to find effective dynamics of a localized oscillatory wave written in the form of a geometric optics ansatz

\[
v(x, t) = b(x, t) e^{i S(x, t)/\varepsilon} + O(\varepsilon). \tag{14}
\]
If initially \( b(0) = b_0 \) is localized near position \( x_0 \), and the frequency of initial oscillation is \( \xi \), i.e., \( S(0) = S_0 = \xi \cdot x, \) we obtain a new wave at time \( t \) approximately localized near the Lagrangian particle \( x(t) = \varphi_t(x_0) \). Plugging this ansatz into the Euler system (12) one reads off the leading order evolution of the amplitude \( b \) and phase \( S \) in the Lagrangian coordinates of the underlying field \( u_0 \):

\[
\frac{dS}{dt} = 0, \quad \frac{db}{dt} = -\partial u_0(x)b + 2(\xi, \partial u_0(x)b) \frac{\xi}{|\xi|^2},
\]

where \( \xi = \nabla S \) is the frequency covector. To write the system in closed form we replace the transport of \( S \) by the transport of the frequency vector, and the resulting system reads

\[
\begin{cases}
\frac{dx}{dt} = u_0(x), \\
\frac{d\xi}{dt} = -\partial^\top u_0(x)\xi, \\
\frac{db}{dt} = -\partial u_0(x)b + 2(\xi, \partial u_0(x)b) \frac{\xi}{|\xi|^2},
\end{cases}
\tag{15}
\]

supplemented by initial conditions \( x(0) = x_0, \xi(0) = \xi_0, \)
\( b(0) = b_0, \) and the incompressibility constraint \( b_0 \cdot \xi_0 = 0. \) This is a so-called bicharacteristic-amplitude system (BAS for short). From a dynamical viewpoint the first two equations represent a bicharacteristic flow on the tangent bundle of the fluid domain \( \Omega = T^*\mathbb{T}^n \), denoted \( \chi_t(x, \xi) \), and the last amplitude equation represents evolution of a vector \( b \) on the fiber bundle over \( \Omega \) with fibers given by orthogonal planes \( \pi^{-1}(x, \xi) = \{ b : b \cdot \xi = 0 \} \). Thus, \( b(x_0, \xi_0, b_0, t) = B_t(x, \xi)b_0 \) is a cocycle family of maps over the flow \( \chi \).

The asymptotic expansion of the Euler semigroup \( G_t \), defined by (12)–(13), is dominated by the \( B \)-cocycle playing the role of the principal symbol

\[ G_t = \Phi_t \text{Op}[B_t] + K_t, \]

where \( \Phi \) is the Leray projection onto the divergence-free fields, \( \Phi v = v \circ \varphi_{-t} \), \( \text{Op}[B_t] \) is the leading order pseudodifferential operator, and \( K_t \) is a similar operator of order \( -1 \). For the high frequency waves the frequency localization of pseudodifferential operator \( \text{Op}[B_t] \) leads precisely to the ansatz (14) which becomes justified a posteriori.

The shortwave instabilities can now be studied by looking into the Lyapunov spectrum of the BAS whose maximal exponent is given by

\[
\mu = \lim_{t \to \infty} \frac{1}{t} \log \sup_{x_0, \xi_0, b_0 |\xi_0| = 1, |b_0| = 1} |B_t(x_0, \xi_0)b_0|. 
\]

The main result of [VF93] establishes a direct relationship between the growth rate of the BAS and the growth rate of the Euler dynamics in the energy space.

**Theorem 2.2.** Let \( \omega_0 \) denote the exponential rate of the semigroup \( G_t \) in \( L^2 \). Then

\[
\omega_0 \geq \mu.
\]

The high frequency asymptotic relationship between \( G_t \) and \( B_t \) makes it possible to relate shortwave instabilities to the essential spectrum of the semigroup. The BAS was found to be fully descriptive of the essential spectrum in later works. For particular flows, however, Theorem 2.2 proved to be extremely versatile in many different situations. For example, for the aforementioned elliptic vortices, locally given by \( u_0 = (a^2 y, -b^2 x, 0) \), the growth of the BAS becomes a Floquet problem over time-periodic elliptic trajectories. This case was also studied in works of Lifschitz and Hameiri around the same time. The amplitudes \( b \) become unstable in directions pointing off of the xy-plane, which is consistent with the empirical observations of Orszag, Patera, and others. A systematic study of the BAS and various dynamic scenarios leading to instabilities was performed in [FV92]. First, in 2D, the quantity \( |b||\xi| \) is conserved. Hence, \( \mu \) is related precisely to the exponential stretching of the underlying field \( u_0 \). Here the cotangent cocycle associated with the \( \xi \)-equation has the same Lyapunov spectrum as that of the tangent cocycle (11). Thus, \( \mu = \omega_0 \) in this case. In particular, all parallel shear flows are shortwave stable. In 3D, the analogue of this law is conservation of the volume

\[
\text{Vol}(b', b'', \xi) = (b' \times b'') \cdot \xi
\]

for any pair of amplitudes \( b', b'' \) over the same frequency \( \xi \). Hence, in 3D we obtain

\[
\mu \geq \frac{1}{2} \omega_0.
\]

In any case, exponential stretching makes the flow spectrally unstable. On the other hand, some integrable flows \( u_0 \), where \( u_0 \times (\nabla \times u_0) = \nabla B \), on nondegenerate level tori of the Bernoulli function \( B \) are found to be stable, namely \( \mu = 0 \). Geometric instability criteria for vortex rings without swirl were provided as well.

In several subsequent works (see [FSV97, FS05] and references therein), Friedlander expanded the geometric optics method to a range of models appearing in geophysics and magnetohydrodynamics. In all these cases the underlying bicharacteristic flow remains the same but the amplitude equation changes according to a simple recipe—it captures the principal symbol of the linearization:

\[
\frac{db}{dt} = a_0(x, \xi)b, \quad \mathcal{L} = -u_0 \cdot \nabla + \text{Op}[a_0] + \text{Op}[a_1] + \cdots.
\]
Thus, for the surface quasigeostrophic equation describing evolution of a potential temperature on a horizontal surface the \( b \)-equation reads

\[
\frac{db}{dt} = i \frac{\xi \cdot \nabla \vartheta_0(x)}{\|\xi\|} b,
\]

where \( \vartheta_0 \) is the underlying steady temperature. In this case the essential spectrum is neutral, \( \mu = 0 \). For density stratified fluids we have

\[
\frac{db}{dt} = \left( 2 \frac{\xi \otimes \xi}{\|\xi\|^2} - id \right) \partial_u(x)b + r \left( id - \frac{\xi \otimes \xi}{\|\xi\|^2} \right) \nabla \Phi(x),
\]

\[
\frac{dr}{dt} = -b \cdot \nabla \rho_0(x).
\]

Here \( \Phi \) is the gravitational potential. The kinematic dynamo falls under the same scheme and yields (11). Camassa-Holm (Euler-\( \alpha \)) gives

\[
\frac{db}{dt} = \left( \frac{\xi \otimes \xi}{\|\xi\|^2} - id \right) \partial_u^T(x)b + \frac{\xi \otimes \xi}{\|\xi\|^2} \partial_u(x)b,
\]

and for inviscid systems of nonrelativistic superconductivity we have

\[
\frac{db}{dt} = \left( 2 \frac{\xi \otimes \xi}{\|\xi\|^2} - id \right) \partial_u(x)b + \left( id - \frac{\xi \otimes \xi}{\|\xi\|^2} \right) B \times b.
\]

Numerous other applications of the theory were found to non-Newtonian fluids also. The reach of the method proved to be truly astonishing.

2.3. From linear to nonlinear instability. Justification of the linearization procedure for inviscid fluids remains a very challenging problem to this day. Providing an explicit bound on the "bad" essential part of the spectrum given by exponent \( \mu \) is a helpful tool to prove a range to sufficient conditions for the analogue of the Lyapunov theory—going from linear to nonlinear instability [FSV97]. In several subsequent works Friedlander established several pioneering results in this direction. First, in 2D if there is a point spectrum (exact eigenvalue) beyond \( \mu \), i.e.,

\[
\mathcal{L} = \lambda \mathcal{V}, \quad \text{Re} \lambda > \mu,
\]

then the underlying steady flow \( u_0 \) is unstable in the energy norm [VF03]. Construction of flows with oscillatory laminar regions that fulfill this condition have been provided in Friedlander’s works with Yudovich. In the region outside of the essential spectrum, in fact one can also construct unstable invariant manifolds by analogy with finite-dimensional theory and dissipative systems as was done later in works of Lin and Zeng. Next, for the linearized Navier-Stokes system

\[
\frac{\partial u}{\partial t} = -u_0 \cdot \nabla u - u \cdot \nabla u_0 - \nabla p + \varepsilon \Delta u, \quad \nabla \cdot u = 0
\]

both in 2D and in 3D, those dominant eigenvalues reappear for small viscosities \( \varepsilon \to 0 \) in a strong spectral limit: for any eigenvalue of the linearized Euler system with \( \text{Re} \lambda > \mu \) and \( \varepsilon \) sufficiently small the Navier-Stokes system gains point spectrum in a vicinity of \( \lambda \) with the same multiplicity, and moreover the Riesz projection \( \mathbf{P}^\perp \) corresponding to those spectral subspaces near \( \lambda \) tends to that of the Euler equation \( \mathbf{P}^\perp \) in the uniform operator topology. This result proves to be particularly interesting in view of the fact that the nonlinear Navier-Stokes system inherits instability in \( L^3 \) (and in fact any \( L^p \) for \( p > 1 \)) from the linearization, a classical result of Yudovich. Thus, any steady flow in 3D that has inviscid spectrum beyond \( \mu \) becomes nonlinearly unstable in the vanishing viscosity sense.

3. Dyadic Models

One way to gain an understanding of certain aspects of the equations of fluid motion is to introduce toy models which share properties with the actual equations, but are simpler to analyze. During the last two decades, the work of Friedlander has shaped studies of so-called dyadic models of the fluid equations, which simulate the energy cascade through dyadic frequency shells.¹ In these models, the nonlinearity of the Euler equations \( (u \cdot \nabla)u \) is simplified so that only local interactions between neighboring scales are considered. However, simplifications of the nonlinear term vary, and as a consequence the models differ. Some of the first examples of models of this type were derived by Desnyanskiy and Novikov in the context of oceanography, and by Gledzer, whose model was subsequently generalized by Ohkitani and Yamada (and is now known as the GOY model).

The dyadic models that Friedlander explored are designed to share with the actual equations of fluid motion the scaling of the nonlinear term in 3D (which we motivate in the next subsection) and the following properties:

- A skew-symmetry property of the nonlinear term,

\[
\langle (u \cdot \nabla)u, u \rangle_{L^2(\Omega^3)} = 0. \tag{18}
\]

- Conservation of energy for the classical solutions to the Euler equations,

\[
\|u(\cdot, T)\|_{L^2}^2 = \|u_0\|_{L^2}^2, \tag{19}
\]

which is a consequence of (18) and the divergence-free condition, as can be seen by pairing the Euler equation (1) with \( u \) in the \( L^2 \) sense.

- Decay of energy for classical solutions to the Navier-Stokes equations (4)–(6),

\[
\|u(\cdot, T)\|_{L^2}^2 = \|u_0\|_{L^2}^2 - 2 \nu \int_0^T (-\Delta u, u). \tag{20}
\]

¹Specifically, a \( j \)-th dyadic shell refers to a region where the Fourier frequency \( \xi \) lies in the annular domain \( 2^{j-1} \leq |\xi| \leq 2^j \). The fluid velocity in the \( j \)-th dyadic shell is modeled with a single representative, \( u_j \).
Broadly speaking, dyadic models provide a framework for studying specific aspects of turbulence theory, while being mathematically accessible. Moreover, in some instances these models motivated results on actual equations of fluid motion, as was the case in e.g. [CCFS08].

3.1. Introducing a dyadic model. Let us now recall a version of a dyadic model from [CFP07]. This model was inspired by a wavelets model introduced in [KP02] as a tool to help guide a partial regularity result for actual Navier-Stokes equations with hyperdissipation. We start by briefly revisiting the wavelets model.

First, we recall some terminology from [KP02]. A cube $Q$ in $\mathbb{R}^3$ is called a dyadic cube if its sidelength is an integer power of $2$, $2^l$, and the corners of the cube are on the lattice $2^l \mathbb{Z}^3$. Let $D$ denote the set of dyadic cubes in $\mathbb{R}^3$. Let $D_j$ denote the subset of dyadic cubes having sidelength $2^{-j}$. Then the parent of $Q$, denoted by $PQ$, is introduced as the unique dyadic cube in $D_{j(Q)-1}$ which contains $Q$. On the other hand, one defines $C^k(Q)$, the $k$-th-order grandchildren of $Q$, to be the set of those cubes in $D_{j(Q)+k}$ which are contained in $Q$.

The modeling starts by replacing a vector-valued function $u$ by a scalar-valued one. An orthonormal family of wavelets is denoted by $\{w_Q\}$, with $w_Q$ the wavelet associated to the spatial dyadic cube $Q \in D_j$. Then $u$ can be represented as

$$u(x,t) = \sum_Q u_Q(t)w_Q(x).$$

Note that due to spatial localization of $w_Q$,

$$\|w_Q\|_{L^2} \sim 2^{3j(Q)/2}. \quad (21)$$

On the other hand

$$\|\nabla w_Q\|_{L^2} \sim 2^j. \quad (22)$$

Having in mind (21) and (22), a cascade-down operator is defined through its $Q$th coefficient as follows:

$$(C_d(u,v))_Q = 2^{3j(Q)/2}u_{PQ}v_{PQ},$$

with the scaling $2^{3j(Q)}$ that reflects the upper bound on the nonlinear term implied by (21)–(22). Similarly, a cascade-up operator is defined as the adjoint of $C_d(u,v)$ via

$$(C_u(u,v))_Q = 2^{3j(Q+1)/2}u_Q \sum_{Q' \in C(Q)} v_{Q'}. \quad (23)$$

Then the cascade operator is introduced as

$$C(u,v) = C_u(u,v) - C_d(u,v).$$

Having defined the Laplacian as $-\Delta(w_Q) = 2^j w_Q$, one introduces the following model equations:

- Dyadic Euler equation:
  $$\frac{du}{dt} + C(u,u) - \Delta u = 0.$$

By construction of the cascade operators, we have $\langle C_d(u,u),u \rangle = \langle C_d(u,u),u \rangle$, which implies the skew-symmetry property of the operator $C$,

$$\langle C(u,u),u \rangle = 0. \quad (23)$$

A simple consequence of (23) is conservation of energy for the dyadic Euler equations and decay of energy for the dyadic Navier-Stokes equations, at least at the formal level (for sufficiently regular solutions).

The above dyadic models are special cases of the following infinite system of coupled ordinary differential equations, which was studied by Friedlander:

$$\frac{d}{dt}a_j + v2^{2j}a_j - 2^{c(j-1)}a_{j+1}^2 + 2^c a_j a_{j+1} = f_j \quad (24)$$

for $j = 0, 1, 2, \ldots$, where $a_{-1} = 0$, $c$ is a positive parameter related to intermittency, and $\frac{1}{2}a_j^2$ represents the total energy in the frequencies of order $2^j$. The force $f$ is such that $f_0 > 0$ and $f_j = 0$ for all $j > 0$, so that the energy is pumped on low modes.

As we have seen above, the model preserves many features of the fluid equations, while the nonlinearity is simplified by considering only local interactions between scales. Moreover, the choice of the constant $c = 5/2$ ensures that the nonlinearity in the dyadic model obeys the same $L^2$-based estimates as Euler (see (19)) and Navier-Stokes equations (see (20)). Thanks to these $L^2$-based estimates and a certain monotonicity present in the model, a finite time blow-up was exhibited for the inviscid dyadic model [KP05], as well as the viscous dyadic model with some “small” degrees of dissipation [KP05] or large values of $c$ [Che08]. For instance, solutions blow up when $c > 3$, in which case the dyadic model scales as $4^j$-dimensional Navier-Stokes equations. Such a monotonicity property resembles monotonicity of certain quantities present in so-called “cooperative” systems (see for example the work of Palais and the work of Bernoff and Bertozzi where singularities in a modified Kuramoto-Sivashinsky equation were identified). Finite time blow-up in the inviscid case was sharpened by Kiselev and Zlatoš. A three-dimensional vector model for the incompressible Euler equations was introduced in [FP04], which is similar in some features to a discretized approximate model constructed by Dinaburg and Sinai for the Navier-Stokes equations in Fourier space. It was shown in [FP04] that for special initial data the evolution equations of the divergence-free vector model reduced to the scalar dyadic Euler system and finite time blow-up occurs in this model for the three-dimensional incompressible Euler equations. This was a brief snapshot of

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results for dyadic models around 2005, when Susan Friedlander initiated the study of phenomena related to turbulence at the level of dyadic models.

3.2. Onsager’s and Kolmogorov’s conjectures. Up to now we discussed conservation of energy at a formal level, i.e., for sufficiently regular solutions to Euler equations (1). However one might wonder about the minimal level of regularity of a solution to the Euler equation that guarantees conservation of energy. In fact this seemingly naive question is connected with the statistical theories of turbulence developed by Kolmogorov (1941) and Onsager (1949). In their seminal works, it is suggested that an appropriate mathematical description of three-dimensional turbulent flow is given by weak solutions of the Euler equations which are not regular enough to conserve energy. Onsager conjectured that for the velocity Hölder exponent $h > 1/3$ the energy is conserved and that this ceases to be true for $h \leq 1/3$. This latter phenomenon is now called turbulent or anomalous dissipation. Kolmogorov’s theory predicts that in a fully developed turbulent flow the energy spectrum $E(|k|)$ in the inertial range is given by

$$E(|k|) = c_0 \varepsilon^{2/3} |k|^{-5/3},$$

where $\varepsilon$ is the average of the energy dissipation rate.

While the rigidity part of Onsager’s conjecture (namely the regime corresponding to the conservation of energy) has been well understood due to works of Eyink and Constantin-E-Titi—prior to works on dyadic models—the flexibility part represented a challenge for a long period of time. Thanks to advances in the method of convex integration due to De Lellis-Székelyhidi, the flexibility part of the Onsager conjecture for the Euler equation has been very recently settled by Isett, and by Buckmaster-De Lellis-Székelyhidi-Vicol for dissipative solutions. However, the state of the puzzle regarding the flexible part of conjecture was completely open back in 2007. In that context, the dyadic model (24) provided a mathematical laboratory for addressing the phenomena predicted by Onsager and Kolmogorov.

More precisely, in [CF07, CFP10] Friedlander et al. showed that the inviscid ($\nu = 0$) dyadic model possesses a unique fixed point $\tilde{a}$, whose energy spectrum $S(\kappa) \sim \varepsilon^{2/3} \kappa^{-8/3}$, which is just on the borderline of the Sobolev space $H^{5/6}$, where the $H^{5/6}$ Sobolev space is equipped with the norm

$$\|a\|_{H^{5/6}}^2 = \sum_{j=0}^{\infty} 2^{2j} a_j^2.$$

This showed that all the solutions of the dyadic Euler model stop satisfying energy equality at some time (which

resolved Onsager’s conjecture in the negative direction), and the long-time behavior is exactly as predicted by Kolmogorov’s theory of turbulence, but with extreme (or what we now call fully intermittent) energy spectrum. In fact, as it was observed in [CF09], the dyadic model (24) covers the whole intermittency range $d \in [0, 3]$, where the intermittency dimension $d$ is connected to the parameter $c$ as $c = 1 + 3^d - \frac{8}{3}$ (see Subsection 3.3). Then

**Theorem 3.1 ([CFP07, CFP10]).** The following hold for the dyadic system (24) in the inviscid case $\nu = 0$:

1. Every regular solution (defined to be a solution with bounded $H^{5/6}$ norm) satisfies the energy equality.
2. There exists a unique fixed point $\tilde{a}$ to (24), which is a global attractor. The fixed point is not in $H^{5/6}$. In fact, it lies exactly in the space $B_{2,3}^{1/6}$ (defined in (31) below), which takes into account intermittency.
3. The energy spectrum of the fixed point

$$S(\kappa) \sim \varepsilon^{2/3} \kappa^{-8/3},$$

where $\tilde{a}_0 f_0$ is the energy input rate, that corresponds to the anomalous energy dissipation rate.
4. Every solution blows up in finite time in $H^{5/6}$ and in $B_{3,3}^{1/6}$ for any $\varepsilon > 0$.
5. The $H^s$ norms of every solution are locally square integrable in time for $s < 5/6$, and every solution eventually dissipates energy.

We note that the relevance of the Sobolev exponent $5/6$ stems from three copies of modeled velocity “sharing” the scaling of the localized coefficient $2^{5/2}$ in the skew-symmetry property for the dyadic nonlinear term (23). The existence of a global attractor for an inviscid system, at first, seems surprising. However it is exactly consistent with the concept of anomalous or turbulent dissipation conjectured by Onsager.

The relation between the fixed points of inviscid and viscous dyadic models is as follows.

**Theorem 3.2 ([CF09]).** The following hold for the dyadic system (24) in the viscous case $\nu > 0$:

1. The global attractor for the viscous dyadic model is a fixed point $\tilde{a}^\nu$.
2. The fixed point of the viscous system $\tilde{a}^\nu$ converges in $L^2$ to the fixed point of the inviscid system $\tilde{a}$ as $\nu \to 0$. Moreover, the energy dissipation rate converges to the anomalous energy dissipation rate of the inviscid system, i.e.,

$$\lim_{\nu \to 0} \varepsilon^\nu = \langle f, \tilde{a}\rangle = \varepsilon,$$

where

$$\varepsilon^\nu = \nu \|\tilde{a}^\nu\|_{H^{5/6}}^2 = \langle f, \tilde{a}^\nu\rangle$$

is the energy dissipation rate.
3.3. From the dyadic model to the full equations. One of the main features of the dyadic Navier-Stokes model, the forward energy cascade, leads to the question of whether solutions satisfy the energy equality. The nonlinear term in the dyadic model is skew-symmetric by construction, and hence one immediately obtains
\[ \frac{1}{2} \frac{d}{dt} \sum_{j=0}^{\infty} a_j^2 = -\Pi_j - \nu \sum_{j=0}^{\infty} 2^{2j} a_j^2 + \sum_{i=0}^{j} f_i a_i, \]  
(26)
where the flux is defined as
\[ \Pi_j = 2^{5j} a_j^2 a_{j+1}, \]
where we again chose \( c = 5/6 \). Passing to the limit as \( j \to \infty \), it follows that every solution \( a \in L^3(0, T; H^{5/6}) \) satisfies the energy equality
\[ \sum_{i=0}^{\infty} a_i^2(t) = \sum_{i=0}^{\infty} a_i^2(t_0) + \int_{t_0}^{t} \left[ -\nu \sum_{j=0}^{\infty} 2^{2j} a_j^2 + \sum_{i=0}^{j} f_i a_i \right] ds \]
(27)
for all \( 0 \leq t_0 \leq t \leq T \). Surprisingly, this result was not known for the fluid equations at that time, so Friedlander and collaborators extended it to the Navier-Stokes equations in [CSF12].

As we have seen in Section 3.2, a simplified dyadic model (24) mimicked highly nontrivial predictions for real equations, raising the following questions. First, can we rigorously justify the derivation of the model? Second, what is the meaning of the parameter \( c \) that affects the energy spectrum, which ranges from classical Kolmogorov’s 5/3 to the extreme 8/3 power law? This started a fruitful series of works on obtaining optimal bounds for the energy flux and incorporating a notion of intermittency in the mathematical studies of fluid equations.

Consider the Navier-Stokes equations (4) for the motion of a three-dimensional incompressible viscous fluid. Define
\[ u_{\leq \lambda_j} = u * \mathcal{F}^{-1}(\psi(\xi^{2})), \]
where \( \psi(\xi) \) is a smooth nonnegative function supported in the ball of radius one centered at the origin and such that \( \psi(\xi) = 1 \) for \( \xi \leq 1/2 \), and \( \mathcal{F} \) is the Fourier transform. The energy flux due to nonlinear interactions through the sphere of radius \( \lambda_j = 2^j \) is defined as (see [CCF08])
\[ \Pi_j = -\int_{\mathbb{R}^3} (u \cdot \nabla)u \cdot (u_{\leq \lambda_j} \cdot u_{\leq \lambda_j}) \, dx. \]
Using the test function \( (u_{\leq \lambda_j} \cdot u_{\leq \lambda_j}) \) in the weak formulation of the Navier-Stokes equations we obtain
\[ \frac{1}{2} \frac{d}{dt} \|u_{\leq \lambda_j}\|^2 = -\Pi_j - \nu \|\nabla u_{\leq \lambda_j}\|^2 + \langle f_{\leq \lambda_j}, u_{\leq \lambda_j} \rangle. \]
(28)
In [CCF08], Cheskidov, Constantin, Friedlander, and Shvydkoy obtained the following new bounds on the nonlinear term in (4):
\[ |\Pi_j| \lesssim \sum_{i=-1}^{\infty} \lambda_{i-j}^{-3/2} \lambda_i \|u_i\|^3, \]
(29)
where \( u_j = u_{\leq j+1} - u_{\leq j} \) is the Littlewood-Paley projection of \( u \). This estimate employing the Littlewood-Paley decomposition produced not only a sharpening of the conditions under which there is no anomalous dissipation, but also provides detailed information concerning the cascade of energy through frequency space. More precisely, it shows that the energy flux \( \Pi \) through the sphere of radius \( \kappa \) is controlled primarily by scales of order \( \kappa \). The estimate also showed that a critical space for solutions in which the energy equality is guaranteed, Onsager’s space, is \( B^{1/3}_{3,\infty} \) for the Navier-Stokes equations, and \( B^{1/3}_{3,0} \) for the Euler equations.

Now define \( a_j = \|u_j\|_2 \), so that \( \frac{1}{2} a_j^2 \) represents the energy in the dyadic shell of radius \( 2^j \). In order to mimic the flux estimate (29), we need to pass from \( L^3 \) to \( L^2 \), which can be done thanks to Bernstein’s inequality:
\[ a_j^3 \lesssim \|u_j\|_3^3 \lesssim \lambda_j^3 a_j^3. \]
(30)
To capture the whole range of possible saturations of the Bernstein inequality, define an intermittency parameter \( d \in [0, 3] \), which, roughly speaking, represents the dimension of the set occupied by eddies, such that
\[ \|u_j\|_3 \sim \lambda_j^{3-d} a_j. \]
Then it is natural to define the Besov norm of \( a \) as
\[ \|a\|_{B^{d}_{3,\infty}} = \sup_j 2^{j(3-d/6)} a_j. \]
(31)
This combined with the locality of (29) motivates the following model for the energy flux:
\[ \Pi_j = \lambda_j^{1+3-d/2} a_j^2 a_{j+1}. \]
(32)
Here \( d = 3 \) corresponds to the so-called Kolmogorov’s regime where eddies occupy the whole space (or lower bound on \( \|u_j\|_3 \) in (30)) and \( d = 0 \) is the case of extreme intermittency (or upper bound in (30)). Motivated by (28), subtracting subsequent equations (26) (for \( j \) and \( j - 1 \)) gives
\[ \frac{1}{2} \frac{d}{dt} a_j^2 = \Pi_{j-1} - \Pi_j - \nu \lambda_j^2 a_j^2 + f_j a_j, \]
(33)
which is exactly the dyadic model (24) with \( c = 1 + \frac{3-d}{2} \) by definition of the flux.
4. The Magnetogeodynamo

The geodynamo is the process by which the rotating, convecting, electrically conducting molten iron in Earth’s fluid core maintains the geomagnetic field against ohmic decay. The convective processes in the core that produce the velocity fields required for this dynamo action are a combination of thermal and compositional convection. A detailed description of the dynamo problem requires the examination of the three-dimensional partial differential equations governing incompressible magnetohydrodynamics (MHD) under the effect of Coriolis, Lorentz, and gravity forces (see the system (34)–(37) below). The system also possesses thermal source terms which model radioactive decay within Earth’s core, and can have an essentially stochastic character. The mathematical statement of the geodynamo problem asks whether there are initial data for the MHD system for which the evolution of the magnetic field grows for a sufficiently long time, i.e., the existence of instabilities. These instabilities are also expected to play a fundamental role in magnetostrophic turbulence and turbulent dynamo theory [ML94].

Due to its complexity, in order to simulate this system, current computational limitations require parameter choices that are several orders of magnitude larger than what is physically realistic. It is therefore reasonable to attempt to gain some insight into the geodynamo by considering a reduction of the full MHD equations to a system that is more tractable, but still maintains some of the key physical features. The magnetogeostrophic (MG) equation proposed by Moffatt and Loper [ML94] (see (37) and (41) below) is one such model, which has gained significant interest in the mathematical community, mostly due to Friedlander’s work on this subject. During the past decade Friedlander and her collaborators have gone from laying down the mathematical foundations of the MG equations, to proving delicate results about the long-time dynamics of solutions, the instability of its steady states as it relates to the geodynamo, and to rigorously deriving the model from the small parameter regime postulated in its physical derivation.

4.1. Derivation of the MG model. For simplicity of notation, assume that the axis of rotation and the gravity are aligned in the direction of the Cartesian vector $e_3$. Moffatt and Loper [ML94] furthermore assume that the magnetic field is the sum of an underlying purely toroidal constant field $B_0e_2$ and a perturbation field $b(x, t)$. The fluid velocity vector field is denoted by $u(x, t)$, while the buoyancy scalar field as $\theta(x, t)$. In the rotating frame of reference the MHD system becomes:

$$R_m \left[ \partial_t b + u \cdot \nabla b - b \cdot \nabla u \right] = e_2 \cdot \nabla b + N^2 \partial_\theta + \nu \Delta u,$$

$$\nabla \cdot u = 0, \quad \nabla \cdot b = 0,$$

$$\partial_t \theta + u \cdot \nabla \theta = x \Delta \theta + S.$$  

(35)

(36)

(37)

Here $S(x, t)$ is a given thermal source due to radioactive decay acting on the system, and naturally may be considered to include stochastic components. The dimensionless physical parameters that appear above are: the inverse Elsässer number $N^2$, the Rossby number $R_o$, the magnetic Reynolds number $R_m$, the inverse Peclet number $\kappa$, and the inverse square of the Hartman number $\nu$.

The physical postulate of the Moffatt-Loper MG model is that slow cooling of the Earth leads to slow solidification of the liquid metal core onto the solid inner core, releasing latent heat of solidification which drives compositional convection in the fluid core. Based on this physical postulate, arguments are given in [ML94] for the appropriate ranges of the aforementioned parameters: $N \approx 1$ and $R_o, R_m \ll 1$. The MG model is obtained by setting $N = 1$ and passing $R_o, R_m \to 0$ in (34)–(36), equations which simplify to

$$e_3 \times u = -\nabla P + e_2 \cdot \nabla b + \partial e_3 + \nu \Delta u,$$

$$0 = e_2 \cdot \nabla u + \Delta b,$$

$$\nabla \cdot u = 0, \quad \nabla \cdot b = 0.$$  

(38)

(39)

(40)

The linear system of equations (38)–(40) determine the vector fields $u$ and $b$ in terms of the scalar buoyancy $\theta$, encoding the vestiges of the physics in the problem: Coriolis force, Lorentz force, and gravity. Further vector manipulations of (38)–(40) give the expression

$$[(e_3 \cdot \nabla)^2 + (\nu \Delta^2 - (e_2 \cdot \nabla)^2)]u = (e_3 \cdot \nabla)(e_3 \times \nabla \theta) - [\nu \Delta^2 - (e_2 \cdot \nabla)^2] \nabla \times (e_3 \times \nabla \theta)$$

(41)

which allows us to compute $u$ as a function of $\theta$, under the model’s self-consistency assumption that both $\theta$ and $u$ have zero vertical mean. Note that all the differential operators appearing in (41) have constant coefficients. Thus, it is convenient to rewrite (41) as $u = M_\theta \varphi$, where $M_\theta$ is a vector Fourier multiplier operator with associated symbol $\hat{M}_\theta$ given by

$$\hat{M}_\theta(k) = (k_2 k_3 |k|^2 - k_1 k_3 (k_2^2 + \nu |k|^4)) D_\nu(k)^{-1},$$

$$\hat{M}_\varphi(k) = (-k_1 k_3 |k|^2 - k_2 k_3 (k_2^2 + \nu |k|^4)) D_\nu(k)^{-1},$$

$$\hat{M}_\varphi(k) = ((k_1^2 + k_3^2)(k_2^2 + \nu |k|^4)) D_\nu(k)^{-1},$$

$$D_\nu(k) = |k|^2 k_3^2 + (k_2^2 + \nu |k|^4)^2$$

(3)

(4)

The styles of $\nu$ and $k$ are speculative, but likely very small. For the moment we keep them as free parameters, which allows us to pass $\nu, k \to 0$ later on in the analysis.

Note that while the physically relevant boundary for the Earth’s fluid core is a spherical annulus, for the purposes of studying the mathematical properties of the MG equations we simply consider periodic boundary conditions.
for \( k = (k_1, k_2, k_3) \in \mathbb{Z}_+^3 := \mathbb{Z}^3 \setminus \{k_3 = 0\} \). On \( \{k_3 = 0\} \) we define \( M_{ij}(k) = 0 \) for all \( j \in \{1, 2, 3\} \), since \( \theta \) and \( u \) have zero vertical mean.

In summary, for \( \nu, \kappa \geq 0 \), the magnetogastrophic MG\(_{\nu,\kappa}\) equation is the nonlinear advection-diffusion equation (37), in which the incompressible velocity field \( u = M[\theta] \) is given by the constitutive law (41). Some of the key properties of the MG\(_{\nu,\kappa}\) equation are that it is three dimensional, its diffusion is given by the classical Laplacian (when \( \kappa > 0 \)), and the symbol \( \hat{M}(k) \) is orthogonal to the wavevector \( k \) and is an even function of it. Most importantly, the nature of the operator \( M_{ij} \) changes dramatically between the inviscid case \( \nu = 0 \) (when \( M_{ij} \) is an unbounded operator) and the dissipative case \( \nu > 0 \) (when \( M_{ij} \) is a smoothing operator). This latter fact plays a crucial role in the analysis of MG\(_{\nu,\kappa}\).

It is instructive to compare the MG\(_{\nu,\kappa}\) equation to more classical hydrodynamic models in the canon of nonlinear active scalar equations, such as the 2D surface quasi-geostrophic (SQG) and its dissipative versions. The SQG equation has received tremendous interest in the mathematical community over the past decades, through works of Constantin, Wu, Cordoba, Caffarelli-Vasseur, Kiselev-Nazarov-Volberg, and many many others. The recent results in the analysis of the SQG equation have played a fundamental role in developing the mathematical foundations for the MG\(_{\nu,\kappa}\) model.

4.2. The inviscid model \( \nu = 0 \). In Friedlander’s original work on this subject [FV11], it is shown that when \( \nu = 0 \) there exist regions of Fourier space where \( M_{ij}(k) \) is unbounded and may even grow as fast as \( |k| \) when \( |k| \to \infty \). Thus, in this worst-case scenario \( M_{ij} \) acts as an order one Fourier multiplier, and so the map \( \theta \mapsto u = M[\theta] \) effectively loses one derivative. Since the evolution of \( \theta \) in (37) satisfies the maximum principle (when \( S = 0 \)) one may prove an a priori bound on \( \theta \in L^p_tL^\infty_x \); this is the strongest norm of \( \theta \) which is a priori bounded uniformly in time. In turn, in view of the aforementioned properties of \( M_{ij} \), we may only deduce a bound on \( u \) in \( L^\infty_tBMO^{-1}_x \), i.e., \( u \) behaves as the divergence of a \( BMO \) skew-symmetric matrix. The crucial observation is that in three dimensions advection-diffusion equations with incompressible \( L^p_tBMO^{-1}_x \) drifts are critical, meaning that the natural parabolic scaling of equations leaves the size of the rescaled drift velocity \( u \) unchanged in this norm. When the initial datum \( \theta_0 \) or the forcing term \( S \) are large, it is notoriously difficult to establish the global existence of smooth solutions for such problems, and indeed, prior to [FV11] this was an open problem. Using a variant of the parabolic De Giorgi iteration and the incompressibility of \( u \), Friedlander and the last author have proven the following.

**Theorem 4.1** ([FV11]). Let the initial data \( \theta_0 \in L^2 \cap L^\infty \) and \( \theta \in L^\infty(0, \infty; L^2) \cap L^2(0, \infty; H^1) \) be a weak solution of MG\(_{0,\kappa}\), where \( \kappa > 0 \). Then, for any \( t_0 \), there exists \( \alpha > 0 \) such that \( \theta \in C^{\alpha/2}(t_0, \infty; C^\alpha) \).

The proof of Theorem 4.1 starts by using De Giorgi iteration to establish the boundedness of the weak solutions to MG\(_{0,\kappa}\). Proving Hölder regularity requires a more delicate argument since the drift velocity \( u \) is the divergence of a \( BMO \), rather than \( L^p \), matrix. By essentially using the divergence-free nature of \( u \) and by appealing to the John-Nirenberg inequality, one may however prove a suitable \( L^p \)-based Caccioppoli inequality, which is the key ingredient in De Giorgi’s improvement of the oscillation lemma. Note that while Theorem 4.1 only establishes global Hölder continuity of weak solutions to MG\(_{0,\kappa}\), a posteriori one may deduce these solutions are \( C^\infty \) smooth for \( t > 0 \), and thus unique.

Having established the global existence of smooth solutions for MG\(_{0,\kappa}\) with \( \kappa > 0 \), Friedlander and her collaborators have turned their attention to proving the existence of instabilities for the nonlinear model, since this is after all what the geodynamo problem asks for. Building on the ideas discussed in Section 2, Friedlander was able to prove that for \( m \geq 1 \), the operator corresponding to MG\(_{0,\kappa}\) linearized about the steady state \( \Theta_0 = \sin(m \pi x_3) \), with associated \( U_0 = 0 \) and forcing \( S = \kappa m^2 \sin(m \pi x_3) \), has unstable point spectrum. Moreover, the largest eigenvalue has real part which is at least as large as \( 2^{-2}\kappa \) once \( \kappa \) is taken to be sufficiently small and \( m \ll \kappa^{-1} \). As in Section 2, a careful analysis shows that this linear instability implies that solutions to the full nonlinear MG\(_{0,\kappa}\) are Lyapunov nonlinearly unstable: initially small perturbations of \( \Theta_0 \) grow exponentially in time, which is consistent with the dynamo instabilities.

Friedlander, jointly with Rusin and the last author, has obtained a number of further results concerning the MG\(_{0,\kappa}\) model. For instance, the system MG\(_{0,0}\) is Hadamard ill-posed in Sobolev spaces, but local well-posedness is recovered if one adds back a dissipative operator of the type \( \kappa(\Delta)^\gamma \) with \( \gamma \geq 1/2 \). For further results in the inviscid case, we refer the interested reader to the review paper [FRV14].

4.3. The viscous model \( \nu > 0 \). The nature of the MG\(_{\nu,\kappa}\) equations changes dramatically when considering the viscous case \( \nu > 0 \). To see this, return to the symbol \( \hat{M}_{ij} \) defined implicitly by (41). For \( \nu > 0 \), instead of having an unbounded symbol, one may show that \( |k|^2|\hat{M}_{ij}(k)| \lesssim 1 \) for all \( k \in \mathbb{Z}^3 \). Thus, the map \( \theta \mapsto u = M[\theta] \) is smoothing of order two in the viscous case; a regularization that is even stronger than the Biot-Savart law. Thus, an a priori estimate on \( \theta \in L^p_tL^\infty_x \) (natural in view of the maximum principle for (36)), yields a bound for \( \nabla^2 u \in L^p_tL^p_x \) for any \( p < \infty \). In particular, the Lipschitz norm of \( u \) is a priori
controlled, globally in time, and as for classical ODEs, one may thus hope that the system is globally well-posed even when the diffusivity parameter $\kappa$ vanishes. This problem was recently resolved by Friedlander jointly with Suen.

Theorem 4.2 ([FS15]). Consider $\nu > 0$ and $\kappa \geq 0$. Assume that $\theta_0 \in L^2$ has zero mean. Then, there exists a unique global weak solution $\theta \in BC((0, \infty); L^2)$ with $u \in C((0, \infty); W^{2,3})$ of the MG$_{\nu, \kappa}$ equation.

The remarkable fact about the above result is that it holds even for $\kappa = 0$. It is also shown in [FS15] that solutions to MG$_{\nu, \kappa}$ converge in the vanishing viscosity limit $\nu \to 0$ towards solutions of MG$_0, \kappa$ for $\kappa > 0$. Moreover, there is no anomalous dissipation of energy for (37) in the vanishing diffusivity limit: for any $T > 0$ and $\theta_0 \in H^1$, we have that

$$\lim_{\kappa \to 0} \int_0^T \int_{\mathbb{R}} |\nabla \theta_{\kappa, \nu}|^2 \, dx \, dt = 0,$$

where $\theta_{\kappa, \nu}$ denotes the unique solution of MG$_{\nu, \kappa}$ guaranteed by Theorem 4.2. The above result addresses the question of magnetogeostrophic turbulence raised by Moffatt and Loper [ML94]. Concerning the geodynamo problem, using techniques similar to those in Section 2, it was proven by Friedlander and Suen in [FS15] that the dissipative MG$_{\nu, \kappa}$ equation sustains exponentially growing dynamo-type instabilities.

4.4. Singular limits for the magnetogeodynamo in a stochastic setting. Another direction of Friedlander’s work in magnetohydrodynamics concerns the Moffatt-Loper model (34)–(37) in the stochastic setting, where the source term $S$ in thermal evolution equation (37) is specified as a Gaussian white noise. This probabilistic setting of the model interprets white noise driven terms as a heat source which “continuously regenerates the statistically stationary temperature distribution throughout the core” as described by Moffat and Loper [ML94]. As such, an important feature of Friedlander and her collaborators’ work in the stochastic setting is to analyze statistically invariant states, i.e., to study invariant measures of the associated Markovian dynamics. More broadly, such measures play an important role in the study of turbulence as they provide a framework for identifying robust statistical quantities in turbulent flows.

In a joint work with Földes, the second coauthor, and Richards [FFGHR17], Friedlander considered the singular parameter limits $R_o, R_m \to 0$ for (34)–(35), with these limits being carried out in terms of the corresponding invariant measures. Roughly speaking, the work [FFGHR17] establishes that statistically robust quantities of the full MHD system with $0 < R_o, R_m \ll 1$ are well approximated by those measured using the formal limit system $R_o = R_m = 0$. More precisely, we summarize this result as follows.

Theorem 4.3 ([FFGHR17]). Consider (34)–(37) with $\nu, \kappa > 0$, in the presence of a stochastic source term of the form

$$S(x, t) = \sum_{k \in \mathbb{Z}, m \in \{0, 1\}} \alpha_{k, m}(x) W^{k, m}(t),$$

where $\alpha_k(x) := \cos(k \cdot x)$, $\alpha_k(x) := \sin(k \cdot x)$, $\alpha_k \in \mathbb{R}$ are amplitudes, and $[W^{k, m}]$ is a collection of independent white noise processes. Subject to the nondegeneracy (hypoellipticity) condition that $\alpha_{(1,0),m}, \alpha_{(0,1),m}, \alpha_{(0,0),m}$ are nonzero for $m = 0, 1$, the limit equation (34)–(37) when $R_o = R_m = 0$ has a unique statistically invariant state $\mu$ which is achieved at an exponential rate; cf. (44) below. For any collection of statistically invariant states $\{\mu_{R_o, R_m}\}_{R_o, R_m > 0}$ and any suitably regular observable $\Phi$ of the dynamics, we have

$$\left| \int \Phi(u, b, \theta) \, d\mu_{R_o, R_m} - \int \Phi(u, b, \theta) \, d\mu \right| \leq C(R_o + R_m)^\gamma,$$

where the constants $\gamma, C > 0$ are independent of $R_o, R_m > 0$.

An interesting feature of the above result is that the estimate (43) is independent of possible nonuniqueness in the approximating statistics. Here we observe that the formal limit system when $R_o = R_m = 0$, i.e., MG$_{\nu, \kappa}$, is an active scalar equation with a smoothing constitutive law of order two, and therefore classically yields a Markovian dynamic. On the other hand, for positive values of $R_o, R_m$, it is not clear that (34)–(37) is well posed or that the associated statistically invariant states $\mu_{R_o, R_m}$ are unique; for positive $R_o, R_m$, the invariant states $\mu_{R_o, R_m}$ are considered as stochastic analogues (i.e., martingale solutions) of stationary Leray weak solutions.

The strategy in [FFGHR17] turns on establishing a spectral gap condition is suitable in a Wasserstein metric for the Markovian dynamics for the limit system. Here, one considers

$$W_p(\mu, \nu) := \inf_{\Gamma \in \mathcal{C}_{\mu, \nu}} \int \rho(U, V) \, \Gamma(dU, dV),$$

where $\rho$ is taken to be a certain metric, topologically equivalent to $L^2$, but which punishes elements far from the origin, and $\mathcal{C}_{\mu, \nu}$ is the set of couplings of $\mu$ and $\nu$. “Weak Harris” mixing results for Markovian systems which are adapted to such topologies $W_p$ were developed by Hairer, Mattingly, Kuksin, Shirikyan, and others in the early 2000s. By leveraging such modern variants of Harris’ classical theorems one may establish bounds of the type

$$W_p(\mu, \nu) \leq C e^{-\varepsilon \beta} W_p(\mu, \nu).$$

A crucial step in the analysis leading to (44) is to study the delicate interactions of the stochastic source terms $S$ in (42) with the nonlinear portion of the drift $u \cdot \nabla \theta$ present in (41), in order to establish a certain Hörmander-type hypoellipticity condition. This analysis produces a form of...
smoothing in the Markovian dynamics, the so-called asymptotic strong Feller condition of Hairer and Mattingly.

The bound (44) crucially affords a reduction of the study of the convergence of the stationary states in (43), to the establishing of bounds between positive and limit solutions in the parameters \( R_0, R_m \) at a fixed finite time \( t_s > 0 \). The finite time convergence analysis produces interesting challenges due to a phase space mismatch between the full system when \( R_0, R_m > 0 \) and the active scalar equation representing the limit. This formally suggests a multitime scale analysis to correctly approximate solutions at an initial layer in time. One insight in [FFGHR17] is that, beyond the initial layer, there is no need to correct the dynamics in order to obtain a bound similar to (43) for the thermal components of the dynamics; these decay rates may then be transferred to bounds on velocity and magnetic components.

A recent work of Tao exploits the blow-up mechanism of the dyadic model and adds the energy cascade delays to break the 4D barrier that result in a construction of an “averaged” 3D Navier-Stokes model which fulfills all the same energy estimates as the actual Navier-Stokes, but blows up in finite time—a long-awaited result demonstrating that the energy method alone is not enough to resolve the Clay Problem for 3D Navier-Stokes.

The MG model has sparked research in many directions, and in particular it was one of the first active scalar equations to which the convex integration method was adapted. It played a crucial step in understanding the role of symmetries in the Fourier multiplier for the scalar-to-velocity constitutive law, as a mechanism for generating wild solutions.

Friedlander’s contribution to the ongoing work on the Onsager conjecture laid the basis for the mathematical formalization of the physical concepts of intermittency and accumulation set for the forward energy cascade in turbulent flows. These concepts proved to be experimentally measurable and were well received in the physical community. Friedlander’s analytical results on the energy law remain sharp to date and were extended to fluids models with boundaries, to inhomogeneous and compressible fluids, and even to general hyperbolic conservation laws.

5. Impact of Friedlander’s Work
It is hard to overestimate the significance of Friedlander’s research and her impact on the work of so many mathematicians, including the authors of this survey. The geometric optics method has developed into a powerful tool to study instabilities for a broad range of fluid models. Further advances in this area have led to a full description of the essential spectrum of the 2D and 3D Euler system. This, in turn, made it possible to apply methods from dynamical systems to construct invariant manifolds near unstable equilibria.

Friedlander’s results on the long-time behavior of dyadic models represent a prototype of the “dream scenario” for the full Euler equations—finite time blow-up or regularization that leads to the Onsager-critical regularity for solutions with any initial data. The regularization property of the nonlinear term has been investigated further starting with the work of Barbato, Morandin, and Romito.

References


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Dusa McDuff has led three mathematical lives. In her early twenties she worked on von Neumann algebras and established the existence of uncountably many different algebraic types of \( \text{II}_1 \)-factors.\(^1\) After an inspiring six months studying with Gel’fand in Moscow and a two-year postdoc in Cambridge studying topology, she wrote a “second thesis” while working with Graeme Segal on configuration spaces and the group-completion theorem, and investigated the topology of various diffeomorphism groups, in particular symplectomorphism groups. This brought her to symplectic geometry, which was revolutionized around 1985 by Gromov’s introduction of \( J \)-holomorphic curves.

In this short text I will describe some of McDuff’s wonderful contributions to symplectic geometry. After reviewing what is meant by “symplectic” I will mostly focus on her work on symplectic embedding problems. Some of her other results in symplectic geometry are discussed at the end. More personal texts about Dusa can be found in [11b]. Parts of this text overlap with the “Perspective” in [11b] written jointly with Leonid Polterovich.

1. **Symplectic**

There are many strands to and from symplectic geometry. The most important ones are classical mechanics and algebraic geometry. I do not list these strands but refer you
to [17b] and to my 2018 survey in the Bulletin. Here, I simply give the definition.

Definition 1. Let M be a smooth manifold. A symplectic form on M is a nondegenerate closed 2-form $\omega$. A diffeomorphism $\varphi$ of M is symplectic (or a symplectomorphism) if $\varphi^* \omega = \omega$.

The nondegeneracy condition implies that symplectic manifolds are even-dimensional. An example is $\mathbb{R}^{2n}$ with the constant differential 2-form

$$\omega_0 = \sum_{i=1}^{n} dx_i \wedge dy_i.$$ 

Other examples are surfaces endowed with an area form, their products, and Kähler manifolds.

If you begin your first lecture on symplectic geometry like this, you may very well find yourself alone the following week. You may thus prefer to start in a more elementary way. Let $\gamma$ be a closed oriented piecewise smooth curve in $\mathbb{R}^2$. If $\gamma$ is embedded, assign to $\gamma$ the signed area of the disc $D$ bounded by $\gamma$, namely $\text{area}(D)$ or $-\text{area}(D)$, as in Figure 1.1.

![Figure 1.1. The sign of the signed area of an embedded closed curve in $\mathbb{R}^2$.](image1)

If $\gamma$ is not embedded, successively decompose $\gamma$ into closed embedded pieces as illustrated in Figure 1.2, and define $A(\gamma)$ as the sum of the signed areas of these pieces.

![Figure 1.2. Splitting a closed curve into embedded pieces.](image2)

Definition 2. The standard symplectic structure of $\mathbb{R}^{2n}$ is the map

$$A(\gamma) = \sum_{i=1}^{n} A(\gamma_i), \quad \gamma = (\gamma_1, ..., \gamma_n) \subset \mathbb{R}^{2n}.$$ 

A symplectomorphism $\varphi$ of $\mathbb{R}^{2n}$ is a diffeomorphism that preserves the signed area of closed curves:

$$A(\varphi(\gamma)) = A(\gamma) \quad \text{for all closed curves } \gamma \subset \mathbb{R}^{2n}.$$ 

A symplectic structure on a manifold $M$ is an atlas whose transition functions are (local) symplectomorphisms, and a symplectomorphism of $M$ is then a diffeomorphism that preserves this local structure.

The standard symplectic structure of $\mathbb{R}^{2n}$ is thus given by assigning to a closed curve the sum of the signed areas of the $n$ curves obtained by projecting to the coordinate planes $\mathbb{R}^2(x_i, y_i)$. And a symplectic structure on a manifold is a coherent way of assigning a signed area to sufficiently local closed curves. The equivalence of the two definitions follows from Darboux’s Theorem. Around every point of a symplectic manifold $(M, \omega)$ there exists a coordinate chart $\varphi$ such that $\varphi^* \omega = \omega$.

The group of symplectomorphisms of a symplectic manifold is very large. Indeed, for every compactly supported smooth function $H : M \times [0, 1] \to \mathbb{R}$ each time-$t$ map $\varphi^t_H$ of its Hamiltonian flow is a symplectomorphism. Symplectomorphisms of this form are called Hamiltonian diffeomorphisms. The Hamiltonian flow is the flow generated by the vector field $X_H$ implicitly defined by

$$\omega(X_H, \cdot) = -dH(\cdot).$$

For $(\mathbb{R}^{2n}, \omega_0)$ one has $X_H = J_0 \nabla H$, where $J_0$ is the usual complex structure on $\bigoplus_{i} \mathbb{R}^2(x_i, y_i)$.

2. Symplectic Embedding Problems

By Darboux’s Theorem, symplectic manifolds have no local invariants beyond the dimension. But there are several ways to associate global numerical invariants to symplectic manifolds. One of them is by looking at embedding problems. Take a compact subset $K$ of $(\mathbb{R}^{2n}, \omega_0)$. By a symplectic embedding $K \to M$ we mean the restriction to $K$ of a smooth embedding $\varphi: U \to M$ of an open neighborhood of $K$ that is symplectic, $\varphi^* \omega = \omega_0$. In this case we write $K \xrightarrow{\varphi} (M, \omega)$. For every $K$, the largest number $\lambda$ such that the dilate $\lambda K$ symplectically embeds into $(M, \omega)$ is then a symplectic invariant of $(M, \omega)$. Seven further reasons to study symplectic embedding problems can be found in my Bulletin article.

2.1. The Nonsqueezing Theorem. Now take the closed ball $B^{2n}(a)$ of radius $\sqrt{a/\pi}$ centered at the origin of $\mathbb{R}^{2n}$. (The notation reflects that symplectic measurements are 2-dimensional.) By what we said above, there are very many symplectic embeddings $B^{2n}(a) \xrightarrow{\varphi} \mathbb{R}^{2n}$. However, none of them can make the ball thinner, as Gromov proved in his pioneering 1985 paper.

Nonsqueezing Theorem. $B^{2n}(a) \xrightarrow{\varphi} B^2(A) \times \mathbb{R}^{2n-2}$ only if $a \leq A$.

The identity embedding thus already provides the largest ball that symplectically fits into the cylinder $B^2(A) \times \mathbb{R}^{2n-2}$ of infinite volume. While there are many forms
of symplectic rigidity, this theorem is its most fundamental manifestation. The theorem shows that some volume-preserving mappings cannot be approximated by symplectic mappings in the $C^0$-topology.

In [95a], Lalonde and McDuff generalized the Non-squeezing Theorem to all symplectic manifolds.

**General Nonsqueezing Theorem.** For any symplectic manifold $(M, \omega)$ of dimension $2n - 2$,

$$B^{2n}(a) \xrightarrow{s} (B^2(A) \times M, \omega_0 \oplus \omega) \text{ only if } a \leq A.$$

Every good tool in symplectic geometry can be used to prove the Nonsqueezing Theorem. However, the technique of $J$-holomorphic curves used by Gromov is the most influential one, and also the most important tool in McDuff's work.

An almost complex structure $J$ on a manifold $P$ is a smooth collection \{J_p\}_{p \in P}, where $J_p$ is a linear endomorphism of $T_pP$ such that $J_p^2 = -\text{id}$. The “almost” indicates that such a structure does not need to be a complex structure, i.e., does not need to come from a holomorphic atlas. Not all symplectic manifolds admit complex structures, but they all admit almost complex structures. A $J$-holomorphic curve in an almost complex manifold $(P, J)$ is a map $u$ from a Riemann surface $(\Sigma, j)$ to $(P, J)$ such that

$$du \circ j = J \circ du.$$

This equation generalizes the Cauchy–Riemann equation defining holomorphic maps $\mathbb{C} \rightarrow \mathbb{C}^n$. In this text, the domain of a $J$-holomorphic curve will always be the usual Riemann sphere, namely the round sphere $S^2 \subset \mathbb{R}^3$ whose complex structure $j$ rotates a vector $v \in T_pS^2$ by $\frac{v}{2}$. Even in this case, it is usually impossible to write down a $J$-holomorphic curve for a given $J$. But this is not a problem, since one usually just wants to know that such a curve exists.

Now assume that $\varphi : B^{2n}(a) \xrightarrow{s} B^2(A) \times \mathbb{R}^{2n-2}$. Choose $k$ so large that after a translation the image of $\varphi$ is contained in $B^2(A) \times (0, k)^{2n-2}$. Compactifying the disc to the sphere $S^2(A')$ with its usual area form of area $A' > A$ and taking the quotient to the torus $T^{2n-2} = \mathbb{R}^{2n-2}/k\mathbb{Z}^{2n-2}$, we then obtain a symplectic embedding

$$\Phi : B^{2n}(a) \xrightarrow{s} S^2(A') \times T^{2n-2} =: (P, \omega),$$

where $\omega$ is the split symplectic structure on the product $P$. We will see that $a \leq A'$. Since $A' > A$ was arbitrary, Gromov's theorem then follows.

Denote by $J_0$ the usual complex structure on $B^{2n}(a) \subset \mathbb{C}^n$, and let $J$ be an almost complex structure on $P$ that restricts to $\Phi(J_0)$ on $\Phi(B^{2n}(a))$ and that is $\omega$-tame, meaning that $\omega(v, J_0v) > 0$ for all nonzero $v \in TP$. Such an extension exists, since $\omega$-tame almost complex structures can be viewed as sections of a bundle over $P$ whose fibers are contractible.

**Lemma.** There exists a $J$-holomorphic sphere $u(S^2)$ through $\Phi(0)$ that represents the homology class of $S^2(A')$.

This existence result follows from Gromov's compactness theorem for $J$-holomorphic curves in symplectic manifolds. The key point for the proof of the compactness theorem is that $J$ is $\omega$-tame, implying that $J$-holomorphic curves cannot behave too wildly. The compactness theorem implies the lemma because the class of $S^2(A')$ is primitive in $H_2(P; \mathbb{Z})$.

![Figure 2.1. The geometric idea of the proof.](image-url)
that reduced the General Nonsqueezing Theorem essentially to the case proved by Gromov. The most important and influential of these is a multiple folding construction. Its simple version was used in [95a] to show how the General Nonsqueezing Theorem implies that for all symplectic manifolds Hofer's metric on the group of compactly supported Hamiltonian diffeomorphisms is nondegenerate and hence indeed a metric; see §3.3. These two theorems were the first deep results in symplectic geometry proven for all symplectic manifolds.

2.2. Ball packings. We next try to pack a symplectic manifold by balls as densely as possible. Taking the open ball \(B^4(1)\) as target, let

\[ p_k = \max \left\{ k \frac{\text{Vol}(B^4(a))}{\text{Vol}(B^4(1))} \mid \prod_k B^4(a) \hookrightarrow \hat{B}^4(1) \right\} \]

be the percentage of the volume of \(B^4(1)\) that can be filled by \(k\) symplectically embedded equal balls. Then:

\[
\begin{array}{c|cccccccc}
 k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \geq 9 \\
p_k & \frac{1}{2} & \frac{3}{4} & 1 & \frac{20}{25} & \frac{24}{25} & \frac{63}{64} & \frac{288}{289} & 1 \\
c_k & 1 & 2 & 2 & \frac{5}{2} & \frac{5}{2} & \frac{8}{3} & \frac{17}{6} & \sqrt{k} \\
\end{array}
\]

The lower line gives the capacities

\[ c_k = \inf \left\{ A \left| \prod_k B^4(1) \overset{s}{\hookrightarrow} \hat{B}^4(A) \right. \right\} \]

that are related to the packing numbers \(p_k\) by \(c_k^2 = \frac{k}{p_k^2}\). This table was obtained for \(k \leq 5\) by Gromov, for \(k = 6, 7, 8\) and \(k\) a square by McDuff and Polterovich [94b], and for all \(k\) by Biran. This result is a special case of the following algebraic reformulation of the general ball packing problem

\[
\prod_{i=1}^k B^4(a_i) \overset{s}{\hookrightarrow} \hat{B}^4(A).
\]

Ball Packing Theorem. An embedding (2.2) exists if and only if

(i) (Volume constraint) \(A^2 > \sum_{i=1}^k a_i^2\)

(ii) (Constraint from exceptional spheres)

\[ A > \frac{1}{d} \sum_{i=1}^k a_i m_i \]

for every vector of nonnegative integers \((d; m_1, \ldots, m_k)\) that solves the Diophantine system

\[ \sum_{i=1}^k m_i = 3d - 1, \quad \sum_{i=1}^k m_i^2 = d^2 + 1 \]

and can be reduced to \((0; -1, 0, \ldots, 0)\) by repeated Cremona moves.

Here, a Cremona move takes a vector \((d; m_1, \ldots, m_k)\) with \(m_1 \geq \cdots \geq m_k\) to the vector

\[(d'; m') = (d + \delta; m_1 + \delta, m_2 + \delta, m_3 + \delta, m_4 + \delta, \ldots, m_k),\]

where \(\delta = d - (m_1 + m_2 + m_3)\), and then reorders \(m'\).

Before discussing the proof, we use the theorem to obtain table (2.1). If \((d; m_1, \ldots, m_k)\) is a solution of (DE), then

\[(3d - 1)^2 = \left( \sum_{i=1}^k m_i \right)^2 \leq k \sum_{i=1}^k m_i^2 = k(d^2 + 1),\]

that is,

\[(9 - k)d^2 - 6d + (1 - k) \leq 0.\]

For \(k \leq 8\), this equation has finitely many solutions \(d\), and so (DE) has finitely many solutions for \(k \leq 8\). They are readily computed:

\[(1; 1, 1), (2; 1^5), (3; 2, 1^3, 1^2), (4; 2^2, 1^5), (5; 2^2, 1^3, 1, 1), (6; 2^2, 2^2, 2^2), (7; 2^2, 2^2, 2^2, 1), (8; 2^2, 2^2, 2^2, 1, 1, 1).\]

Thus the symplectic blow-up of \(C^2\) at the origin \(0\) is obtained by replacing 0 by all complex lines in \(C^2\) through 0. At the topological level, this operation can be done as follows. First remove from \(C^2\) an open ball \(B^4\). The boundary \(S^3\) of \(C^2 \setminus B^4\) is foliated by the Hopf circles \(\{ (\alpha z_1, \alpha z_2) \mid \alpha \in \mathbb{S}^1 \}\), namely the intersections of \(S^3\) with complex lines. Now \(Bl(C^2)\) is obtained by replacing each such circle by a point. The boundary sphere \(S^3\) becomes a 2-sphere \(CP^1\) in \(Bl(C^2)\) of self-intersection number \(-1\), called the exceptional divisor. The manifold \(Bl(C^2)\) is diffeomorphic to the connected sum \(C^2 \# CP^2\).

This construction can be done in the symplectic setting. If one removes \(B^4(a)\) from \(R^4\), then there exists a symplectic form \(\omega_a\) on \(Bl(R^4)\) such that \(\omega_a = \omega_0\) outside a tubular neighborhood of the exceptional divisor \(CP^1\) and such that \(\omega_a\) is symplectic on \(CP^1\) with \(\int_{CP^1} \omega_a = a\). Given a symplectic embedding \(\varphi : B^4(a) \to (M, \omega)\) into a symplectic 4-manifold, we can apply the same construction to \(\varphi(B^4(a))\) in \(M\) to obtain the symplectic blow-up of \((M, \omega)\) by weight \(a\).
There is also an inverse construction. Given a symplectically embedded \(-1\)-sphere \(\Sigma\) in a symplectic 4-manifold \((M, \omega)\) of area \(f_\omega\omega = a\), one can cut out a tubular neighborhood of \(\Sigma\) and glue back \(\mathbb{B}^4(a)\), to obtain the “symplectic blow-down” of \(M\).

Now let \(M_k\) be the smooth manifold obtained by blowing up the complex projective plane \(\mathbb{P}^2\) in \(k\) points. Its homology \(H^2(M_k; \mathbb{Z})\) is generated by the class \(L\) of a complex line and by the classes \(E_1, \ldots, E_k\) of the exceptional divisors. Let \(\ell, e_i \in H^2(M_k; \mathbb{Z})\) be their Poincaré duals. Every symplectic form \(\omega\) on \(M_k\) defines a first Chern class \(c_1(\omega)\), namely the first Chern class of any \(-\ell\)-tame almost complex structure. If we take a symplectic form \(\omega\) on \(M_k\) constructed as above via \(k\) symplectic ball embeddings (that always exist if the balls are small enough), then \(-c_1(\omega)\) is Poincaré dual to the class \(K := -3L + \sum E_i\). Define \(C_k(M_k) \subset H^2(M_k; \mathbb{R})\) to be the set of classes represented by symplectic forms with \(-c_1(\omega) = PD(K)\).

Compactifying \(\mathbb{B}^4(A)\) to \(\mathbb{P}^2(A)\) with its usual Kähler form integrating to \(A\) over a complex line, and using that the classes \(E_i\) can be represented by symplectic \(-1\)-spheres, McDuff and Polterovich [94b] obtained

\begin{enumerate}[Step 1.]
  \item There exists an embedding \(\prod_{i=1}^k \mathbb{B}^4(a_i) \to \mathbb{B}^4(A)\) if and only if the class \(a := A\ell - \sum a_i e_i \in H^2(M_k; \mathbb{R})\) lies in \(C_k(M_k)\).
\end{enumerate}

Our symplectic embedding problem is thus translated into a problem on the symplectic cone \(C_k(M_k)\). In Kähler geometry, the problem of deciding which cohomology classes can be represented by a Kähler form has a long history and is quite well understood. The solution of our symplectic analogue, however, needs different ideas and tools: call a class \(E \in H^2(M_k; \mathbb{Z})\) exceptional if \(K \cdot E = 1\) and \(E^2 = -1\), and if \(E\) can be represented by a smoothly embedded \(-1\)-sphere.

If \(\omega\) is a symplectic form on \(M_k\) with \(-c_1(\omega) = PD(K)\), then any \(\omega\)-symplectic embedded \(-1\)-sphere represents an exceptional class. Seiberg–Witten–Taubes theory implies that the converse is also true: every exceptional class \(E\) can be represented by an \(\omega\)-symplectic embedded \(-1\)-sphere. This implies one direction of

\begin{enumerate}[Step 2.]
  \item \(a = A\ell - \sum a_i e_i \in H^2(M_k; \mathbb{R})\) lies in \(C_k(M_k)\) if and only if \(a^2 > 0\) and \(a(E) > 0\) for all exceptional classes.
\end{enumerate}

This equivalence is remarkable. Of course, a necessary condition for a class \(\alpha\) with positive square to have a symplectic representative is that \(\alpha\) evaluates positively on all classes that can be represented by closed symplectically embedded surfaces (and in particular on spheres). But the equivalence says that this is also a sufficient condition, and that it is actually enough to check positivity on spheres.

To prove the other direction in Step 2, one starts with an embedding of \(k\) tiny balls of size \(\varepsilon a_i\) and then changes the symplectic form on \(M_k\) in class \(A\ell - \varepsilon \sum a_i e_i\) in such a way that these balls look large. This can be done with the help of the inflation method of Lalonde–McDuff [94a, 96a].

**Inflation Lemma.** Let \((M, \omega)\) be a closed symplectic 4-manifold, and assume that the class \(C \in H^2(M; \mathbb{Z})\) with \(C^2 \geq 0\) can be represented by a closed connected embedded \(J\)-holomorphic curve \(\Sigma\) for some \(\omega\)-tame \(J\). Then the class \([\omega] + s PD(C)\) has a symplectic representative for all \(s \geq 0\).

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2.2}
\caption{The ray in the symplectic cone provided by the Inflation Lemma.}
\end{figure}

I explain the proof for the case \(C^2 = 0\). In this case, the normal bundle of \(\Sigma\) is trivial, and we can identify a tubular neighborhood of \(\Sigma\) with \(\Sigma \times D\), where \(D \subset \mathbb{R}^2\) is a disc. Pick a radial function \(f(r)\) with support in \(D\) that is non-negative and has \(f_D f = 1\). Let \(\beta\) be the closed 2-form on \(M\) that equals \(\beta(z, x, y) = f(r) \, dx \wedge dy\) on \(\Sigma \times D\) and vanishes outside of \(\Sigma \times D\). Then \(\beta = PD(\Sigma)\) and the forms \(\omega + s \beta\) are symplectic for all \(s \geq 0\). Indeed,

\[(\omega + s \beta)^2 = \omega^2 + \frac{s}{2} \omega \wedge \beta + s^2 \beta^2 \geq 0,
\]

where for the middle term we used that \(\omega|_{\Sigma}\) is symplectic.

Now take an embedding \(\prod_{i=1}^k \mathbb{B}^4(a_i) \to \mathbb{B}^4(A)\) of tiny balls. By Step 1 we know that the class \(\alpha_\varepsilon := A\ell - \varepsilon \sum a_i e_i\) has a symplectic representative \(\omega_\varepsilon\). We wish to inflate this form to a symplectic form in class \(\alpha\). A first try could be to inflate \(\omega_\varepsilon\) directly in the direction \(\alpha - \alpha_\varepsilon\) to get up to \(\alpha\). But this does not work, because

\[(\alpha - \alpha_\varepsilon)^2 = \frac{1}{2} \sum (1 - \varepsilon) a_i e_i)^2 = -(1 - \varepsilon)^2 \sum a_i^2 \leq 0.<p>However, assuming for simplicity that \(A\) and the \(a_i\) are rational, Seiberg–Witten–Taubes theory implies that there exists an integer \(n\) such that the Poincaré dual of \(n\alpha\) in \(H^2(M_k; \mathbb{Z})\) can be represented by a connected embedded \(J\)-holomorphic curve for a generic \(\omega\)-tame \(J\). We can thus inflate \(\omega\) in the direction of \(n\alpha\) and obtain that the classes \(\alpha_\varepsilon + sn\alpha\) have symplectic representatives for all \(s \geq 0\).

Rescaling these forms by \(\frac{1}{sn+1}\), we obtain symplectic forms in classes \(A\ell - \frac{sn+\varepsilon}{sn+1} \sum a_i e_i\) that are as close to \(\alpha\) as we like.
usually $J$-holomorphic curves are used to obstruct certain symplectic embeddings, like in the Nonsqueezing Theorem. In the Inflation Lemma and Step 2, however, other $J$-holomorphic curves are used to construct symplectic embeddings.

Step 3. $\alpha^2 > 0$ and $\alpha(E) > 0$ for all exceptional classes if and only if (i) and (ii) in the theorem hold.

Much of this step is rewriting: the inequality $\alpha^2 > 0$ translates to the volume constraint (i), and the inequality $\alpha(E) > 0$ for an exceptional class translates to (ii) if we write $E$ in the natural basis, $E = dL - \sum_i m_i E_i$. We are thus left with showing that a class $E = (a; m_1, \ldots, m_k)$ that satisfies (DE) is exceptional exactly if it reduces to $(0; -1)$ under Cremona moves. This follows from a combinatorial argument of Li and Li; see [12b].

2.3. Ellipsoids. We now look at embeddings of 4-dimensional ellipsoids

$$E(a, b) = \left\{ (z_1, z_2) \in \mathbb{C}^2 \mid \frac{\pi|z_1|^2}{a} + \frac{\pi|z_2|^2}{b} \leq 1 \right\}.$$  

This is the ellipsoid in $\mathbb{C}^2$ whose projections to the coordinate planes are closed discs of area $a$ and $b$. Again we take as target a ball, and after rescaling study the function

$$c(a) = \inf \{ A \mid E(1, a) \xrightarrow{s} \mathbb{B}^4(A) \}, \quad a \geq 1.$$  

Since this function is continuous, we can assume that $a$ is rational. Then each leaf of the characteristic foliation on the boundary of $E(1, a)$ is closed. Proceeding as before, we compactify $\mathbb{B}^4(A)$ to $\mathbb{C}P^2(A)$ and, given an embedding $E(1, a) \xrightarrow{s} \mathbb{B}^4(A) \subset \mathbb{C}P^2(A)$, remove the image and collapse the remaining boundary along the characteristic foliation. But now this yields a symplectic orbifold with one or two cyclic quotient singularities, coming from the special leaves in the two coordinate planes. It may be difficult to reprove the results from Seiberg–Witten–Taubes theory used in the last section in such a space. McDuff in [9] simply circumvented the singularities by using a version of the Hirzebruch–Jung resolution of singularities. She removed a bit more than the ellipsoid by successively blowing up finitely many balls, thereby producing a chain of $J$-spheres. Inflating this chain she reduced the problem $E(1, a) \xrightarrow{s} \mathbb{B}^4(A)$ to the ball packing problem (2.2):

$$E(1, a) \xrightarrow{s} \mathbb{B}^4(A) \iff \prod_{i=1}^k \mathbb{B}^4(a_i) \xrightarrow{s} \mathbb{B}^4(A), \quad (2.3)$$

where the $a_i$ are given by

$$(a_1, \ldots, a_k) = : w(a) = (1, \ldots, 1, \frac{w_1}{\ell_0}, \frac{w_1}{\ell_1}, \frac{w_2}{\ell_1}, \frac{w_2}{\ell_2}, \ldots, \frac{w_N}{\ell_N}, \ldots)$$

with the weights $w_i > 0$ such that $w_1 = a - \ell_0 < 1$, $w_2 = 1 - \ell_1$, $w_3 < w_1$, and so on. For instance, $w(3) = (1, 1, 1)$ and $w(\frac{11}{4}) = (1, 1, \frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. The multiplicities $\ell_i$ of $w(a)$ give the continued fraction expansion of $a$. For $a \in \mathbb{N}$, (2.3) specializes to

$$E(1, k) \xrightarrow{s} \mathbb{B}^4(A) \iff \prod_{k} \mathbb{B}^4(1) \xrightarrow{s} \mathbb{B}^4(A). \quad (2.4)$$

The ball packing problem $\prod_k \mathbb{B}^4(1) \rightarrow \mathbb{B}^4(A)$ is thus included in the 1-parameter problem $E(1, a) \xrightarrow{s} \mathbb{B}^4(A)$.

The function $c(a)$ was computed in [12b] with the help of (2.3). The volume constraint is now $c(a) \geq \sqrt{a}$. Recall that the Fibonacci numbers are recursively defined by $f_0 = 1, f_0 = 0, f_{n+1} = f_n + f_{n-1}$. Denote by $g_n := f_{2n-1}$ the odd-index Fibonacci numbers, hence $(g_0, g_1, g_2, g_3, g_4, \ldots) = (1, 1, 2, 5, 13, \ldots)$. The sequence $g_0 := \frac{g_{n+1}}{g_n}, g_n$ converges to $\tau^2$, where $\tau := \frac{1 + \sqrt{5}}{2}$ is the Golden Ratio. Define the Fibonacci stairs as the graph on $[1, \tau^4]$ made from the infinitely many steps shown in Figure 2.5, where $a_n = \frac{g_n}{g_{n+1}}$ and $b_n = \frac{g_{n+2}}{g_{n+1}}$. The slanted edge starts on the volume constraint $\sqrt{a}$ and extends to a line through the origin.

Ellipsoid Embedding Theorem.

(i) On the interval $[1, \tau^4]$ the function $c(a)$ is given by the Fibonacci stairs.

(ii) On the interval $] \tau^4, (\frac{17}{6})^2]$ we have $c(a) = \sqrt{a}$ except on nine disjoint intervals where $c$ is a step made from two segments. The first of these steps has the vertex at $(7, \frac{8}{3})$ and the last at $(8, \frac{17}{6})$.

(iii) $c(a) = \sqrt{a}$ for all $a \geq (\frac{17}{6})^2$. 

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Thus the graph of \( c(a) \) starts with an infinite completely regular staircase, then has a few more steps, but for \( a \geq (\frac{17}{6})^2 = \frac{1}{36} \), the graph of \( \gamma \) starts with an infinite completely regular staircase, then has a few more steps, but for \( a \geq (\frac{17}{6})^2 = \frac{1}{36} \), is given by the volume constraint.

The theorem better explains the packing numbers \( c_k \) in Table (2.1) in view of the equivalence (2.4); cf. the blue dots in Figure 2.4. The embedding constraint at \( b_n \) comes from a really exceptional exceptional sphere, namely one in an exceptional class \( E = (d; m) \) such that \( m \) is parallel to the weight expansion \( \omega(b_n) \).

When McDuff first looked at ellipsoid embeddings in [09], it was not clear at all that this would unveil an interesting fine structure of symplectic rigidity. Many more infinite staircases have been found by now, and results of Usher show that a simple-looking problem, such as for which \( A \) does the ellipsoid \( E(1, a) \) embed into the polydisc \( B^2(A) \times B^2(bA) \), is already so intricate that we shall probably never know the answer for all \( b \). McDuff in [11a] and independently Hutchings proved that

\[
E(a, b) \overset{s}{\rightarrow} E(c, d) \iff N_k(a, b) \leq N_k(c, d) \quad \text{for all } k,
\]

where \( (N_k(a, b)) \) is the nonincreasing sequence obtained by ordering the set \( \{ma + nb \mid m, n \in \mathbb{Z}_{\geq 0}\} \). Using his embedded contact homology (ECH), Hutchings had associated with every starshaped subset \( K \subset \mathbb{R}^4 \) a sequence of numbers \( c_k(K) \) that are monotone with respect to symplectic embeddings, and for an ellipsoid these ECH capacities equal the above sequence, \( c_k(E(a, b)) = N_k(a, b) \). The McDuff–Hutchings theorem (2.5) therefore implied that ECH-capacities are a complete set of invariants for the problem of embedding one 4-dimensional ellipsoid into another.

These results are all in dimension four, and until recently, not much was known about higher-dimensional symplectic embedding problems beyond the Nonsqueezing Theorem. The reason is that in dimension four, \( J \)-holomorphic curves are a much more powerful tool, because of positivity of intersections of such curves, and because one usually finds them with the help of the 4-dimensional Seiberg–Witten–Taubes theory. However, 4-dimensional ellipsoid embeddings also opened the door for understanding certain symplectic embedding problems in higher dimensions. I describe how they led to packing stability in all dimensions. Given a connected symplectic manifold \( (M, \omega) \) of finite volume, let \( p(M, \omega) \) be the smallest number (or infinity) such that for every \( k \geq p(M, \omega) \) an arbitrarily large percentage of the volume of \( (M, \omega) \) can be covered by \( k \) equal symplectically embedded balls. Table (2.1) shows that \( p(B^6) = 9 \). The finiteness of \( p \) is now known for many symplectic manifolds, and in particular for balls in all dimensions. In this case, the ingredients of the proof are:

(1) McDuff’s observation from [09] that the ellipsoid \( E(1, \ldots, 1, k) \) can be cut into \( k \) equal balls.

(2) The Ellipsoid Embedding Theorem for \( E(1, a) \overset{s}{\rightarrow} B^4(A) \) and the McDuff–Hutchings theorem (2.5).

(3) Ellipsoids admit a suspension construction: for any vectors \( a, b, c \),

\[
E(a) \overset{s}{\rightarrow} E(b) \implies E(a, c) \overset{s}{\rightarrow} E(b, c).
\]

For instance, (2) yields that

\[
E(1, k) \overset{s}{\rightarrow} E(k^{1/3}, k^{2/3}) \text{ and } E(1, k^{2/3}) \overset{s}{\rightarrow} B^4(k^{1/3})
\]

for all \( k \geq 21 \).

Together with (1) and (3) we thus obtain that for these \( k \)

\[
\prod_k B^6(1) \overset{s}{\rightarrow} E(1, 1, k) \overset{s}{\rightarrow} E(1, k^{1/3}, k^{2/3}) \overset{s}{\rightarrow} B^6(k^{1/3})
\]

and hence \( p(B^6) \leq 21 \). Already Gromov had proved that \( p(B^6) \geq 8 \). Is it true that \( p(B^6) = 8 \)?
A symplectic manifold $(M, \omega)$ is rational if a multiple of $[\omega]$ takes rational values on all integral 2-cycles. All closed rational symplectic manifolds have packing stability. The additional ingredient in the proof is that every such manifold can be completely filled by an ellipsoid.

2.4. Connectivity. For a compact set $K \subset \mathbb{R}^{2n}$ and a 2n-dimensional symplectic manifold $(M, \omega)$ let $\text{Emb}_\omega(K, M)$ be the space of symplectic embeddings $K \to (M, \omega)$, with the $C^\infty$-topology. The results discussed above tell us for some $K$ and $(M, \omega)$ whether this space is empty or not. In the latter case one may study its topology. The first task is to see whether $\text{Emb}_\omega(K, M)$ is connected or not.

We first take $K$ to be a ball $B^{2n}(a)$. Then $\text{Emb}_\omega(B^{2n}(a), M)$ need not be connected. The first counterexample was Gromov's camel theorem: for $2n \geq 4$ the camel space in $\mathbb{R}^{2n}$ with eye of width 1 is the set

$$C^{2n} = \{x_1 < 0\} \cup \{x_1 > 0\} \cup B^{2n}(1).$$

Now take $a > 1$ and define the two embeddings $\varphi_\pm : B^{2n}(a) \to C^{2n}$ by $\varphi_\pm(z) = z \pm v$, where $v$ is a multiple of $\frac{\partial}{\partial x_1}$ such that $\varphi_+$ takes $B^{2n}(a)$ to the right half space and $\varphi_-$ takes $B^{2n}(a)$ to the left half space; see Figure 2.6. Then $\varphi_+$ and $\varphi_-$ are not isotopic.

Now consider the two paths of symplectic embeddings $\varphi^t : B^4(1) \to \mathbb{B}^4(4), \varepsilon \leq t \leq 1$, defined by restricting the embeddings $\varphi_s$. Reparametrizing the inverse of the path $\varphi^t$ on $s \in [0, 1]$ and the path $\varphi^s$ on $s \in [1, 2]$ we obtain a path of symplectic embeddings into $\mathbb{B}^4(A)$ that connects $\varphi_1$ and $\varphi_2$. After compactifying $\mathbb{B}^4(A)$ to $\mathbb{C}P^2(A)$ by adding the line $\mathbb{C}P^1$ and by symplectically blowing up the images, this path of embeddings gives rise to a path of symplectic forms $\omega_s$ on $\text{Bl}(\mathbb{C}P^2)$ in class $[\omega_0]$ as shown in Figure 2.7. As in §2.2 apply symplectic inflation to each form $\omega_s$, deforming $\omega_s$ to a form $\tilde{\omega}_s$ in class $[\omega_0] = [\omega_2] = A\ell - e$. Finally blow down the exceptional divisor for each $s$ to get a path of embeddings $\tilde{\varphi}_s : \mathbb{B}^4(1) \to \mathbb{C}P^2(A)$. Since each exceptional divisor is disjoint from $\mathbb{C}P^1$, each ball $\tilde{\varphi}_s(\mathbb{B}^4(1))$ lies in $\mathbb{C}P^2(A) \setminus \mathbb{C}P^1$, and by a theorem of McDuff from [90] this set with the symplectic form obtained after blow-down is indeed symplectomorphic to $\mathbb{B}^4(A)$; cf. §3.2 below. Furthermore, $\tilde{\varphi}_s$ connects $\varphi_1$ with $\varphi_2$.

McDuff's theorem that $\text{Emb}_\omega(B^4(1), \mathbb{B}^4(A))$ is connected is sharp in the following sense. There are convex subsets $K$ of $\mathbb{R}^4$ with smooth boundary that are arbitrarily close to a ball and such that $\text{Emb}_\omega(K, A\ell)$ has at least two components, for some $\lambda > 1$. For the cube $C^4(1) = B^2(1) \times B^2(1)$ the space $\text{Emb}_\omega(C^4(1), \mathbb{B}^4(3))$ even has infinitely many connected components.

3. Other Contributions to Symplectic Geometry

3.1. Construction of (counter)examples. McDuff again and again has given explicit constructions that are both elementary and profound. You can grasp them quickly, yet they can be used to do many other things. We already encountered some of them in the previous section, and I now mention three more. A simply connected symplectic manifold that is not Kähler. While Kähler manifolds are symplectic, the converse is not true. For instance, the odd-index Betti numbers
of compact Kähler manifolds are even. The first example of a compact symplectic manifold that is not Kähler was found by Thurston; his example \((M, \omega)\) is a \(T^2\)-bundle over \(T^3\) with \(b_1(M) = 3\). In [84], McDuff constructed simply connected examples. She symplectically embedded the Thurston manifold \(M\) into \(\mathbb{C}P^3\) and showed that the symplectic blow-up of \(\mathbb{C}P^3\) along \(M\) is simply connected and has \(b_2 = b_1(M) = 3\).

Cohomologous symplectic forms that are not diffeomorphic. Consider two symplectic forms \(\omega_0\) and \(\omega_1\) on a closed symplectic manifold that are cohomologous. If these two forms can be connected by a smooth path \(\omega_t\) of cohomologous symplectic forms, then there exists an isotopy of \(M\) deforming \(\omega_0\) to \(\omega_1\), by Moser’s trick. This is the case, for instance, for any two area forms on a closed surface. In general, however, there may be no such path, as McDuff showed in [87].

Take \(M = S^2 \times S^2 \times T^2\) with the standard split symplectic form \(\omega\) giving all factors area one. Define

\[
\Psi(z, w, (s, t)) = (z, R_{z,t}(w), s, t),
\]

where \(R_{z,t}\) is the rotation by angle \(2\pi t\) of the round sphere about the axis through \(z, -z\). Then the symplectic forms \(\omega_k = (\Psi^*)^k \omega, k \in \mathbb{Z}_{\geq 0}\), are all cohomologous, and they can be joined by a path of symplectic forms, but by no such path in the same cohomology class. The last part of the statement was McDuff’s first result obtained by using \(J\)-holomorphic curves. She studied the space of \(\omega_k\)-tame \(J\)-holomorphic spheres in the class of the first \(S^2\)-factor, and showed that such spheres wrap \(k\)-times around the second factor. Since a diffeomorphism isotopic to the identity acts trivially on homology, an isotopy deforming \(\omega_k\) to \(\omega\) can therefore only exist if \(k = \ell\).

Also in [87] McDuff improved this construction to obtain cohomologous symplectic forms on an \(8\)-dimensional symplectic manifold that are not even diffeomorphic.

Disconnected contact boundaries. The natural boundaries of symplectic manifolds are those of contact type, meaning that there exists a vector field \(X\) defined near \(\partial M\) that is transverse to \(\partial M\), pointing outwards, and is conformally symplectic: \(L_X \omega = d_{\partial M} \omega = \omega\). Symplectic manifolds with boundary of contact type are analogous in many ways to complex manifolds with pseudoconvex boundaries, as was shown by Eliashberg and Gromov in 1989. In the latter situation, the boundary is always connected. However, McDuff [91b] explicitly constructed a compact symplectic 4-manifold whose boundary is of contact type and disconnected.

She starts with the cotangent bundle \(T^* \Sigma\) over a closed orientable surface of genus \(\geq 2\), endowed with its canonical symplectic form \(d\lambda\), where \(\lambda = \sum p_i dq_i\). Also take a Riemannian metric of constant curvature on \(\Sigma\). It induces the connection 1-form \(\beta\) of the Levi-Civita connection on \(T^* \Sigma\) and the radial coordinate \(r\) on the fibers. Then one finds smooth functions \(f, g\) on \([0, \infty)\) such that on the annulus bundle \(\{x \in T^* \Sigma | \frac{1}{2} \leq r(x) \leq 1\}\) the 2-form

\[
\omega = df(r) \beta + g(r) \lambda
\]
is symplectic and makes the boundary of contact type.

3.2. The structure of rational and ruled symplectic 4-manifolds. The classification of compact complex surfaces is an old and beautiful topic in complex and Kähler geometry. Before 1990, the only result on symplectic 4-manifolds in this direction was a theorem of Gromov for the complex projective plane. In [90], McDuff proved the following generalization. If a closed symplectic 4-manifold contains a symplectically embedded sphere with nonnegative self-intersection number, then it is symplectomorphic to either \(\mathbb{CP}^2\) with its standard Kähler structure, to a ruled symplectic manifold, or to a symplectic blow-up of one of these manifolds.

Here, a ruled symplectic 4-manifold is the total space of an \(S^2\)-fibration over a closed oriented surface with a symplectic structure that is nondegenerate on the fibers.

In later work with Lalonde [96a], McDuff classified ruled symplectic surfaces. If \((M, \omega)\) is a ruled symplectic 4-manifold, then \(\omega\) is determined up to symplectomorphism by its cohomology class and is isotopic to a standard Kähler form on \(M\). Li–Liu complemented this result by showing that if \((M, \omega)\) is the total space of an \(S^2\)-fibration over a closed surface, then there is a ruling of \(M\) by symplectic spheres, i.e., \((M, \omega)\) is a ruled symplectic manifold. See [96b] for a survey on this classification.

An elementary but important point in the proofs is that two cohomologous symplectic forms \(\omega_0\) and \(\omega_1\) are diffeomorphic if they tame the same almost complex structure \(J\).

Indeed, all the cohomologous forms \(\omega_t = (1-t)\omega_0 + t\omega_1\), \(t \in [0, 1]\), tame \(J\) and hence are symplectic, and therefore \(\omega_0\) and \(\omega_1\) are diffeomorphic by Moser’s argument.

These works showed that symplectic geometry has something to say about 4-manifolds, and they helped establish symplectic geometry as one of the core geometries.

McDuff has also done much interesting work on the topology of the group of symplectomorphisms for several classes of symplectic manifolds. For \(S^2\)-fibrations over \(S^2\) with any symplectic form she has in particular shown in joint work with Abreu [00] that two symplectomorphisms are isotopic through symplectomorphisms whenever they are isotopic through diffeomorphisms.

3.3. Hofer geometry. Recall that any symplectic manifold \((M, \omega)\) locally looks like the standard symplectic vector space of the same dimension. Our dear geometric intuition from everyday life, so useful in Riemannian geometry to see distances and curvatures, is thus useless in symplectic geometry. However, on the automorphism group \(\text{Ham}^c(M, \omega)\) of Hamiltonian diffeomorphisms that
are generated by compactly supported functions, there is a bi-invariant Finsler metric, that to some extent serves as a substitute for the absence of local geometry in \((M, \omega)\). Given a time-dependent Hamiltonian function \(H : M \times [0, 1] \to \mathbb{R}\) with compact support, take the integrated oscillation
\[
\|H\| := \int_0^1 \left( \max_{x \in M} H(x, t) - \min_{x \in M} H(x, t) \right) dt.
\]
For \(\varphi \in \text{Ham}^c(M, \omega)\) now define
\[
d(\varphi, \text{id}) = \inf_H \|H\|,
\]
where \(H\) runs over all compactly supported Hamiltonian functions whose time-1 flow map is \(\varphi\). Then \(d(\varphi, \psi) = d(\varphi \psi^{-1}, \text{id})\) defines a bi-invariant metric on \(\text{Ham}^c(M, \omega)\). The only difficult point to check is nondegeneracy. This was done by Hofer for \(\mathbb{R}^{2n}\) by variational methods, by Polterovich for tame rational symplectic manifolds by using a rigidity property of Lagrangian submanifolds, and for all symplectic manifolds by Lalonde–McDuff [95a] who used their General Nonsqueezing Theorem discussed in §2.1 and the following symplectic folding construction.

**Figure 3.1.** The symplectic folding construction, schematically.

Assume that \(\varphi \neq \text{id}\). Then we find a symplectically embedded ball \(B^{2n}(a)\) in \(M\) such that \(\varphi(B^{2n}(a)) \cap B^{2n}(a) = \emptyset\). Now consider the ball
\[
B^{2n+2}(a) \subset B^2(a) \times B^{2n}(a) \subset \mathbb{R}^2 \times M,
\]
and assume that \(\varphi\) is generated in time 1 by \(H\). In three steps, the small green ball \(B^{2n+2}(\frac{a}{2}) \subset B^{2n+2}(a)\) is folded on top of its complement, as illustrated in Figure 3.1. First, one views \(B^{2n+2}(a)\) as a \(B^2(a)\)-fibration over the disc \(B^2(a)\) and separates the small fibers from the large ones. In the key step, one then uses the flow \(\phi_{||H||}^t, t \in [0, 1]\), to lift the small green ball. The projection of the red band in the image of \(\lambda\) to \(\mathbb{R}^2\) has area \(\|H\|\). Since \(\varphi(B^{2n}(a))\) is disjoint from \(B^{2n}(a)\), one can thus turn the green ball over the blue part to obtain an embedding
\[
B^{2n+2}(a) \xrightarrow{\sigma} B^2(a) \times B^2(a) \times \mathbb{R}^2 \times M.
\]
Hence \(\|H\| \geq \frac{a}{2}\) by the General Nonsqueezing Theorem. You can readily grasp the construction from the three figures on pages 473, 474, 475 of [17b]. Symplectic folding found many other applications to symplectic geometry.

In [95b], Lalonde and McDuff made a deep study of geodesics in this Finsler geometry on \(\text{Ham}^c(M, \omega)\), that lead to completely new geometric intuitions on this group. It then became possible to think about Hamiltonian dynamics in geometric terms such as geodesic, conjugate point, cut-locus, etc. See Polterovich’s book from 2001 for more on this.

**The books.** While in her papers Dusa shows herself to be an impressive and creative problem solver, the two books she wrote with Dietmar Salamon were (and remain) crucial for the foundation of symplectic geometry and its dissemination. *Introduction to symplectic topology* [17b] explains the methods and results of the field in clear and modern geometric language. A key for the success of this book is that it discusses the main results in the most important and typical cases, without striving for generality. I was very lucky that this book came out (in 1995) just when I wanted to learn the subject. It is one of the main reasons for the transformation of the then small community of symplectic geometers into a large family. In the third edition from 2017, the authors added a chapter with 54 open problems, proving that symplectic geometry is not a closed chapter but rather remains an exploding field.

The second book [12a] provides a rigorous foundation of the theory of \(J\)-holomorphic curves and explains their applications to symplectic topology. The exposition is so precise and to the point that one can often just cite the result one needs. This book transformed the formerly somewhat romantic theory of \(J\)-holomorphic curves into a well-established tool of enormous impact.

In addition to her incredible mathematical legacy, Dusa’s extraordinary generosity to young (and not so young) researchers, her enthusiasm and her (sometimes overwhelming) energy, and her heartfelt commitment in scientific, political, and social issues should be recognized.

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[11b] celebratio.org/McDuff_D


Credits

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Felix Schlenk
Navigating an Uncharted Path: The Life and Legacy of Dr. Gladys B. West

In 2018, the Air Force recognized mathematician Gladys B. West with an Air Force Space and Missile Pioneers Award for her contributions over a career that spanned decades, from the first days of the space race. Notably, she had a hand in what is now a ubiquitous tool, the global positioning system. The use of satellite data to accurately assess position presents rich mathematical challenges, in part because of the difficulty in modeling the Earth itself. In awarding West, the Air Force noted her "increasingly refined calculations for an extremely accurate geodetic Earth..."
model, a geoid, optimized for what ultimately became the Global Positioning System (GPS) orbit. Without that model, and regular updates thereto, the extraordinary positioning, navigation and timing accuracy of GPS would be impossible to achieve” [1]. The authors had an opportunity to interview West in August of 2020 to learn more about her experiences as a trailblazing mathematician. In this article, we provide a brief introduction both to West and to her mathematical contributions at the US Naval Surface Warfare Center.

West envisioned a life beyond her surroundings. At the closing of her interview with the authors, Dr. West said that, “mathematics creates a precise pathway” to finding solutions to many problems [3]. With conviction and precision, she used her vision and her mathematical skills as a compass to chart a path unlike any other she had seen. Her journey began simply with the dream of experiencing a life different than what she knew growing up on a farm in rural Virginia. As an African American girl born in 1930 at the beginning of the Great Depression, it was a bold desire. “I continued to dream beyond my wildest imagination, and no matter how inconceivable those dreams, I still had hope for a better life ahead,” Dr. West explains in her memoir, It Began with a Dream [2]. Published in 2020, her autobiography chronicles her remarkable trajectory, as she became a pivotal contributor to a system that is now used worldwide. A high school geometry class gave her the first inkling that mathematics might be a part of the route to something different. “I was starting to realize early that this newly found love for geometry was something that could help me find that road I had dreamed about, the road that would take me far away from that farm,” shares West [2].

The Early Years

During an era of Jim Crow in a segregated South, educational and professional opportunities for African Americans were limited, but West found clues and inspiration for her next steps from the people and environment around her. She found solace in her imagination and in simple moments like the melodic beauty of raindrops on the tin roof of her home during rainy days. “I always made the best of my days, though, by continuing to dream and imagine that I would be doing some other kind of work when I grew up” [2]. She had deep respect and admiration for her parents, in particular for her mother, Ma Macy, whose work ethic, organizational skills, and commitment to family and community made her a great role model in West’s eyes. West says, “My mother worked part time, and one way of helping her was to learn new things and then do them too” [3]. But a labor-intensive life of working as a farmer was not a life she wanted for herself. West shares, “I remember my whole life struggling and planning and moving from one thing to another and wanting something more” [3]. When a fellow classmate, Dorothy Bates, a couple of years her senior, graduated from high school and continued on to college, she knew a college education was a possibility and aspired to do the same.

West attended high school at Dinwiddie Training School after completing the first years of her education in a single classroom school with one teacher. West earned top grades, in particular in her science and math courses, and was told by her teachers she was “college material.” But with no way to afford college, it wasn’t clear how she would achieve that goal. When she learned that the top senior in her high school was guaranteed a scholarship to college, she was motivated to earn that spot and successfully became valedictorian of her class. When it came time for her to select a college major she thought about what was most familiar to her. “Since I had been on the farm I probably was going to major in Home Economics, that’s where I knew everything,” says West [3]. Given her academic track record, however, her teachers urged her to consider math, a major they felt would be challenging and one that not all students could be successful pursuing. West reflects, “I knew that I liked the orderliness of math, the preciseness of it, the neatness of it. All of that fit my personality.” West humbly ventured to say that she was good at math “not because I was so smart, but because I worked at it” [3]. These early years reveal the role of teachers in identifying and encouraging her interest in mathematics.

College and Life-Changing Mentoring

West attended Virginia State College (now University), a Historically Black College in Ettrick, Virginia. VSU was founded in 1882, during a period when over ninety HBCUs were founded, primarily in Southern states, to provide higher educational opportunities for freed descendants of African slaves in the US [4]. Already-established colleges and universities in the US would not be legally required to integrate until 1954 with the landmark ruling of Brown v. Board of Education.1 When asked about the role of HBCUs in educating Black students West says, “At the Black colleges you have more roots there...you have more of an opportunity to relate to more people who are affecting your life” [3]. While in college, West joined the sorority Alpha Kappa Alpha (AKA), the first sorority

1 Even before Brown v. Board of Education, predominantly white institutions of higher education in the United States that had slowly opened their programs to Black and female graduate students in mathematics were not also open to hiring such faculty. In studying the history of these trailblazers—the first few Black men and women to earn PhDs in mathematics in the United States—it is easy to find mathematicians such as William Claytor or Evelyn Boyd Granville who, after postdocs elsewhere, took positions at HBCUs at least in part because predominantly white institutions refused to hire them. See Patricia Kenschaft’s article “Black men and women in mathematical research” [14], for more on the paths of many of the first Black mathematicians in the United States.
established for African American women. “You have Black colleges supporting sororities that brought Black people coming together to serve each other and the world,” West reflects, “You have the feeling that you are there to help and dig in” [3]. This model of service and community would stay with her throughout her career. Interestingly, it was a fellow VSU alum and AKA who first shared the news of West’s contributions to the development of GPS with the local press [2].

West also received life-changing mentorship from Professors John and Louise Hunter at VSU. The Hunters were remarkable in their own right. John Hunter was the third African American to earn a PhD in physics (Cornell University, BS in electrical engineering from MIT). Louise Hunter was the second African American and first African American woman to earn a doctorate from the University of Virginia, which was in the School of Education, after earning Bachelor’s and Master’s degrees in mathematics from Howard University. To West, they were the first model of a “power couple,” and their relationship foreshadowed her long-time marriage and partnership with fellow mathematician Ira West. The combination of her hard work and their sage advice and guidance helped her to successfully navigate college. After her graduation in 1952, she taught high school mathematics, and then with the encouragement of the Hunters, she returned to earn her Master of Science degree in mathematics in 1955. She was beginning to teach high school again when an application resulted in a new job offer: a position as a mathematician conducting research for the Navy.

**Becoming a Mathematician for the US Navy**

The US Naval Proving Ground in Dahlgren, now the Naval Surface Warfare Center Dahlgren Division (Dahlgren), in Virginia, became the primary site of US Naval computing in 1948 (the year West had graduated from high school and three years after the end of WWII). After an executive order issued in 1955 by President Eisenhower that banned discrimination in Federal hiring practices, the door was open to hiring African Americans in Federal agencies. While the surrounding communities were still segregated (making dormitory housing on the naval grounds a necessity for Black employees), Gladys West credits her forward-thinking manager, Ralph Niemann, for opening the doors at Dahlgren to Black mathematicians. “He had great vision and foresight, bringing in those big computers, which required lots of mathematicians and scientists. Niemann believed that recruiting women and minorities into the workforce was one way to bring in people with very strong skills who may have been overlooked elsewhere. At a time when so many were overlooking those who had looked like me, Niemann gave me a chance, the chance of a lifetime” [2].

But just because the doors were open at Dahlgren didn’t mean it was a trivial endeavor to walk through those doors and stay. “We came to Dahlgren in the early 1950s, and we were the ones integrating the Naval Proving Ground. So we knew there would be a lot of hardships and discrimination we were going to face,” recalls West [3]. Her commitment to the bigger picture of this opportunity—not just for herself but for those who would be coming after her—guided her steps. The work was challenging enough, but additionally, this was accomplished in a time of state-sanctioned segregation and a time when not many women could be found working in scientific or technical roles. West shares in her memoir, “I wanted to adapt to this fascinating, new environment and succeed as soon as I could. I knew my work was cut out for me. As a double minority like I was, at a time where opportunities for women and people who looked like me were scarce, I felt there would be a lot of folks counting on me. There were other women of color who were coming behind me, the Hunters, and my family back home, and I absolutely wanted to make them proud” [2]. Her mathematical abilities, coupled with her work ethic and mindset, positioned her to tackle the complex problems being explored at Dahlgren while navigating an environment where racial and gender barriers persisted. West became adept at converting challenges to fuel her motivation to excel during her 42-year career at Dahlgren.

**The Need for a Global Positioning System**

Dr. Gladys West’s long and distinguished career as a mathematician at Dahlgren coincided with an unprecedented period of synergy between computational mathematicians and physicists. In the wake of the Russian launch of Sputnik, the Space Race had begun, and applications involving

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2In 1971 Hunter-McDaniel Hall was built at VSU to house the science departments and was named in part for Dr. John M. Hunter who served as former head of the Physics Department, Director of the Graduate School, and Dean of the College [5].
satellites were inspiring new research frontiers. “We had some really high-powered physicists and engineers and all who would be working the equations,” recalls West [3]. It is easy to imagine that determining location was of major interest, and the development of the global positioning system became a focus. Dahlgren was interested in determining how they could use satellite data to map as precisely as possible specific locations on the planet. This required geodesy, a branch of mathematics focused on estimating the shape of the Earth. For example, when geodesic circles are introduced to students, the Earth is often modeled as a sphere, or slightly better, as an ellipsoid because of flattening at the poles, but the reality is much more complex. To achieve a high level of precision, one which today we have come to take for granted from GPS readings on handheld devices, the Naval researchers had their work cut out for them. Even over water, where we might naively think that questions of topography were simpler, they had to incorporate the effects of tides, atmospheric conditions, and imperfect satellite readings over the ocean. It was necessary to understand the expertise of researchers dealing with all of these areas of physics and engineering. “They had [hired] mathematicians because they felt that we could adapt to the other sciences,” says West [3].

These complex problems were being tackled just as computing technology was taking off. Dahlgren acquired the latest technology in computing, the Mark II and Mark III, which in the 1950s meant the Naval researchers had access to a computer which took up the size of a room, the most powerful one on the planet during that decade. By the late 1950s, IBM had built the Naval Ordnance Research Calculator (NORC) which was installed at Dahlgren. These machines were a remarkable contrast to the desktop calculators of the time. West notes, “It was completely new to us. We had never seen a big machine like that. All we had seen was a Marchant Calculator that sits on the desk” [3] which they used to compute range tables for ballistics. West had to create programs that would translate the scientific ideas into executable algorithms, coded in zeros and ones, which would be entered on punch cards by other staff members—indeed, it took a team of humans to implement this cutting edge technology once the mathematicians had done their work. “We had punch cards back in that day. We would write out what we wanted punched on the cards, and when we first began it was coded in zeros and ones. What you wrote up was given to the key-punchers to punch it out on the cards for you; they gave it to another group to verify what was punched. Then it came back to you to put it in the tray, set it up, and it goes off to the computer where a whole group of operators access your punch cards. Sometimes they’d call you to see if you wanted to watch it, to see whether it blows up or it goes. That was old time; it’s much easier now” [3]. Finding an error could mean combing through iterations of data in detail, unlike today’s debugging where a specific error message may already point the way to the correction. West delighted in recalling a moment when she was able to pinpoint an error, “Thinking about the satisfaction of cracking the problem, finding the error, and also the team being excited about it, that was a nice moment” [3].

With the advent and availability of computers, and advances that contributed to the accurate modeling of physics at the speed of light, it became feasible to look at truly hard problems, ones that required precision in measurements as well as delicate computational modeling. Understanding Satellite Altimetry (SA), which measures the time taken for an electromagnetic (radar) wave or pulse to travel from a satellite to the sea surface and back, had resulted in new opportunities in military and scientific applications. It served as one of the core disciplines in Gladys West’s mathematical efforts during the decades of the 1970s and 1980s. The results of these efforts ended up having significant military impact, but it also became an enabling factor that ushered in the current era of oceanographic studies from space.

**Tackling Positioning Data**

During a period of about a dozen years, Dr. West and her group at Dahlgren developed a successful mathematical and computational methodology to process radar altimeter observations. In a series of papers, which appeared between 1979 and 1986 (see references [6]–[9]), a set of intricate approaches and solutions were introduced and presented, which later also served as the backbone for the creation of GPS. In today’s terminology, we refer to the type of problems that arise in SA and GPS as multiscale/multiphysics problems, whose modeling requires dealing with features with large variations in scales, both temporally and spatially, informed by data that is often very noisy and not directly related to the quantities being modeled. In processing SA data, one needs to take into account the effects of the atmosphere, ocean currents and tides, and the planet’s rotation, among other features. Satellite Altimetry is based on the seemingly simple notion that a wave’s travel time reveals the distance it has travelled. While this notion is based on knowing the speed of the wave, which in this case is the speed of light, it is equally dependent on one’s ability to measure accurately how a wave interacts with the medium it travels through. The bulk of West’s work concentrated on recovering

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5Such machines at Dahlgren were the descendents of Mark I, or IBM’s Automatic Sequence Controlled Calculator. Programmed by leading researchers such as Grace Hopper, who had also come to the Navy in the 1940s, the Mark I was used by John von Neumann for military applications including the development of nuclear weapons. Hopper’s career served as inspiration; West and other naval researchers were aware of her highly significant role in the development of computing, as well as her breaking of gender barriers.
information about sea surface height measurements that were collected from a variety of satellites launched in the 1970s and 1980s, with the goal of mapping anomalies in the geoid, anomalies due to a variety of parameters and forces, notably variations in density and bathymetry, and the impact of currents and tides in the oceans. As an aside, in the case of GPS, where typically one needs to measure distances from at least four satellites to accurately locate the position of a receiver, additional mathematical complications arise. Satellites move fast enough that the laws of special relativity are at play, and, as if that is not enough to deal with, the curvilinear nature of satellite orbits introduces the need for computations that can only be handled by taking into account general relativity effects. It cannot be overemphasized how much today’s accuracy of locating our position to within a few meters owes itself to the computations that were pioneered and tested at Dahlgren many years before GPS became a reality.

As mentioned earlier, one of the applications that motivated Dr. West and her group at Dahlgren was to discover an accurate way to use SA data to describe the geoid, the equipotential surface of the Earth’s gravity field. Identifying this surface, which is a good approximation of the mean sea level in the hypothetical case of no fluid motion in the oceans, is an old problem whose formulation has been addressed by many great mathematicians including Gauss, Stokes, and Laplace. If the Earth were a perfect sphere whose density depended only on radial distance, then its gravitational potential \( \phi_1 \) has the form

\[
\phi_1 = \frac{GM}{r},
\]

where \( G \) is the universal gravitational constant, \( M \) is the total mass of the planet, and \( r \) stands for distance. Additionally, the rotation of the planet about its axis induces the centrifugal acceleration \( \Omega \times (\Omega \times \mathbf{r}) \), where \( \Omega \) and \( \mathbf{r} \) are the rotation and position vectors, respectively. The latter quantity has the potential \( \phi_2 \), that is,

\[
\Omega \times (\Omega \times \mathbf{r}) = \nabla \phi_2.
\]

It turns out that \( \phi_2 = -\frac{1}{2}r^2 \Omega^2 \), where \( r_\perp \) is the perpendicular distance to \( \Omega \) and \( \Omega = |\Omega| \). Therefore the idealized potential \( \Phi \) is just the sum of \( \phi_1 \) and \( \phi_2 \):

\[
\Phi = \frac{GM}{r} - \frac{1}{2}r_\perp^2 \Omega^2.
\]

The geoid is a level surface of \( \Phi \).

This derivation of the geoid is of course overly simplistic, a rough, first-order approximation of the real surface. In addition to Earth not being a perfect sphere, the density distribution over the planet causes anomalies whose structure is actually quite important for many applications. Figure 2 shows a sketch of a reference ellipsoid together with the geoid and its anomalies superimposed, and the various geometric quantities one must consider in a satellite measurement. The 2013 text by R. Sanso and M. Sideris [10] has a comprehensive introduction to geoid determination, ranging from various boundary-value problem formulations for the governing partial differential equations that are relevant for forward and inverse problems, to data-based techniques using satellite data that have become available since the Dahlgren efforts of the 1970s and 1980s.

Returning to Dr. West’s work, the group at Dahlgren did not have access to the computational frameworks that are available today and so they chose to address the problem of determining geoid anomalies directly by concentrating on the available satellite data at the time, data obtained from the Geodynamics Experimental Ocean Satellite (GEOS-3), SEASAT, and Geodetic Satellite (GEOSAT). These data were, on the one hand, noisy due to measurement errors that are very common in such systems, but more importantly, the data carried signatures of interactions with various atmospheric and oceanic phenomena that needed to be treated to get an accurate approximation of the mean sea level. As outlined in detail in [6] through [9], the Dahlgren team’s approach of using a Kalman filter smoothing process based on a third-order Markov process (see [9], in particular) ended up being the right tool for processing the radar altimeter data. This led to highly accurate sea level altitude measurements, with less than 10 centimeters of error.
The 1986 paper [9] contains the full story, the statistical algorithms, details of the GEOSAT data files, and the Kalman filter smoother. Among these major tools that are meant to deal with messy data, one finds an algorithm for computing the sea surface height \( N_i \) (see Figure 2) from the radar altimeter data \( h_i \): \( N_i \) is the height of the geoid at time \( t_i \), the average sea surface height above the reference ellipsoid that defines the hypothetical sea surface when the fluid is standing still, and \( h_i \) is the measured distance from the satellite to the ocean surface, which is computed from data after painstaking corrections for atmospheric effects. Figure 2 shows a schematic of the position of the satellite, the reference ellipsoid \( N_i \), and several other relevant quantities which will be introduced shortly [12].

It turns out that the determination of \( N_i \) requires first finding \( \phi_i \), the geodetic latitude, which is a solution of the equation
\[
\tan \phi_i = g(\tan \phi_i),
\] (1)
where
\[
g(x) = \frac{1}{\rho_i} \left[ z_i + \frac{ae^2 x}{\sqrt{1 - (e^2)x^2}} \right].
\] (2)
Here \((x_i, y_i, z_i)\) is the position of the satellite, \( a \) is the mean radius of Earth, taken as 6378137 meters in [9], \( \rho_i = \sqrt{x^2_i + y^2_i} \) is the distance of the satellite to the polar axis of the ellipsoid, and \( e \) is the eccentricity of the reference ellipsoid: \( e = \sqrt{2f - f^2} \) and \( f = 1/298.257223563 \).\(^6\)

Equation (1) is then viewed as a fixed point problem, and West applied an iteration algorithm that converged to the correct solution: beginning with an initial guess of \( \tan \phi_i^{(1)} = \frac{z_i}{\rho_i} \), the iteration \( \tan \phi_i^{(k+1)} = g(\tan \phi_i^{(k)}) \) is continued until the tolerance \(| \tan \phi_i^{(k+1)} - \tan \phi_i^{(k)} | \leq \tau \), where \( \tau = (1 + \tan^2 \phi_i^{(k)}) \times 10^{-9} \), is achieved. Once \( \phi_i \) is determined, one can compute the height \( H_i \) since
\[
\sin \phi_i = \frac{z_i}{\eta_i + H_i},
\]
and \( \eta_i \), the height of the reference ellipsoid above the equatorial plane, is given by
\[
\eta_i = \frac{a(1 - e^2)}{\sqrt{1 - e^2 \sin^2 \phi_i}}.
\]
The quantity \( N_i \), the desired sea surface height, is now determined from \( N_i = H_i - h_i \).

Today, accurate measurement of sea surface height is one of the key tools in atmospheric oceanography and an important metric for measuring variability in the global climate. Subsequent to the earlier work at Dahlgren, the resulting accurate gravity models developed by the National Geospatial-intelligence Agency (NGA) and others are vital for military applications and for many commercial enterprises.

**Barriers and Breakthroughs**

While advances in GPS gained widespread utility once made public, research teams worked on a wide range of projects over West’s 42 years at Dahlgren, the details of which were classified. Yet in the midst of these technological advances and the excitement of the progress being made on the work at Dahlgren, and regardless of the quality of their work, West and other Black mathematicians faced additional professional hurdles. “You can usually tell when something is happening ‘cause there’s a little feeling inside,” West recalled [3]. We may be familiar with both the explicit indignities of Jim Crow and implicit biases that persist today, but many of these hurdles had cascading effects. A simple explicit example was the way in which racism affected the researchers’ work travel. Because many hotels, especially in the South, operated as “Whites only,” Black employees simply were not provided the same professional travel opportunities; this in turn limited professional development and the chance to lead projects, not to mention the opportunity to develop deeper ties with coworkers. Unsurprisingly, given that it was the 1950s, women were less likely to be considered for supervisory positions. West shared her perspective on how she navigated these workplace challenges, “We had come from an earlier place in which things were so poor, so scarce, and so having found this good job, we weren’t going to run away from it just because someone turned their head [at us], you know?...There is a sort of downside but I guess I used it for inspiration purposes” [3].

\(^6\)For an excellent and accessible mathematical introduction to the various concepts in geodesy in general, and details about some of the notation in Figure 2, see the 1997 text by Gilbert Strang [11], Chapter 14.
Nevertheless, over time West came to lead projects analyzing satellite data, including as a project leader for GEOS-3, mentioned above, and, by the late 1970s, as a project manager for SEASAT [1]. Other work recognized by the Navy includes research in the early 1960s, when West and a small team of colleagues took advantage of the NORC and the new era of computational power to study the orbit of Pluto relative to Neptune, nicknamed “Project 29V,” for which the group received a Naval achievement award.

In her memoir, West reflects on her mentor Louise Hunter’s career, observing that Hunter, already a professor and with a Master’s degree in mathematics, was driven to pursue a doctorate degree as well, becoming the first Black woman with a graduate degree from UVA. “It seemed like she still had something to prove, and maybe she felt like she was carrying the weight of other women on her shoulders. She probably wanted to be respected equally to the others in her profession, who were almost always men” [2]. Although West herself had already served as a leader in various capacities both at and even outside of work, as a school board member and as a member of the credit union board, she returned to school for a PhD in Public Administration, having earned a Masters in this field in 1973. Through an agreement with Virginia Tech and Dahlgren, she further continued her graduate work while working full time, and after her retirement, completed her dissertation, earning her doctorate in 2000.

The Dream for the Next Generation

There is much we can learn from West’s life about pursuing dreams that may appear unattainable. Her legacy lies not only in her mathematical contributions but also in how she exemplifies life lessons such as persisting even in the face of challenges. It has been seventy years since West was first hired as a mathematician at Dahlgren, and yet many women today can still relate to the challenges of navigating mathematical environments in which they are the only one or one of only a few women present. How can this experience become the exception instead of the norm? We learn from West’s life that her talent, discipline, and sense of teamwork learned during her early days helped her to persist in mathematics. Equally important was having teachers that recognized her abilities, mentors that guided her decision making, and employers who provided opportunities by giving her a chance. In the midst of her accolades and honors in recent years, she has retained a spirit of supporting and inspiring others by publishing her memoir and sharing her story with young people. Furthering her legacy, The Dahlgren Museum established The Ira and Gladys West Scholarship Fund [13], named after her and her husband of sixty-three years, Ira West. Donations will fund college scholarships for students in King George County, a rural area with many first-generation students and home to the Dahlgren base. Her life has made navigating the path towards a successful future a little easier for the next generation.

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References

Read about the mathematical research, inspiring stories, and advice of these AMS members and more women researchers and role models at www.ams.org/women-18


Credits

The opening image and Figures 1 and 3 are courtesy of the West family.
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Ask Good Questions: Becoming a Teacher of Statistics

Stacey Hancock

Last year, Dr. Allan Rossman, Professor of Statistics at Cal Poly–San Luis Obispo, started a new blog about teaching introductory statistics called *Ask Good Questions*. In his inaugural post, he writes,

I suspect that we teachers spend too much of our most precious commodity—time—on creating presentations for students to hear and writing exposition for them to read. I think we serve our students’ learning much better by investing our time into crafting good questions that lead students to develop and deepen their understanding of what we want them to learn.

[All19]

When I first started teaching, I modeled my courses off of the courses from which I learned—lecture during class time, write on the board while students take notes, then assign readings and homework outside of class. Years later, my teaching style, especially for the introductory statistics course, has flipped. I now spend more class time on in-class activities and class discussion than lecture, and “lecture” has been moved outside of the classroom in the form of readings, videos, and short content quizzes for understanding. Though fully “flipping” a course can be a daunting task, and it may not be for everyone, bringing good questions into the classroom is one of the most powerful ways to become an effective teacher of statistics.

The Importance of Statistical Thinking

In 2016, the American Statistical Association endorsed the revised Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report [GAI16]. These six recommendations, with two new emphases in the first recommendation, provide broad guidelines for both what

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Early Career

to teach in introductory statistics courses and how to teach those courses:
1. Teach statistical thinking.
   - Teach statistics as an investigative process of problem-solving and decision-making.
   - Give students experience with multivariable thinking.
2. Focus on conceptual understanding.
3. Integrate real data with a context and purpose.
4. Foster active learning.
5. Use technology to explore concepts and analyze data.
6. Use assessments to improve and evaluate student learning.

These recommendations embody the theme “ask good questions.” (Incidentally, Allan Rossman was also on the revision committee for the GAISE College Report.) In particular, statistical thinking is built on the practice of asking good questions. Statistical thinking skills allow students to consume statistical information from a critical eye, recognizing the inherent variability in data and possible sources of bias.

Consider the following example. The Daily Star article “Eating chocolate can help you LOSE weight, shock study discovers” reports on a German study that “found that eating chocolate can reduce your waistline, lower your cholesterol and help you sleep” [Lau18]. The study randomly assigned volunteers to three diets, one of which included consuming 42 grams of dark chocolate per day. During the study, the chocolate group lost 10% more weight than the low-carb group, and this difference was statistically significant. As it turns out, however, the study was designed to demonstrate how easily bad science can turn into big headlines.

A savvy statistical thinker would ask: What was the sample size? How many hypothesis tests did they run? The study was conducted on 5 men and 11 women. One of the volunteers was dropped from the study, leaving 15 participants. Such a small sample size ensures high variability in statistics such as the mean weight lost. The number of tests? Eighteen. Even if none of the 18 null hypotheses were false, these researchers guaranteed themselves a 1 – (1 – 0.05)18 = 60% chance that at least one p-value would be less than 0.05, resulting in a false positive. Just by chance, that false positive happened to be the difference in average weight loss between the chocolate group and the low-carb group.

Journalist John Bohannon explains how he and his colleagues used small sample sizes and p-hacking to find statistically significant benefits of chocolate in data from an actual clinical trial in his Gizmodo article, “I fooled millions into thinking chocolate helps weight loss. Here’s how” [Joh15]. Bohannon and colleagues published their results in the fee-charging open access journal International Archives of Medicine—the paper was accepted for publication within 24 hours—and through a catchy press release, the paper was picked up by media outlets such as The Daily Star. (The careful reader may notice that the news article in The Daily Star was published in 2018, whereas the Gizmodo article explaining the bad science was published in 2015. Once bad science is out, it is extremely difficult to retract!)

We use this pair of articles as a case study in our introductory statistics course during the second week of the semester to underscore the importance of statistical thinking. Bohannon notes that some of the online comments on these news stories posed questions the reporters should have asked—questions about the validity of Bohannon’s credentials (the Institute of Diet and Health website that he made up), or why calories were not counted [Joh15]. By focusing our statistics courses around statistical thinking rather than a list of statistical recipes, we are cultivating those readers who critically called out the reporters.

**Correlation, Not Causation**

As another demonstration of the GAISE recommendations in practice, consider the well-known phrase, “correlation does not imply causation.” Though this phrase is often repeated, making causal conclusions from correlational relationships is deeply rooted in the human psyche. We only need to look at a few news headlines to find evidence of our natural causal thinking: “Music lessons improve kids’ brain development,” “Diet of fish ‘can prevent’ teen violence,” or “The gender pay gap is largely because of motherhood” [The06, Gab03, Cla17]. In his book Thinking, Fast and Slow, Daniel Kahneman writes,

> We easily think associatively, we think causally, but statistics requires thinking about many things at once, which is something that [the fast part of our brain] is not designed to do. [Kah11, p. 13]

At the heart of the “correlation does not imply causation” principle is multivariable thinking—thinking about the relationships between more than two variables at once. Multivariable thinking is the key to spotting confounding variables in observational data.

What thought first comes to mind when you read the following statement: “People who wear sunscreen have a higher rate of skin cancer than people who do not”? When I pose this fun fact to my statistics class, I hear a variety of surprised reactions: Is that true? How is that possible? Does sunscreen contain cancer-causing chemicals? I then ask, “what other variable are we missing here?” After a pause, someone will eventually say “sun exposure.” People wear sunscreen because they are going to be out in the sun. If you are not wearing sunscreen, chances are you aren’t planning on being exposed to the sun for an extended period of time. Sun exposure is a confounding variable—it is related to both the explanatory variable (whether one wears sunscreen) and the response variable (development of skin cancer). We cannot conclude that sunscreen causes
Recognizing confounding variables in observational studies takes practice, and this skill, I would argue, is the most valuable skill a student should gain from an introductory statistics course. Traditionally, this course did not go beyond two variables. The curriculum would start with one categorical variable, then one quantitative variable, then move to a difference in proportions, a difference in means, and finally, regression. The modern introductory statistics course, however, engages students with multivariable data sets from the beginning, exploring relationships between multiple variables through data visualization.

Getting Started in the Classroom
The two best pieces of advice that I was given as a new teacher of statistics were: (1) start small, and (2) use existing materials. Like myself, you may be used to the lecture-style classroom, and the thought of fostering active learning may seem uncomfortable and overwhelming. Start with introducing an activity into a single class. Try it out. See how it goes. Slowly, you can begin introducing more activities, adapting course materials to your own teaching style, and eventually, you will build a statistics course which exemplifies the GAISE recommendations.

When looking for existing materials, the GAISE College Report is an excellent place to start. Indeed, the six GAISE recommendations can apply to any statistics course, not just at the introductory level, and the report contains a vast number of classroom activities, projects, data sets, and assessment items. The Consortium for the Advancement of Undergraduate Statistics Education (CAUSE) website, https://www.causeweb.org, has a collection of resources for the undergraduate statistics education community from syllabi and cartoons to discussions and suggestions on how to teach simulation-based inference. When you read an interesting news article on a recent study, bookmark the article and save the bookmarks in a folder of potential case studies for your course. I also highly recommend checking out Allan Rossman’s Ask Good Questions blog [All19].

I’ll conclude with one last quote, by the American writer Alice Wellington Rollins:

The test of a good teacher is not how many questions he can ask his pupils that they will answer readily, but how many questions he inspires them to ask him which he finds it hard to answer. [Ali98, p. 339]

This quote appeared in the Journal of Education in 1898, yet it continues to be relevant today. In asking our students good questions, our ultimate goal is to transfer that skill to our students.

References

Contributing to Open Education: Why, How, and What I am Doing

Mine Dogucu

My social media timeline has reminded me that five years ago I was reading Bolstad’s Introduction to Bayesian Statistics [Bol07]. I was a graduate student back then. Now I am a professor teaching statistics and data science. I am currently

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cowriting the book Bayes Rules! An Introduction to Bayesian Modeling with R [JOD21] that is open access.¹ I share my teaching materials on my course websites (e.g., https://www.introdata.science) and keep a blog on teaching data science. Five years ago, if someone had told me that my teaching materials would be accessible to the public on the internet, I would not have believed them as I knew very little about open education. Today my personal rule is that if I have any teaching materials sitting on my computer that could potentially benefit others, I should make them publicly available. My way of contributing to open education is sharing my teaching materials in the public domain. I will share my experiences from the last five years on how I embraced open education, started to contribute to it, the tools and resources I use, and why I would recommend it, especially to early-career instructors.

Why Open Education?
My interest in open education first started with using open access books in my courses. I have mostly taught at public institutions and found textbooks, supplemental websites, and other educational materials to be unaffordable for my students. Early on I decided to assign my students open access books (e.g., OpenIntro Statistics, a collection of open access introductory statistics books) or assign them books that are accessible electronically through the school library when possible.

Beyond textbooks, I came across numerous faculty websites with teaching materials from their courses. Have you ever taught a course for the first time and a colleague gave you all their teaching materials from the time that they taught the course? The first time anyone teaches a course they may want to know how everyone else is teaching the same course, how others are ordering the topics, and what resources others are using. We may not necessarily use their materials but even knowing what they are doing can help us shape our ideas. Finding faculty websites with course materials in the public domain was beneficial for me to develop my own. Having benefited from open work of others I decided to share my work openly as well.

Open education contributions, whether it is course websites, blogs, or books, not only help learners. Sharing our teaching in the public domain can help us treat teaching as a collective action, moving us all forward together. We can learn from each other’s examples, illustrations, jokes, and analogies. We can use them, reuse them, and improve them.

Getting Out of your Comfort Zone
When we talk about doing anything out in the public as teachers or scholars, whether it is open science or open education, we often talk about the tools we need and the best practices. Even when I knew the tools and the best practices I could not get myself to share teaching materials publicly. As scholars, we are expected to share our work with the public. However, doing this early in my career was a lot harder.

What has helped me get out of my comfort zone was finding a community that appreciated my work and used my work that I shared publicly. In my case, this supportive network was R Ladies, a global organization that supports gender minorities in the R community. When I gave a talk on missing data to the R Ladies New York chapter right after finishing my PhD, it was the first time I shared everything related to my talk: the data, the code, and the presentation. A few months later, a community member messaged me asking me to remind her of the link for the materials because this was the best missing data resource for them. I wrote my PhD dissertation on missing data and I know that my slides were not necessarily the best resource on the topic but this correspondence made me realize that many missing data resources are behind a paywall and considering the way I introduced the topic, my talk might be a useful resource for this person.

An instructor does not necessarily have to share all of their teaching materials with the public. It is not a dichotomy of whether one embraces open education or not. Sharing a single figure may help other learners and instructors. For instance, Allison Horst generously shares her illustrations in the public domain such as the one in Figure 1. I have used her illustrations in my teaching multiple times.

For any readers who are trying to find the courage to get out of their comfort zone, I would suggest that you start taking one small step at a time, find a supportive community, and focus on the impact you would be making.

¹(Dogucu) The first five chapters of the book are available at https://bayesrulesbook.com

Figure 1. Derivative 5 by Allison Horst is licensed under CC BY 4.0.

Tools and Resources
I find having a single environment for all my work helps keep my workflow organized. For me, this environment is R Markdown [XAG18] and I understand that for many readers this may be LaTeX. I use R Markdown to create my course materials (presentations, handouts, exams, etc.), write my journal articles, run data analyses, build websites, write a book, and keep a blog. In the past, I used to do most of my work in a LaTeX environment; however, over the years
I switched to R Markdown mainly because I teach R Markdown in my courses and I wanted the tools I use and the tools I teach to be consistent. I can teach R Markdown even in my introductory-level classes. I should also note that I do write LaTeX equations and use LaTeX templates within R Markdown when needed so I have not given up LaTeX.

As I started contributing to open education, I had to learn about copyright and licenses. Learning about licenses has also helped me understand what I can and cannot use from others’ resources. I would suggest you check what I report here with a lawyer if needed as anything I provide is not legal advice. For my work, I use Creative Commons licenses. Depending on the work, if needed I put some limitations on how the work is shared publicly. I sometimes use the share-alike license which requires that if someone uses or edits the work, they must share it with the same license. I often use the noncommercial license which prevents commercial use of my work. One can also use a combination of these licenses. For instance, our aforementioned Bayesian book is shared with Creative Commons Attribution-NonCommercial-No Derivatives 4.0 International License.

Sharing my teaching beyond my classroom has made me more considerate of accessibility, a topic that I am still learning about. I started using screen readers to test whether my materials are readable by screen readers. A screen reader reads what is on the screen out loud. This is important technology for the visually impaired. Using a screen reader has made me aware of writing alternate text for images so that the alternate text of the image can be read by screen readers. I also started using colors from the Okabe and Ito color-palette to make figures that are friendly to colorblind people [O102]. I still have to learn a lot about accessibility.

I also found social media to be a beneficial resource in finding open education resources and disseminating my work, as well as meeting other statisticians and data science educators. I connected with many people from around the world on Twitter. For instance, I met one of my coauthors on Twitter and this connection has resulted in coauthorship of our Bayesian book.

More importantly, if all my teaching materials were sitting on my computer it would have only helped my students. The internet provides a medium for us to publish work that does not necessarily fall under the traditional publication umbrella such as journal or book publication. For instance, I started my blog [https://www.DataPedagogy.com](https://www.DataPedagogy.com) over the summer with the intention that I would share my thoughts—longer than a tweet, shorter than a manuscript—on data, pedagogy, and data pedagogy. What I share on this blog would not have made it to the public otherwise. Even for more traditional publications such as books, one can publish a work in progress or update published books online which can make the online version of the book stay more current than the hard-copy book.

Sharing work publicly has also provided me with newer opportunities and introduced me to newer networks. Due to my blog, I was invited to write this piece for the AMS Notices and similarly a blog post I wrote is to be published in the American Statistical Association’s AMSTAT News. This is my first connection ever to the AMS network and I feel fortunate about that. Through this connection, I even got to learn about one more collection of open education resources at AMS Open Math Notes.

In the last five years I have changed as a teacher and a scholar in many ways. Open education has been a big part of this change. I have benefited from both using open education resources and contributing my own. I would recommend contributing to open education to anyone, especially to early-career professionals who are looking for ways to share their work and to connect with other educators.

**What Is in it for the Instructor?**

Sharing my teaching openly has benefited me as well. In academia the impact of someone’s work is usually measured by the number of publications and citations. Contributing to open education has taught me that I can define my own impact of my work which helps with my well-being regardless of how academia measures my impact. My websites have been visited by thousands of people from more than 100 countries. I do not think I would have reached the same number of people with only journal publications as a junior faculty. I do not think even physical copies of our book when published would make it to 100 countries.

More importantly, if all my teaching materials were sitting on my computer it would have only helped my students. The internet provides a medium for us to publish work that does not necessarily fall under the traditional publication umbrella such as journal or book publication. For instance, I started my blog [https://www.DataPedagogy.com](https://www.DataPedagogy.com) over the summer with the intention that I would share my thoughts—longer than a tweet, shorter than a manuscript—on data, pedagogy, and data pedagogy. What I share on this blog would not have made it to the public otherwise. Even for more traditional publications such as books, one can publish a work in progress or update published books online which can make the online version of the book stay more current than the hard-copy book.

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In the last five years I have changed as a teacher and a scholar in many ways. Open education has been a big part of this change. I have benefited from both using open education resources and contributing my own. I would recommend contributing to open education to anyone, especially to early-career professionals who are looking for ways to share their work and to connect with other educators.

**References**


**Credits**

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Creating an Actuarial Option for Students is Within your Reach

Susan Staples

Introduction

There has been a clear trend in recent years towards an emphasis on career opportunities at colleges and universities in the US. According to a 2019 US News & World Report article, “Prospective students and parents increasingly assess schools for how well they help undergrads gear up for the job market,” and “over 85% of college freshmen reported that getting a ‘better job’ was a very important factor in their decision to go to college, compared with about 72% a decade earlier” [1]. Accordingly, that magazine, as well as The Princeton Review, now has metrics to rank the career placement of alumni and career centers of schools.

For classically trained mathematicians such as myself, designing or modifying a mathematics curriculum that can accommodate such a shift in emphasis may seem bewildering and daunting, at least at the outset. In this article, I briefly describe the implementation of a concentration in actuarial science within a thoroughly mainstream mathematics curriculum. Such an adaptation requires the addition of less than a handful of new courses and therefore preserves the overall tenor of the original curriculum. At the same time, an actuarial concentration prepares students for a possible career that currently enjoys wide availability, good earning potential, high levels of job satisfaction, and favorable market projections. In addition, the few new courses offer instructors the opportunity to develop applications that enliven many of the calculus and probability topics which students often find boring or of little value. Above all, I will attempt to demonstrate that such a concentration is already within reach for most schools. In fact, the industry itself provides many useful resources that schools may tap into, which I will discuss also.

Actuarial Science and Career Opportunities

Actuaries do not make their living by consulting moldy, hundred-year-old mortality tables. They study and quantify risk by building and analyzing mathematical models, and are at work in many sectors of the economy, with employers from insurance companies, financial institutions, consulting firms, the retirement industry, and government agencies. Actuarial jobs are routinely listed among those with excellent earning potential and high levels of job satisfaction. Moreover, the Bureau of Labor Statistics predicts that “employment of actuaries will be 20% higher in 2028 than it was in 2018, a job growth rate that is much faster than the 5% average among all occupations” [2].

The development of an actuary’s career, which proceeds from the first credentialed status of “associate” and later to the status of a “fellow” of an actuarial society, can be a lengthy process. Completion of the associate level includes seven exams, while the fellow level consists of a total of 10 or 11 exams depending on the accreditation track pursued. However, the passing of the first two exams, usually referred to as Exam P (for probability) and Exam FM (for financial mathematics), is sufficient in most cases to launch a career. I will discuss the mathematics involved in these two preliminary exams below. Further information related to the actuarial societies will be provided in the resources section.

Mathematics Courses in a Basic Actuarial Curriculum

Statistics, probability, and interest theory courses are central to the design of an introductory actuarial curriculum. The aforementioned Exam P and Exam FM are concerned with the last two of these mathematical areas; coverage of the topics in these two exams form the core mathematics of an introductory actuarial program. A well-designed actuarial degree plan also includes classes in economics, finance, and accounting in addition to the key mathematics courses.

Most departments of mathematics already have a first class on probability that covers both discrete and continuous settings. Depending on the scope and depth of the existing probability class in the curriculum, either one probability class alone or a two-semester sequence in probability would treat all the necessary topics in probability. For this reason, I will focus my comments on interest theory, which likely needs to be added to the existing mathematics curriculum. Again, some departments cover the interest theory syllabus in a one-semester three or four credit class, while others may use two quarters or two semesters to complete the curriculum. (See the resources section for more information on the course syllabi.)

Interest theory is a standard part of financial mathematics, and a course on the subject usually assumes a background of two semesters of single-variable calculus. The AMS/MAA Textbooks series publishes one of the most popular textbooks on interest theory: Mathematical Interest Theory by Vaaler, Harper, and Daniel. A significant number of financial models and algorithms are developed in such a course and mastery of these models and their associated formulas is necessary for Exam FM. Problems are multistep in set up, exacting in detail, and often involve clever observations. Of course, these clever observations themselves can be classified and learned with practice. The flavor of teaching interest theory may be likened to that of teaching second semester calculus, especially integration techniques, sequences, and series.
Not only do interest theory problems provide applications of calculus to finance, but they also offer opportunities for students to hone their problem-solving skills on integration, and sequences and series. Here are a few illustrative examples. An early concept discussed in the course is the notion of the effective rate of interest over a time period. The effective rate of interest is constant in the setting of compound interest as well as four other classical models. However, the effective rate of interest is a function that varies with time in a number of other standard models, including simple interest. Launching from an understanding of the effective rate of interest and taking an appropriate limit, a notion akin to an instantaneous rate of interest called the force of interest arises. Integral calculus is employed to build a model for how money accumulates in settings where the force of interest is not constant. Many other models and examples throughout the course depend on sequence and series techniques to solve problems. Students are often surprised to learn that there are practical uses for series and that these examples move far beyond a mere determination of whether or not a series diverges or converges. A later topic in the course relies on utilizing Taylor series to determine how to build a portfolio that is so-called “immunized” or protected against fluctuations in interest rates. Careful analysis of the second-degree Taylor polynomial fit generated by the immunization model leads to a remarkable result! In special settings the prescribed fit protects the portfolio for all interest rate fluctuations and not merely for small changes in rates. With this model the details that undergird the theory of convergence of Taylor series come to life in an interesting application.

Resources
There are two actuarial societies in the United States, the Society of Actuaries, SOA (https://www.soa.org/), and the Casualty Actuarial Society, CAS (https://www.casact.org). Both the SOA and CAS offer support to faculty members in developing an actuarial curriculum. Specific curricular goals including syllabi and suggested texts for these courses are provided by the SOA at https://www.soa.org/education/exam-req/edu-asa-req/. The syllabi featured on the website are remarkably detailed, containing information from outlines of overall objectives all the way down to individual section numbers of recommended textbooks. These tips can be very useful, particularly for a new program. The websites also publish old exams and exam study materials. The problems on the exams are challenging for students and national passing rates are low. Therefore, I recommend that instructors familiarize themselves with these exam materials, especially as they develop a new program.

The actuarial field also features a number of organizations that support students and enhance diversity initiatives. The CAS welcomes student members to join CAS Student Central (https://www.casstudentcentral.org/) and the SOA facilitates the Candidate Connect community for aspiring actuaries (https://www.soa.org/future-actuaries/soa-candidate-connect-features/). Several other groups offer scholarship and mentoring opportunities focused on increasing diversity in the actuarial field. The International Association of Black Actuaries (https://www.blackactuaries.org/), the Organization of Latino Actuaries (https://www.latinoactuaries.org/), and the Gamma Iota Sigma Solutions for Authenticity Inclusion and Diversity (https://www.gammaiotasigma.org/gammasaid/) are three such resources. To put in perspective the level of underrepresentation of minorities in the actuarial field here are some quick estimates. A representative from IABA offers a rough approximation that historically 1% of credentialed actuaries are Black, and representatives from OLA provide the estimate that over the last ten years about 2% of new members to the CAS or SOA are Latinx. Most actuaries enter the profession after an undergraduate degree and continue their studies on the job instead of pursuing graduate school. Thus, an actuarial career provides a more affordable entry point into a high-profile profession for many first-generation college students.

If starting an actuarial option at your university has piqued your interest, the SOA provides a checklist of requirements for a program to be formally listed as a University or College with an Actuarial Program (UCAP) (https://www.soa.org/institutions/). The community of actuarial educators welcomes newcomers and the UCAP listing includes a contact person for each actuarial program. You can also meet other actuarial educators online in the MAA Connect Actuarial Educators group. Faculty members creating an actuarial program can also find support from two programs sponsored by the societies: the CAS University Liaison Program and the SOA University Support Actuary Program.

Concluding Remarks
Directing the TCU actuarial program for twenty years has been a most rewarding experience for me. Dozens of talented TCU graduates have well-established careers in the actuarial industry and a few have gone on to graduate studies in related areas. At our annual actuarial career fair, a number of alumni actively recruit our students. I wish the same success to you, if you venture down the road to create an actuarial option at your own institution.

References
Early Career

Trust Your Instincts When Opportunity Arises

Noah Giansiracusa

At several key points in my education and career I’ve felt a tension between what I thought I should be doing and what I really wanted to be doing professionally—but a lesson I’ve learned over and over is that what you should be doing professionally is whatever you really want to be doing. I’ll try to convince you of this notion throughout this column, but whether you should follow this advice perhaps depends on whether you want a conventional career or are open to a potential career adventure.

Without exception, whenever I have thrown caution to the wind and followed my passions rather than following more traditional paths, I believe I have come out of it more successful, not to mention happier, than I would have otherwise. The first couple of times this happened I thought it was just serendipity, that the random thing I did against my better judgment somehow, by pure chance, turned out to be professionally beneficial beyond anything I could have anticipated. But it’s happened so many times now that I don’t think it’s simply luck. Here are some overarching reasons why I now believe your career will grow more by trusting your instincts and following your passions than by just doing what you think the profession expects of you:

1. You will be more motivated to work hard, and the product of your work will likely be more creative and inspired, if you are working on something you are passionate about.
2. Your applications (for jobs, promotions, grants, etc.) will stand out more and be more memorable if your career path doesn’t look identical to everyone else’s.
3. Networking is one of the most important activities you can do at any career stage, and unusual opportunities almost always end up introducing you to people you otherwise would not have met.

I think the easiest way to convince you of this is simply to walk you through various junctures in my life where I did what I wanted to even though at the time I thought this was the “wrong” thing to do.

After one year of grad school I wasn’t sure if it, or math more generally, was really for me. Over that first summer rather than studying for exams or doing a directed reading with a potential adviser, as I “should” have done, I decided to test the waters outside of academia and lay the first steps for a possible escape route. I looked for an internship, and ultimately found one that appealed to me on a variety of levels—a public policy fellowship at the National Academies in DC—and went for it even though mathematically this would set me back a whole summer. As grad school progressed I became more interested in math and open to academia, so when graduation came around I put this idea of a career in public policy out of mind and did the usual thing of applying for postdocs. To my surprise, I landed one that was far more prestigious than anything I ever expected or felt I had a chance at. To be quite honest, I was puzzled by why I got it.

A year into that postdoc, in a conversation with my mentor, Bernd Sturmfels, over pizza and beer he encouraged me to try doing some applied and interdisciplinary math, but I pushed back and said I only have a limited time as a postdoc to establish myself professionally so feel I should keep focusing on the pure math I had done in grad school that got me this postdoc. Sturmfels then said, bluntly, that he read dozens and dozens of applications from algebraic geometers that all looked the same, competent and excited to share the latest theorem they had proven, but the reason he selected my application over the others wasn’t because my thesis was better than theirs (it wasn’t!) nor that I was a stronger mathematician overall (I certainly wasn’t!), it was because in grad school I had spent a summer doing a public policy fellowship in DC and that showed him that I was willing to take chances professionally and think outside the box and connect with people in other fields.

I was shocked when he told me this, that the activity I thought set me back in math was actually what set me apart from other mathematicians. This casual comment that my postdoc mentor almost certainly doesn’t remember making has been one of the most influential insights of my career—whether consciously or not, it has been at the heart of almost every professional decision I have made since that moment. (I don’t think I’ve ever told you this before, Bernd, so now is my chance: thank you for your wisdom and candor.)

Later in my postdoc, my research was hitting some snags—progress was grinding to a halt and relationships with collaborators were starting to fray. I felt that I should persever and push through this rough patch, but I was having trouble getting myself to do so. Meanwhile, an
intriguing opportunity arose. My brother is also a mathematician, a topologist, and we always thought it would be fun to work together, but we’re not really in the same field so we hadn’t tried to do so yet. Tropical geometry was gaining a lot of attention at the time and seemed like a setting where we could perhaps combine our algebro-geometric and topological backgrounds, but neither of us knew anything about tropical geometry so we’d have to start by learning the basics first. My intuition said this would be fun and so satisfying to finally work with my mathematician brother, but the grown-up voice in my head said this is a distraction from the actual research I should be doing. Thankfully I ignored that latter voice.

Working with my brother was indeed very fun and refreshing! We got to learn a new area together, and we both felt so uninhibited in this work because it wasn’t our “serious” research; I think this helped us produce more creative ideas than either of us would have been able to in the fields we were actually trained in. The first paper we wrote together, just for a fun diversion from actual work, is the best paper I’ve written to date. Not only that, but we’ve been able to continue working in this area for years now and we’ve both met so many mathematicians we didn’t previously know. Now I realize I was misunderstanding how the balance works: taking up a new project/collaboration/field might take some time away from your other work, but don’t worry about that, you’ll always find time for the work you need to do—meanwhile you’ll double the size of your network, the number of math papers you can read, the number of areas you can work in, etc. You gain far more than you lose. But only do this if you really want to. If your inner voice says to try something new and different in your research program, then you should listen and do it.

At my first tenure-track job the university ran a first-year seminar program where faculty in any department can teach a class on pretty much any topic they propose. And I mean anything: one colleague in another department taught a seminar on bee-keeping, not because that was necessarily advisable) path through multiple academic jobs in which each time I felt I had landed in paradise and would never move; then within two or three years external circumstances led me to walk away from what I believed was my one true dream job. But suffice it to say every new job you’ll ever consider, whether it’s in academia or not, whether it’s a step up or a step down in prestige, will provide you with a wealth of new opportunities. And if you trust your instincts and pursue the opportunities that really appeal to you—especially the ones that you deep down yearn for, knowing you’d love to do them even though some of your mentors and peers may judge you skeptical for doing so—then you’ll find new happiness and success in your job and never look back with regrets.

I’m an algebraic geometer with a PhD thesis on moduli spaces (advised by a mentor who has patiently supported and believed in me despite what must have looked from the outside like a sequence of self-sabotaging career moves: thank you, Dan Abramovich, for your superhuman devotion to math and mentorship) yet I now teach data science at a business school. It’s not where I thought I’d end up, but somehow I felt this was a really fun teaching opportunity to math and mentorship (mostly because I wanted to read the fascinating book *Math on Trial* by Leila Schneps and her daughter Coralie Colmez, and figured this would give me the incentive to do so). Not only was it really fun, as I had expected, but so many good things came from this that I didn’t expect at all! I got to meet Leila in person and chat with her about the amazing work she’s been doing as an expert witness in court cases involving statistics and she convinced me to dig deeper into the field. (I don’t think I’ve told you this before: thank you, Leila, for your encouragement and infectious curiosity.) This nonstandard math class even had a direct, positive impact on my regular algebraic geometry career: when applying for NSF grants you need to include outreach, and it’s often hard to think of outreach that is suitable for pure math projects, but I was able to develop this math and law class into an outreach program that I include in my grant proposals and, for better or worse, on more than one occasion that’s been the highest-rated part of the proposal!

This math and law topic soon spread from the classroom and outreach to my actual research and led to new papers and excellent collaborations with students. I even gave a colloquium talk to a room that while mostly comprising mathematicians included one law professor too. The law professor slept through most of my talk, but seeing him there in the audience was still one of the highlights of my career. He and I had a nice conversation afterwards, and I remembered another piece of sage advice Sturmfels imparted during my postdoc: don’t do interdisciplinary math thinking you’ll solve hard problems in other fields and revolutionize distant disciplines by bringing in powerful math, but do talk to scholars in other fields and listen to their ideas and learn about the problems that interest them, for even just having such conversations is a big success.

I’ll skip over the details of my circuitous (and not necessarily advisable) path through multiple academic jobs in which each time I felt I had landed in paradise and would never move; then within two or three years external circumstances led me to walk away from what I believed was my one true dream job. But suffice it to say every new job you’ll ever consider, whether it’s in academia or not, whether it’s a step up or a step down in prestige, will provide you with a wealth of new opportunities. And if you trust your instincts and pursue the opportunities that really appeal to you—especially the ones that you deep down yearn for, knowing you’d love to do them even though some of your mentors and peers may judge you skeptical for doing so—then you’ll find new happiness and success in your job and never look back with regrets.
but I love it! New colleagues to meet, new topics to learn and teach, new opportunities around every corner.

My plan for my first year here was to lay low and learn the ropes. Best not to overextend myself, nor to stand out any more than an algebraic geometer already does in a business school. At least that’s what the grown-up voice in my head said. By now you can probably guess what happened when I got an announcement from the dean’s office that there’s a university-wide curricular reform endeavor underway and proposals for new interdisciplinary courses are sought from all departments, especially courses that bridge the divide between the business departments and the arts & sciences departments. Of course I wisely recognized that I should leave this for others who are more qualified and prepared than me. Even though it’s something I thought I’d really enjoy, I resisted temptation and focused on what I had been trained to do and what I believed I was hired to do. Wait, have you been paying attention at all?! Of course I once again threw caution to the wind and dove in head first.

My chair and dean were very supportive: they sat me down and explained why I shouldn’t get distracted from my primary obligations nor lose sight of my research-oriented tenure expectations, but that if this is something I really want to do then I have their blessings. I joined an incredible group of colleagues developing a course on discerning Fact from Fiction. It was a great way for a new faculty member like me to meet faculty outside my home department. And it was really stimulating working on the course proposal with them. But when the proposal got approved, nobody on the team had time for it—some were busy chairing their departments, others were serving on time-consuming committees, and others were already doing overload teaching from other approved course proposals. So, I ignored the voice of caution in my head and convinced my chair and dean and proposal team that I was up for the task, even though I most certainly had not convinced myself of that. And as ill-conceived as this was on my part, it’s been one of the best decisions of my career (along with all the other poor decisions I’ve made in the past).

First of all, prepping for the class meant I got to read lots of really interesting books that I otherwise would never have found the time to read. Even though I didn’t feel too qualified to teach the material at first, I found it so fascinating that it was almost effortless to speak about it in class each time. Since this section of the class was offered in my department of mathematical sciences, I tailored the overall Facts and Fictions class to be about Truth and Lies in Data. Even though this was a class meant mostly for first-year students, teaching it helped me feel more like an actual data science professor instead of just a math professor faking his way into data science.

The class was taught in Spring 2020, so the pandemic turned it remote halfway through. But since this was a nonstandard math class it wasn’t the prerequisite for anything so I felt free to cover pretty much whatever I wanted, totally unconstrained by curricular norms—so as soon as the lockdown struck, we focused the class almost entirely on the pandemic. Every day something data-related about COVID-19 was in the news so the class became an outlet to discuss and try to make sense of the whirlwind of coronavirus chaos together. I learned so much from the class and from my students. Afterwards my university’s alumni magazine included this class in an article about teaching during the pandemic (https://www.bentley.edu/magazine/case-study-covid). And while I’m mostly used to teaching math classes with, say, 25–35 students, this class had only 10 students and was mostly discussion-based—which meant not only was it rather refreshing, but it was less additional work to teach than I expected.

I got so absorbed in the material from that class that afterwards I decided to try writing a general audience book on the material. I know I should be focusing on writing research papers instead to help with my tenure case and things like that, but at this point in my life I’ve learned to trust that inner voice (the fun, ambitious, enthusiastic one, not the overly cautious grown-up one) and to follow my professional passions with abandon. Right now I’m very happily teaching the second iteration of this Truth and Lies in Data class. Writing the book has been more engaging and exciting than I ever imagined—and the synergy between teaching the class and writing the book has been very rewarding. I’ve even enjoyed learning about the publishing industry (the book, titled How Algorithms Create and Prevent Fake News, is due out soon). Since the class is being taught remotely anyway due to the pandemic, I decided to post condensed versions of the lectures publicly online on my YouTube channel ([https://www.youtube.com/channel/UC2WFIBiNxDialcxp2JddNnA](https://www.youtube.com/channel/UC2WFIBiNxDialcxp2JddNnA)) which has helped me develop a public-facing side of my job, which is something I always wanted to do but, well, never thought I really should because it’s not the standard path.

But now I know: do what you want to do, not what you should do—you’ll do better work this way. Don’t try to guess what people reading your applications (jobs, promotions, grants, etc.) will want to see—do the things you are passionate about and convince the reader that these are worthwhile endeavors. Don’t look around at your peers to see what career paths they have and try to force yourself to have the same one—doing this will make you look like everyone else rather than the unique person that you truly are.

So much of who I am as a mathematician, and who I am as a person, stems from the random opportunities that have arisen along the way. Of all these opportunities, teaching a nonstandard math class is one of the least difficult to get into (it’s a lot easier than changing jobs, believe me!) and yet potentially one of the most rewarding, personally and professionally.

You’ll be surprised and amazed at the places it will take you and the people you’ll meet along the way. And that
brings me to my final expression of gratitude in this article: thank you, Angela Gibney, for editing this AMS column and providing the guidance and support to early-career mathematicians that I have been fortunate enough to receive from you personally for so many years now.

When Life Gives You Lemons, Make Mathematicians

Kira Adaricheva, Ben Brubaker, Pat Devlin, Steven J. Miller, Vic Reiner, Alexandra Seceleanu, Adam Sheffer, and Yunus Zeytuncu

This is a happy story during difficult times. It is a story about how the pandemic led to something good. It also describes a new type of undergraduate summer program. We wish to start a discussion about this new approach, how it could be improved, and whether more people should pursue it. Please reach out to the authors to learn more about their experience and potential opportunities to be involved with such programs.

Our story begins in the Spring of 2020, when the pandemic made it clear that summer programs wouldn’t run as usual. Many students found themselves stuck at home with nothing to do during the summer. Some programs switched to running remotely and tried to help by accepting more participants than usual (including several programs organized by the authors). However, this support was negligible compared to the number of students who were stuck.

After various Zoom and email discussions, we created the Polymath REU program (https://geometrynyc.wixsite.com/polymathreu). This is an undergraduate-level version of the original Polymath program (https://en.wikipedia.org/wiki/Polymath_Project). The goal of the original Polymath project is to solve problems by forming large-scale collaborations between mathematicians. The collaborative work is done on a dedicated wiki site. This project involves long-standing open problems and some of the world’s leading mathematicians. The new program is similar but aimed at undergraduates. It includes modest open problems that do not require significant background. It also involves research mentoring by experts.

The Polymath REU consisted of 12 research projects from a wide variety of mathematical fields. There were 27 research mentors and over 300 undergraduates. The participants came from a wide variety of colleges and universities. There were many participants from top American institutions, from a variety of American institutions we were not familiar with, and from institutions in Mexico, Egypt, the UK, Romania, Israel, Denmark, India, Canada, Portugal, and more.

The program was a success. The exit surveys were quite positive (see Figure 1) and we expect at least 14 resulting manuscripts. We believe that many of these manuscripts will be published in nonundergraduate research journals. An up-to-date status of the manuscripts can be found on the program’s website. Results have already been presented in multiple conferences. After the program ended, some participants started non-Polymath projects with their mentors.

We also had the wonderful pleasure of discovering exceptional students who were not accepted to any standard

Credits

Author photo is courtesy of Noah Giansiracusa.

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Exit Survey Results by Question (1-disagree, 5-agree)

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<th>3</th>
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<td>60</td>
<td>40</td>
<td>20</td>
<td>0</td>
<td>80%</td>
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<tr>
<td>I’m glad I did this program</td>
<td>80</td>
<td>60</td>
<td>40</td>
<td>20</td>
<td>0</td>
<td>80%</td>
</tr>
<tr>
<td>I’d recommend a large-scale REU program like this to a friend</td>
<td>80</td>
<td>60</td>
<td>40</td>
<td>20</td>
<td>0</td>
<td>80%</td>
</tr>
</tbody>
</table>

Figure 1. Some results from the exit survey. The y-axis is the percentage of people who marked that answer.
REU. Some because they were not US citizens and others because they did not have much mathematical background. We found several candidates worth considering for admission to any reputable PhD program who might have been overlooked without the support gained from an experience of this sort. We also recruited students from adjacent fields such as computer science, physics, and mathematics education and utilized their skills in meaningful ways. We believe that the program is likely to have a big impact on the lives of all these students. It exposed their exceptional abilities and will open doors for future opportunities.

The program also created a community. As you might expect, groups of students became close and worked together, discussed grad school applications, mathematical riddles, and so on. We were also not surprised to see participants playing online board games together. We were surprised about a variety of other participant activities: a group for sharing recipes and pictures of dishes they cooked, a group for sharing and discussing music that they like, attempts to practice languages together, and more. These discussions and activities were open to everyone in the program.

It was challenging organizing a new type of program under considerable time pressure that gave us little notice to plan it, and we have learned a lot. Next summer we will change many things, relying on the experience gained. We now share both the good and the bad from this past summer, and welcome suggestions from readers.

The Polymath REU is an Inclusive Program

We accepted all applicants who took a proofs class and had a supportive letter from a math professor. Out of 352 applicants, 303 were accepted. We expected some participants not to be active in the program, but wanted to give everyone a chance. Some participants found it difficult to make progress with the research but contributed in other ways (writing, running the website, or even organizing social events). Rather than setting an expectation for all participants to contribute new findings, we encouraged students to join the program to learn more math and get an idea about what research looks like.

The above raises an issue: who gets to have their name on any published papers? We decided that anyone who spends a reasonable amount of time on a project gets to have their name included. Imitating the original Polymath Project, some of our papers use a pen name such as “Polly Matthews Jr.” (with a list of the authors below). This is not a perfect solution, since some participants contributed considerably more than others. As expected, some projects were pushed forward by a small group of exceptional participants.

We made the above issue clear when first advertising the program, so as not to mislead anyone. Participants who made significant contributions can ask for a letter of recommendation from a mentor. Thus, when applying to grad school or a job, students who contributed more will have stronger support.

The following is our rough and possibly inaccurate estimate:

- About a third of the participants were inactive.
- About a third of the participants were actively participating in the research work.
- The last third were following along but not actively contributing to the research.

Comments in the exit surveys complained about many participants not contributing. This happened for a variety of reasons; for example, some students had to work significantly more hours than they expected to help their family (some students sadly withdrew from the program due to such considerations). Still, we feel our approach remains the best solution, or at least a good start: everyone has the opportunity to participate, it is up to them to choose to do so. While this problem is not unique to our Polymath program, it is something we will consider for future iterations. What can we do, especially at the start of the summer, to increase those who actively participate?

The following remarks from the exit surveys give a flavor for what different participants enjoyed:

- “My favorite part was reading the literature and collecting data that would support or contradict our conjectures.”
- “My favorite part were the people in the program.”
- “This was my first research experience so it was also very nice to see how research is done and to contribute some results to it.”
- “While not being a big contributor to the group, I had fun learning what I could and challenging myself with the exercises.”
- “I really enjoyed the freedom I had to research what interested me the most within my project.”
- “My favorite part was the presentation.”

The Structure of the Program

The program was split into projects, and each participant belonged to just one (though some mentors ran two unrelated projects). We had groups in number theory, combinatorics, complex analysis, convex geometries, commutative algebra, and representation theory. Each project had a main mentor, who not only was an active researcher in the relevant field but also had previous mentoring experience. Most main mentors were also organizers of standard REU programs (we thus wanted a program that mentors could run while still doing previous commitments). Each project included additional mentors, who were mostly postdocs and graduate students. In addition to helping manage the time commitments for the senior mentors, this allowed us to help train some of our junior colleagues in mentoring research.

The program ran for eight weeks during June–August of 2020. The first week of the program was dedicated to introducing the various projects, via Zoom talks, introductory documents, and more. Each participant then ranked their
preferences. This worked better than expected. With a single exception, all participants joined one of their two top choices, and each group had between 19 and 30 students.

Imitating the original Polymath Project, we created a wiki site for the program, but ultimately this did not work well. While the wiki served as a central repository for posting resources such as papers and videos, most of the technical work was done through Overleaf and Zoom.

The more interesting development was suggested by the students—Discord. A Discord server consists of chat rooms, voice channels, and more. Each project had forums for work purposes (usually multiple rooms for different parts of the project). There were also chat rooms for social activities, for the various groups (games, music, food, and so on), chat rooms open only to mentors, and so on. The students used voice channels, mostly to chat with others who happen to be online and schedule working meetings. In the exit survey, the participants marked that they found Discord to be the most helpful resource, and then their peers and the mentors. The wiki page was marked as significantly less helpful than anything else.

We were worried about toxic situations in the Discord chat rooms, such as students disrespecting or discouraging others. We appointed a group of student volunteers to be chat room moderators. It was also rare to have a time when none of the 27 mentors were connected. This worked! We only had a single falling out among students, due to an impatient student. No moderator ever stumbled upon a serious incident and our anonymous complaints form never contained such a complaint.

The program ended with a two-day Zoom conference, where each group presented their results. We let participants invite family, friends, and teachers. We encouraged each group to have multiple speakers, to allow many students to practice their presentation skills. We also encouraged the participants to present their results in other conferences. These include local conferences about undergraduate research and conferences such as the Joint Mathematics Meetings.

**An Anxiety Issue**

Impostor syndrome was a main concern throughout the program. It was caused by students coming from different backgrounds, by some students having previous familiarity with the topic of their project, by the social media phenomenon in which others’ lives or skills seem rosier when presented online, and more. We tried a variety of approaches and will continue to think about how to address this issue.

During the first week of the program, we ran Zoom meetings that introduced the projects. We wanted students to get to know the mentors and the problems before committing to one. On the third day we had a welcoming session, where we mentioned that it’s standard not to understand many parts of a math talk. This was clearly a moment of big relief for many participants. Another such moment happened when we discussed impostor syndrome. We should have stated both at the beginning of the first day.

Every project started with learning the relevant material and thinking about exercises. Some students were having difficulties learning the material but were too embarrassed to tell their mentors. The number of active participants increased when mentors ran additional Zoom sessions to support participants who were struggling.

**The Future**

We suggest the name MOO-REU for a Massive Online Open REU. The Polymath REU program is a MOO-REU, but one can imagine different types of MOO-REU programs. One question is whether there exists a more effective format than the one we used, or perhaps several as one size rarely fits all perfectly.

MOO-REUs are clearly useful during a pandemic, when many students are stuck at home with nothing to do. Are MOO-REUs also useful in more standard times? We believe that the answer is yes, for several reasons:

- The number of available spots in math REUs is currently significantly smaller than the number of qualified applicants. Some people claim that it is easier to get accepted to a PhD program than to an REU program. Even if this is an exaggeration, the current situation seems unhealthy.
- We believe that the most important aspect of MOO-REUs is providing opportunities to students who usually do not get into REUs. In particular, such programs provide opportunities to international students and to students who come from a college and high school that do not have many mathematical activities. This lack of opportunity to demonstrate their abilities makes it unlikely for them to get accepted to a standard REU.
- Participating in a MOO-REU requires a smaller commitment from the participants. It is an opportunity for students who cannot commit to a full-time program. It is also an opportunity for students who are not sure if they should do math research. Such students can be partially active in the program, learn more math, and get a first impression about what research looks like.
- It is also valuable for mentors, providing them an opportunity to help create a research program.

We believe the MOO-REU model can be exported to many theoretical sciences; it would be wonderful if a version could be created for lab sciences, as many of the students who lost summer opportunities hailed from there.

Participants also believe that the program should continue, as the comments below from the exit surveys indicate:

- “I’d like to know when the next one is.”
- “I would really hope this happens again in the future!”
Early Career

• “I really would want REU like this to happen again next summer!”
• “This program was the highlight of my summer so thank you for this cool opportunity : )”
• “This made my summer 10 times better than it was going to be.”

Our Main Conclusion: Make Lemonade
The most important conclusion from this story might not be about the Polymath REU or even about math. It is that one can make a big impact in a difficult situation without having many resources. Sometimes, all you need are good intentions and time.

We were surprised by how simple it was to create a new kind of large-scale program. Tools such as Zoom, MediaWiki, and Discord allowed us to quickly build something large and complicated. The important thing was to have people who want to help and are willing to spend time on the program.

The program owes much of its success to the many dedicated mentors who supported the students. It is unfair that only the main mentors have their names on this article and in some other places. We thus wish to include the names of all other mentors here. This wouldn’t have happened without them: Madina Bolat (University of Illinois, Urbana-Champaign), Galen Dorpalen-Barry (University of Minnesota), Ben Drabkin (University of Nebraska-Lincoln), Nóra Frankl (London School of Economics), Claire Frechette (University of Minnesota), Gent Gjonbalaj (Tufts University), Elizabeth Kelley (University of Minnesota), Surya Mathialagan (MIT), Erin Meger (Université du Québec à Montréal), Clayton Mizgerd (Williams College), Jonathan Passant (University of Rochester), Fei Peng (Carnegie Mellon University), Tudor Popescu (Carnegie Mellon University), Abigail Raz (University of Nebraska-Lincoln), Cody Stockdale (Clemson University), Eric Nathan Stucky (University of Minnesota), Tingting Tang (San Diego State University), Nathan Wagner (Washington University in St. Louis), Nawapan Wattanawanichkul (Bowdoin College).

Credits
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Perseverance and Representation: A Memorial for Katherine Coleman Goble Johnson (1918–2020)

Ranthony A.C. Edmonds and Omayra Ortega

“Lady Mathematician Played Key Role in Glenn Space Flight” splashed across the front page of The Pittsburgh Courier on March 10th of 1962, just a little over a week after nearly 3,500 tons of ticker tape confetti showered upon John Glenn in a presidential caliber parade with thousands of attendants in New York City. While the world celebrated Glenn as a national hero, one of the most influential Black newspapers in the United States with a circulation of a quarter of a million people, praised Katherine Johnson as “one of the most brilliant mathematicians of the present era”; a mother, wife, and career-woman whose work in trajectory analysis played a crucial role in the safe return and extraction of John Glenn during his mission as the first American to orbit the Earth.

When Margot Lee Shetterly released her 2016 text Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race, which inspired the award-winning film, Katherine Johnson’s story was catapulted to the forefront, no longer hidden from the greater American consciousness. It took a popular film, created 60 years after she had completed most of her work, for her story to come to light. Why, when her work was instrumental to the success of NASA’s early space missions, did it take decades for her work behind the scenes to come to the forefront? If her

Figure 1. The Pittsburgh Courier, Pittsburgh, Pennsylvania, Saturday, March 10, 1962, page 19.
Katherine Johnson's story is an incredible one of a humble woman who quietly lived a "normal" life while breaking barriers as a Black female Computer at NASA. She seamlessly slipped between two worlds, a racially integrated hub of scientific innovation at work, and a Virginia Commonwealth that strongly resisted the victories of the Civil Rights Movement. Despite the challenges of her being both Black and female, she used her voice and expertise to help a man travel to the moon at a time when African-American women could not even vote. It is unbelievable!

The themes of perseverance, representation, advocacy, service, and activism add a universality to her story. She had many intersecting identities—as a scientist, a mathematician, an African American, a woman, a mother, and a wife—and many people have connected with different parts of her journey. She is an inspiration and much needed role model for generations of people who have not seen themselves reflected in the scientific community and in history books.

In the epilogue of her Hidden Figures text, Margot Shetterly wrote about how Katherine’s story resonated with people of all races, genders, ages, and occupations. She penned, “Katherine Johnson’s story can be a doorway to the stories of all the other women…whose contributions have been overlooked. By recognizing the full complement of extraordinary women who have contributed to the success of NASA, we can change our understanding of their abilities from the exception to the rule. Their goal wasn’t to stand out because of their differences; it was to fit in because of their talent” [1].

In this article, we celebrate the impact and legacy of Johnson’s life and career. In addition to highlighting the mathematical legacy that came from her thirty-year career at NASA, we focus on the human impact of her story on those who have also felt hidden within the mathematical community.

**Contributions to NASA**

Katherine Johnson started her career at Langley Research Center in the segregated West Computing group under the supervision of Dorothy Vaughan. She was there only two weeks before she was sent to work with the engineers in the Flight Research Division. Six months into the job she found herself assisting on a project that involved exploring why a small propeller airplane had seemingly fallen out of the sky and crashed. Through her meticulous data analysis and computations, engineers were able to discern that the disturbance from a large plane that had flown perpendicular to the path of the propeller had acted like an invisible concrete wall in the air. Her work on this research, along with similar reports, paved the way for changes in air traffic regulation, setting lower bounds on the distance between flight paths. She was quickly recognized by her superiors as an important member of the team, and she was soon promoted to a permanent position in the Maneuver Loads Branch within the Flight Research Division.

Katherine Johnson would continue to gain the respect and attention of colleagues throughout Langley Research Center. She is best known for her work in two NASA initiatives, Project Mercury and Project Apollo. Project Mercury was NASA’s first attempt to put a man into space via orbital flight. One of the most important questions of the orbital mission was the following: what exact path will the spacecraft travel across the Earth’s surface, and where will it land? A big challenge was getting the astronaut to return in the Atlantic Ocean close enough to a retrieval Navy ship to be quickly recovered from the water to safety. Katherine Johnson played a pivotal role in solving this problem. “Tell me where you want the man to land, and I’ll tell you where to send him up.”

Her mastery of analytical geometry led to a groundbreaking research report in 1960 entitled "Determination of azimuth angle at burnout for placing a satellite over a selected Earth position" that answered the question of where the space capsule would land. Her primary collaborator on this report was Ted Skopinski. When it came time to write up their work he remarked, “Katherine should finish the report, she’s done most of the work anyway” [1]. This
marked the first time a woman in the Flight Research Division had received credit as an author of a research report. This was acclaimed in the article written about her in *The Pittsburgh Courier*, where they exclaimed, “Mrs. Johnson co-authored the paper which tracked the rocket cone upon its re-entry into the earth’s atmosphere” [2].

The Apollo mission, with its goal of landing an American on the moon, was even more ambitious than Project Mercury. In order to make the 238,900 mile journey to the moon they would need two vehicles. One command module to reach the lunar orbit and then head back to earth, and a separate vehicle that would descend from the lunar orbit to the moon’s surface. While the astronauts navigated the moon’s surface in the lunar lander, the command module would continue to orbit the moon until the two vehicles met again, a moment called the lunar rendezvous. Johnson worked on the lunar rendezvous, determining the precise time that the lunar lander needed to leave the Moon’s surface in order to redock with the orbit and command module. Her work set the stage for Neil Armstrong to become the first person to set foot on the moon!

From 1963 to 1969 she coauthored four reports on problems related to lunar orbits and various scenarios in case of an emergency. She and her coworkers determined a method to use visible stars to navigate a course back to Earth without computer guidance. This was an important option for astronauts. For example, the Apollo 13 astronauts had to consider such backup methods after an on-board explosion damaged the electrical system during their mission. Johnson continued to collaboratively develop aspects of the space shuttle and Earth resources satellite programs for the rest of her career at NASA. She was awarded NASA’s Group Achievement Awards for her work on Project Apollo and the Lunar Orbiter Project.

Katherine Johnson received numerous honors and awards for her contributions to science throughout her career. In 2015, President Barack Obama awarded Johnson the highest civilian honor, the Presidential Medal of Freedom, for her work on many NASA missions including the first manned space flight, John Glenn’s momentous trip on the Friendship 7. “In her 33 years at NASA, Katherine was a pioneer who broke the barriers of race and gender, showing generations of young people that everyone can excel in math and science, and reach for the stars,” Obama said. In 2019, the Hidden Figures Congressional Gold Medal Act awarded the Congressional Gold Medal to Katherine Johnson and Dr. Christine Darden, and posthumously to Mary Jackson and Dorothy Vaughan.

Dr. Johnny L. Houston, professor of mathematics at Elizabeth City State University, noted many of Johnson’s accomplishments in a previous tribute [3], also appearing in these *Notices*. He reiterated in his correspondence regarding this article, “[T]he Congressional Gold Medal is scheduled to be awarded in her honor in late 2020. She received 13 Honorary Degrees, the last one from the University of Johannesburg, S. Africa. There are four major buildings that bear her name, including two major NASA facilities. The West Virginia State Senate has designated August 26 as a state holiday in her honor. The University of West Virginia and West Virginia State University are jointly planning a special physical facility in her honor (housing her awards/recognitions, artifacts and other items) to be available for viewing by the public and tourists who visit the state of West Virginia in the future. The National Association of Mathematicians (NAM) gave her an impressive Centenarian Award in 2019 at NAM’s 50th Anniversary Celebration in January 2019 at the JMM in Baltimore.”

**Impact on the Mathematical Community**

We are fortunate to bask in the legacy of Katherine Johnson as we live our lives as women, as educators, as parents, and as mathematicians. People of all walks of life can draw inspiration from Johnson’s successes. We asked members of the mathematical community to reflect on how the legacy of Katherine Johnson has impacted their lives.

A special reflection from another hidden figure.

“I stood on the shoulders of Katherine Johnson, Mary Jackson, and Dorothy Vaughn.”

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1 Dr. Darden’s reflections are from private communications with the authors.
These words from Dr. Christine Darden rose prominently from a thoughtful reflection she penned on the impact that Katherine Johnson had on her life. Rather than focus on Johnson’s scholarship, she painted a more human picture of a woman who was a mentor, a jovial spirit, and a community builder dedicated to uplifting African Americans in STEM.

Dr. Christine Darden was the fourth hidden figure in Margot Shetterly’s text. Due to the timeline in which she worked at NASA, she was not featured in the film. This generational gap between Darden and the women who came before her at Langley Research Center shines a light on why she felt that she stood on the shoulders of their legacies. In a sense, the boundaries that Katherine Johnson and others broke paved the way for Dr. Darden’s monumental 40-year career at NASA. She retired in 2007 as an internationally recognized expert in sonic booms, the author of over 60 technical reports, and the first African American to be appointed to NASA Langley’s Senior Executive Service, the top rank in the federal civil service.

In her reflection, Dr. Darden noted the intersections of her life with Johnson’s at the very beginning of her mathematical training. Dr. Darden attended college at what is now Hampton University in Virginia. She knew all of Johnson’s daughters well, and was particularly close to Johnson’s daughter Joylette. During this phase of her life, Johnson was a mentor, encouraging Darden and others to pursue mathematics.

“I attended Hampton Institute in the class of 1962. I had fallen in love with mathematics during my junior year of high school. I came with the intention of majoring in mathematics. Katherine’s oldest daughter, Joylette, was in my class and majored in mathematics. Joylette recommended Katherine Johnson as our ‘Class Mother’ and she spoke to our class at least once or twice during the year. A photo of Katherine with her new husband (James Johnson) was in our Freshman yearbook at a podium speaking to our class. Katherine would also reach out to help several of the church children with their mathematics.”

After Darden graduated from the Hampton Institute, she went on to pursue a Masters degree in applied mathematics from Virginia State College, which is now Virginia State University. Darden was recruited to NASA upon graduating with her Masters degree.

“When I was hired by NASA in 1967 the segregated West Computers at NASA did not exist at that time. I was working with the Computer Section of the High-Speed Aeronautics Division and there were three Computers in that group who had been in the West Computers during the 1950s with Katherine Johnson and Mary Jackson. They explained to me how engineers would bring work to the West Computers to be done. That way, the engineers became familiar with the quality of individual Computer’s work and occasionally, when their offices were pressed for help, the engineering section would ask to borrow particular Computers for long-term help. Katherine was one of those Computers who ended up staying with the engineering group and so was the current supervisor of my office.”

For her first three years at NASA, Darden commuted 70 miles round trip to work each day. After Darden moved closer to the city of Hampton, Johnson’s relationship with Darden as a distant colleague started to shift as Darden became more integrated into the local Hampton community. During this period, Katherine Johnson was not in the front of her class, but in the front of her church. Dr. Darden recounted how she was intentional about helping her get involved.

“Katherine was president of the Carver Senior Choir for many, many years until she dropped out because of health. “
The first day I visited Carver after moving to Hampton, VA, in 1970, I sat in the back of the church with my [three] daughters. When church was over, Katherine came out of the choir, walked all of the way to the back of the church and invited me to come to choir rehearsal on Wednesday evening—which I did.

“After I had been there about four years, [Katherine] invited me to join her to sing at funerals at Carver which were usually held during lunch time on a weekday. I did join, and we were able to help out during our lunch hour without taking leave. I visited Katherine’s office several times and I met her coworkers and the boss. In fact, Katherine’s boss attended several of our Carver Choir Concerts at her invitation. She seemed to have a great relationship with her coworkers.”

Dr. Darden also shared other community connections with Katherine Johnson and other colleagues at Langley. They were both active in their local chapter of Alpha Kappa Alpha Sorority, Inc., the first historically African American Greek lettered sorority. Darden joined as an undergraduate at the same time as Katherine’s daughter Joylette. Johnson joined Alpha Kappa Alpha at the age of 15, and was an active participant in the organization throughout her lifetime. In fact, in the *Hidden Figures* text, Shetterly notes that Katherine Johnson watched the media coverage of the Apollo Mission while at a sorority leadership conference [1].

Darden and Johnson also worked together in the National Technical Association (NTA), one of the oldest professional organizations for Black engineers and scientists. Dr. Darden noted some of the details of her service work with Johnson and her close friend Eunice Smith (who also worked at Langley).

“I got to know Katherine and Eunice better by interacting with them both at church and through working with them at the NTA, the first all-Black technical society that had a chartered chapter in Hampton. We technical folk at Langley reinvigorated that chapter at NASA Langley. We sponsored Math Contests for local middle and high school African-American students. We sponsored SAT tutorials for [African-American] students in high school. We sponsored symposia for [African-American] college students in surrounding states to give talks about their summer jobs involving research or other technical activities. The speakers were judged and awarded cash prizes for their presentations.

“At large conferences that NTA sponsored, Katherine, a very outgoing and welcoming person, would meet everyone in the room. Eunice would often be in the background working the logistics of making the conference run. I was probably some place in between. We worked with NTA for many years. Katherine was national treasurer for several terms and I was national secretary for a few years.”

In *Hidden Figures*, Shetterly also noted Johnson’s service work. Just as she had done when their lives intersected in Darden’s course her first year at the Hampton Institute, Johnson continued to make school visits throughout her life. Shetterly remarked that Katherine Johnson praised Christine Darden extensively, saying that “I never go into a school without mentioning Christine” [1]. Dr. Darden has tapped into this same passion for outreach throughout her career. She ended her reflection with encouragement for future generations of STEM.

“Since 2017, I have continued to speak with students and young professionals all over the country encouraging careers in STEM, especially as we hear from Apple and other Silicon Valley companies that they need to hire students from overseas because they cannot find them in America. I tell them they are just as smart. They need to take the right classes and work hard. I share my P4 philosophy which led me to NASA after deciding what my dream career would be. P1—Perceive myself as a mathematician. P2—Plan what classes or experiences I need to have to be a mathematician. P3—Prepare: work the Plan I developed in P2. And P4—Persist. If I run into roadblocks, find a way around them. Solve the problems I encounter! Take risks! But, keep going.”

Reflections from the mathematical community. Dr. Johnny L. Houston had the honor of first meeting Katherine Johnson when presenting her with the Distinguished Service Award at NAM’s Regional Conference in Norfolk, Virginia, in 1996. Years later, he wrote an article for the Notices honoring Katherine Johnson’s legacy entitled, “The

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*These reflections are from private communications with the authors.*
life and pioneering contributions of an African American centenarian: Mathematician Katherine G. Johnson” [3]. Over the last several years he has been in touch with her family, learning more about her life story. In February of 2018 he had the honor of delivering Johnson’s Centenarian Award on behalf of the National Association of Mathematicians (NAM). He was fortunate to be able to attend her homegoing celebration to pay his respects to her legacy. He shared with us some reflections from that day.

“On the morning of March 7, 2020, I attended her Celebration of Life service at the Hampton Univ. Convocation Center in Hampton, VA. Prior to [the] Service beginning, I walk[ed] down front and viewed her body; saying to myself, ‘what a world class giant whose life impacted humanity!’ The service attracted hundreds who came to honor her, family members, friends, celebrities (astronauts, high governmental and NSA officials, etc.).

“She lived such a fascinating and impactful life that during her later years when she began receiving so many recognitions, she raised the question: ‘What is all the fuss about? I was only doing my job.’ She has many more quotations; however, perhaps my favorite is the following: ‘Be anxious to learn. You don’t know everything about everything. There’s always more to be learned—but you have to want to learn it.’ I see this as great advice for future students and scholars.

“It has been my good fortune to relate/communicate personally with her family; especially her daughter, Mrs. Joylette Hylick. I close these Reflections with some rephrasing of words on her Celebration of Life Program, words that seem both profound and appropriate: ‘We know why the caged bird sings.’ The poet Maya [Angelou] reminded us that it sings with a fearful thrill for freedom. Uncaged, an eagle rests upon a mountaintop of dreams. Not soaring because it knows what needs to be done. All this time we did not know about you. Why? Because of your gender or the color of your skin? You are the reason the greatest space project could begin. John Glenn knew that your figures were better than the computer’s best. Now you and co-human computers are no longer hidden figures in NASA’s vast space. The world recently began to offer you honors that were so long overdue. Katherine G. Johnson, our admiration for you is forever sealed. Today, many birds are leaving cages, following great role models like you. We thank you, whole-heartedly, for all that you did for humanity!”

Dr. Jamylle Carter, a professor of mathematics at Diablo Valley College, noted that, “Katherine Johnson just did it, only to be acknowledged for her work in her later years. She lived her life—remarried and raised her children—while so many remained unaware of her accomplishments. Ms. Johnson reminds me of the legacy of Black women mathematicians who continue to live and work with grace, if not recognition.”

Like so many before her, Johnson’s story was almost lost to time. There is a history of erasure of the stories of people of color and the stories of women in this country. Karoline Pershell, Director of Strategy and Evaluation at Service Robotics & Technologies and former Executive Director of the Association for Women in Mathematics, states that, “[l]earning about Katherine Johnson and the
amazing mathematics contributions she made to the space program at a time when the country and her workplace overly made her a second-class citizen was the starting point of realizing how many hidden voices there were across STEM’s history.”

Unfortunately, people of color have few historical examples of themselves thriving in the government or in STEM. Most of us were not aware even of the existence of the group of female African-American Computers powering the United States’ race to the moon. Dr. Jacqueline Brannon-Giles, a professor of mathematics at Houston Community College and Texas Southern University, shares a story from her youth. “I remember a Texas Southern University professor encouraged a few of us to apply to work at NASA. I turned it down because I was young and did not think I could be comfortable with the lack of diversity. After I viewed and studied the movie, I cried because I did not understand how selective exposure in the media and suppression of women could exist, and still exists in America.”

Dr. Rachel Levy, former Deputy Executive Director of the Mathematical Association of America, reflects on time spent at NASA when she was a student researcher. “In the late 1980s as an undergraduate I participated in a practicum at NASA under the direction of Dr. Sylvia Washington. More than thirty years later, hearing Margot Lee Shetterly and Dr. Christine Darden talk about Katherine Johnson and her colleagues in Hidden Figures helped me re-examine my experiences at NASA from a new perspective. I had noticed that Dr. Washington was probably Black (I didn’t ask her) but had never thought about what it might have been like for her to work up to a leadership role at NASA. Now I have.”

Dr. Tanya Moore, founder and managing partner at Intersecting Lines, as well as the cofounder of the Infinite Possibilities Conference, further posits that, “Katherine Johnson’s legacy serves as a reminder to pursue excellence no matter the circumstance or situation you find yourself in. There can be bravery and beauty in the willingness to share your talents, knowledge and creativity for a larger mission and purpose beyond yourself. I am grateful for her life, her brilliance and her humanity.”

Valerie Jean-Pierre, a post-baccalaureate student at Medgar Evers College in New York City, saw Johnson’s story as a revelation. “As a Black mathematics student, it has been inspiring to see a Black woman become a mathematician and be a part of history. Even in my darkest hour, I know I can persevere and not only achieve greatness for myself, but aid others on their path.”

Dr. Ronald E. Mickens, professor of physics at Clark University, reminds us that, “…these successes occurred because of her personal desire and will to make this happen. She defined herself and her capabilities, and would not allow others to do this. This latter quality is her real legacy.”

Dr. Candice Price, an assistant professor of mathematics and statistics at Smith College in Massachusetts, found the fact that these stories could have ever been hidden as a rallying cry. She states, “Katherine Johnson’s strength and legacy inspired me to continue to work towards breaking down barriers, not just for myself but for others. Her story helped motivate the creation of a historical Black mathematicians poster so that stories like Katherine Johnson’s mathematical contribution do not go unknown.”

Christina Eubanks-Turner, an assistant professor at Loyola Marymount University in southern California, shared that, “Katherine Johnson’s story inspired me to persevere and be bold even if I’m the only person of color or female in the room. Her resilience in the face of adversity reminds me that I belong and deserve to be heard.”

Dr. Emille Davie Lawrence, department chair and term associate professor of mathematics at the University of San Francisco, observes a sentiment that every reader can relate to. “Katherine Johnson stood fast against a system that was built to suppress and subdue the career aspirations of Black women. The perseverance she displayed over half a century ago inspires and motivates me to reach for higher heights and even shoot for the moon.”

Brandi M. Adams, a high school mathematics teacher at Destrahan High School in Southeastern Louisiana, uses Johnson’s example to empower her students. “[W]hat are truly the expectations of African Americans in the classroom? Sharing the legacy of Katherine Johnson with my kids opened eyes. It made them think: ‘Yes, I can be good at math. Yes, Black women have done this. Yes, Black women ARE doing this’ [and t]hey are doing this fully, wholeheartedly, intentionally, and intensively.”

Sofía Martínez, an NSF Graduate Research Fellow at Purdue University in Indiana, received one message from Katherine Johnson’s story: “Persevere no matter the obstacle.” Despite this lack of representation,


It is Katherine Johnson’s perseverance that seeds her legacy and will inspire generations of mathematicians, as long as her story continues to be shared.

Reflections from the authors. R. Edmonds: I recall the satisfaction of printing out my homemade name tag for Halloween in 2016—Katherine Johnson, NACA. I pinned it onto my best interpretation of 1950s work attire and spent the day on campus explaining to my classes, and to anyone else who would listen, who Katherine Johnson was and why she was so amazing. The Hidden Figures text had been a gift for passing my comprehensive
exams, and I immediately found myself immersed in the story and amazed at the brilliance and resiliency of the featured women. Here was this phenomenal woman, from the same sorority as me, who also had a passion for mathematics and giving back to her community, that made amazing accomplishments as a scientist despite the lack of representation outside of West Computing amongst her colleagues at Langley Research Center.

This heightened visibility of excellence in STEM by other Black women made a world of difference as a graduate student questioning my place within mathematics. I felt then, and sometimes still do, that even though I love mathematics, I do not fit into the greater mathematical community. Katherine Johnson knew that she belonged, even if she was the “only” along various axes of diversity. I draw inspiration from this personally, and I have used the Hidden Figures story to discuss intersections of mathematics and society, belonging in STEM, and access to mathematical communities through my service, outreach, and teaching. Recently I developed a course at The Ohio State University, Intersections of Mathematics and Society: Hidden Figures, that focuses on the space race as well as diversity and inclusion in mathematical communities.

O. Ortega: In 2017 I had no plans to attend the JMM, but I could not let the small issue of being in another state stop me from an opportunity to hear directly from Christine Darden and Margot Lee Shetterly at the Special Panel Presentation: The Mathematics and Mathematicians Behind Hidden Figures, which was cosponsored by the National Association of Mathematicians. The event was not being streamed, and so I enlisted a close colleague to videoconference with me so that I could still “attend.” With her laptop hidden under her chair, it was hard to hear the panelists through the cacophony of ambient noise, but I persevered through the coughing and seat shuffling to hear their pearls of wisdom.

I needed to learn more about the phenomenal women of the Hidden Figures story. My curiosity was fueled by my own personal struggle of being a woman and member of several marginalized groups in the mathematical sciences. Their successes were critical to my own edification. Katherine Johnson and her contemporaries were humble women who balanced their family lives and careers at NASA at a time when Black American females could not even vote! I am humbled by the balance Katherine Johnson achieved between work, family, and service to her community, and I continue to draw on her exceptional story as I strike this balance in my own life.

R. Edmonds and O. Ortega: For future generations of STEM leaders, those who see themselves reflected in the communities around them and those who don’t, we encourage you to continue your unique path within the mathematical sciences. If there is one lesson that we should take from Katherine Johnson’s model of a life well-lived—one with a healthy balance of loved ones, community, service, career success, purpose, and meaning—it is that we should pursue the dreams that we hold most dear, regardless of what the outside world may say we are destined for. It is important that we each define, for ourselves, what it means to be successful. It is important that we decide, for ourselves, what we can and cannot achieve, and then stick to our plan. The mathematical sciences and their intersections with other disciplines of study provide a wealth of opportunities, and we encourage you to hold tight to your personal goals as you move through your unique path to success. Persevere! Dream Big!

References

Ranthony A.C. Edmonds
Omayra Ortega

Credits
Figure 1 is courtesy of the New Pittsburgh Courier.
Figure 2 is courtesy of Wikimedia.
Figure 3 is courtesy of NASA.
Figures 4 and 7 are courtesy of Dr. Johnny L. Houston.
Figure 5 is courtesy of Dr. Christine Darden.
Figure 6 is courtesy of Betty Bowes.
Figure 8 is courtesy of Edray H. Goins.
Photo of Ranthony A.C. Edmonds is courtesy of JKE Photography.
Photo of Omayra Ortega is courtesy of Timothy Archibald.
AWM at 50 and Beyond

Georgia Benkart, Kristin Lauter, and Sylvia Wiegand

According to the 2016–2017 report on new doctorate recipients [Golbeck-Barr-Rose], women received 29% of the PhDs awarded in mathematics from 1991–2015, but still comprised only 17% of the tenured/tenure-eligible faculty at PhD-granting institutions. The situation is even worse at elite institutions, which typically provide better access to resources to support research activities. It has improved slightly this year, however, as Harvard has just hired two new female full professors for AY 2020/21, bringing its total to three. A study of 435 mathematics research journals found that only 8.9% of editors were women [Topaz-Sen]. Many mathematicians were encouraged by the progress at the International Congress of Mathematicians (ICM) 2014 in Seoul, which highlighted three women—Maryam Mirzakhani, the first woman Fields Medalist, Ingrid Daubechies, the first woman president of the International Mathematical Union, and, Park Geun-hye, the first woman president of South Korea—but the most recent ICM in Brazil in 2018 again saw none of the major research prizes awarded to women. Tables 1–3 show women faculty at elite institutions in 2019–2020, 1991–1992, and 1998–1999.

This year—2021—the Association for Women in Mathematics (AWM) turns 50, March is Women’s History month, and last August we celebrated the hundredth anniversary of women winning the right to vote in the United States (August 18, 1920). Thus it is a good time to take stock of the state of affairs for women in mathematics, and to give an update on the many programs and initiatives launched and run by the AWM on behalf of women and girls in mathematics.

The AWM’s mission is “to encourage women and girls to study and to have active careers in the mathematical sciences, and to promote equal opportunity and the equal treatment of women and girls in the mathematical sciences.” To achieve this mission, the AWM offers programs and activities that support and encourage women in the mathematical sciences at all levels. The AWM is an organization almost entirely powered by volunteers, more than 200 of whom serve on the many committees that run all the AWM programs. As noted in “AWM in the 1990s” [Taylor-Wiegand], which appeared in the Notices close to the AWM’s thirtieth birthday, the AWM’s members and volunteers are passionate about the goals of the AWM. In this article, we focus on the association’s activities and achievements since then, particularly those that the authors are most familiar with. We also provide some observations on how the status of women has changed in the interim.

For many years, leaders of the AWM have tried various approaches to address three major problems confronting women mathematicians: the underrepresentation of women and minorities in research mathematics; the lack of equity in resources, awards, and access; and the barriers to career advancement for women and underrepresented minorities in the mathematical profession. For instance, it is AWM policy that the AWM President write letters to conference organizers who have no women speakers at their conferences and to chief editors of key research journals with no women on their editorial boards. Similarly, AWM Presidents have advocated to the AMS Council for more diversity on the editorial boards of AMS journals and have written to presidents of major institutions to urge them to hire more women into their mathematics faculties.
the AWM received a five-year, $750,000 ADVANCE grant to build and sustain research networks for women in all areas of mathematics.

The AWM ADVANCE Grant: Career Advancement for Women through Research-Focused Networks

The goal of the AWM ADVANCE grant is to create long-term change through communities of women supporting each other in research. Its specific model is to launch Research Networks for Women that can help advance their careers. As a significant step in this direction, in 2015

For comparison, below are the figures from [Taylor-Wiegand]:

<table>
<thead>
<tr>
<th>Department</th>
<th>Tenured</th>
<th>Female</th>
<th>Tenured</th>
<th>Female</th>
<th>Tenured</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>Total</td>
<td></td>
<td>Total</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>24</td>
<td>2</td>
<td>6</td>
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</tr>
<tr>
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</tr>
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<td>38</td>
<td>6</td>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>5</td>
<td>192</td>
<td>27</td>
<td>52</td>
<td>7</td>
</tr>
</tbody>
</table>


The continuing paucity of women faculty at research departments challenges the profession to look for new solutions. Although documenting and calling attention to inequity remain necessary, the AWM must expand its five-decade effort to build community in order to directly support women in mathematics and to proactively increase the opportunities, success, and visibility of women in mathematics. As a significant step in this direction, in 2015

1. AWM Advances!

For comparison, below are the figures from [Taylor-Wiegand]:

<table>
<thead>
<tr>
<th>Department</th>
<th>Tenured</th>
<th>Female</th>
<th>Tenured</th>
<th>Female</th>
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<tr>
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<tr>
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<tr>
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</tr>
<tr>
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<td>11</td>
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<tr>
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<tr>
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<td>36</td>
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</tr>
</tbody>
</table>

*One appointment is half time in physics.

Table 1. Women in mathematics, 2019–2020 [from Cathy Kessel]
a scalable way, and then connect them back to the broader mathematical research community where women are underrepresented.

After five years, the AWM ADVANCE grant has already achieved extraordinary success (see https://awmadvance.org/ for more information):

- launching and supporting 23 Research Networks for Women in all areas of mathematics,
- supporting five to ten Research Collaboration Conferences per year for women in these networks,
- publishing 22 volumes of research articles in the AWM Springer Series,
- supporting AWM workshops at the Joint Mathematics Meetings (JMM) and the SIAM annual meetings to highlight the research from these networks,
- supporting the biennial AWM Research Symposium featuring the work of roughly 350 women researchers in more than 20 special sessions,
- involving more than 2,000 women researchers in mathematics worldwide, and
- creating email and mentoring networks tied to research fields which help to strengthen the pipeline for women in mathematics.

The great success of the WIN (Women in Numbers) Number Theory Network, founded in 2006 by Kristin Lauter, Rachel Pries, and Renate Scheidler, inspired the AWM to initiate Research Networks for Women in other areas of mathematics. Research Networks for Women are formed as a result of Research Collaboration Conferences for Women (RCCWs). RCCWs are week-long conferences, held at mathematics institutes or universities, where junior and senior women come together to work on predefined projects. For example, at the first WIN conference in 2008 at the Banff International Research Station (BIRS) in the Canadian Rockies, 42 women mathematicians—graduate students, postdoctoral researchers, and senior women researchers who had formed groups of five or six several months in advance—spent five days working together on research problems posed in advance. A key factor in the success of the Research Networks is the publication of the articles produced by the working groups at the conference. In order to publish the research, typically the group has to continue working together for some months after the conference, and this helps form lasting bonds and collaborations.

The AWM has expanded the number of RCCWs each year, and has organized follow-up events and infrastructure to sustain and grow the resulting Research Networks.

<table>
<thead>
<tr>
<th>Department</th>
<th>Tenured</th>
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<th>Tenure-Track</th>
</tr>
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<tbody>
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<td></td>
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<tr>
<td>UC Berkeley</td>
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</tr>
<tr>
<td>Total</td>
<td>288</td>
<td>9</td>
<td>183</td>
</tr>
</tbody>
</table>


Figure 1. The first WIN conference, BIRS, 2008.
in various areas of mathematics. The co-PIs on the AWM ADVANCE grant were former AWM Presidents Kristin Lauter and Ruth Charney and former AWM Executive Director Magnhild Lien. Sociologist Erin Leahey served as senior personnel to study the impact of the program, and 2021–2023 AWM President Kathryn Leonard served on the Oversight Committee for the grant.

Several key elements of the Research Networks’ structure foster their success:

- Women help each other through vertical integration—senior women mentor junior women on research problems and in their careers.
- Groups of women collaborators promote their joint work in the broader community by giving research talks and co-organizing conferences with many women speakers.
- Early-career women are encouraged and empowered to co-organize conferences with men and invite their women colleagues.
- Women have more opportunities to participate in the editorial and reviewing process at an earlier career stage through editing the proceedings volumes for the RCCWs.
- Junior and senior women participate in program committees to actively promote the inclusion of women speakers in major conferences and on editorial boards.

In sum, the AWM ADVANCE grant’s Research Networks have created an army of women in many areas of mathematics who are empowered to work on behalf of themselves and other women to change the system by promoting each other’s work in a manner that benefits everyone’s career. They offer a way to circumvent the problem of women spending so much of their professional energy trying to help advance women and minorities that they fall behind male colleagues who have more time for research. In the AWM ADVANCE model, project leaders are essentially rewarded for effective mentoring by having excellent junior research collaborators who further and give increased visibility to the group leaders’ research agendas as well as to their own nascent careers.

Each year since 2012, AWM Workshops at the annual JMM and SIAM meetings bring together women from one of the Research Networks to showcase their work and encourage continued collaboration and mentoring. The AWM also organizes biennial Research Symposia, with high-profile plenary speakers and special sessions in areas linked to the Research Networks.

An AWM committee accepts proposals for new networks twice a year — with deadlines of July 1 and January 1. The committee then helps to refine proposals and match potential networks with a mathematics institute that will host the first collaboration conference for the network. Michelle Manes chaired this committee for three years and assisted in inaugurating many new networks, while also chairing the WIN Steering Committee.

In addition, Katherine Stange, who created the WIN website (https://womeninnumbertheory.org/), has helped Research Networks create their own websites and e-mail mailing listings, which are hosted by the AWM. These websites have increased the visibility of women in research mathematics, both for conference organizers to invite women speakers and for university hiring committees to increase the pool of women candidates.

An external evaluator, Dr. Erin Leahey, Professor of Sociology at the University of Arizona, is studying the efficacy of this model for advancing careers for women in mathematics for the five years of the grant, using workshop surveys (with more than 1,000 participants), conducting a career impact survey (with more than 400 respondents), and analyzing CVs; see [Lauter-Lien].

The WIN conferences have involved more than 250 women in number theory from around the world, were organized by more than 20 distinct women, and produced more than 50 peer-reviewed, published research papers in seven proceedings volumes that have appeared in the Fields Institute Series, the Centre Recherches Mathématiques (CRM) AMS Series, and the AWM Springer Series. When planning for the first WIN conference began, there were three women professors in number theory at top research universities in the US, whereas now there are several dozen women faculty in number theory.

Implementation of the AWM ADVANCE grant award has truly represented the collaborative work of many leading women in mathematics. It was the result of the collective efforts of at least six AWM Presidents and the AWM Executive Director, with inspiration from the Korean Women in Mathematical Sciences (KWMS), and countless organizers of focused Research Networks. The organizers of early RCCWs are listed in Table 5. Since Year 1 of the grant, 25 more RCCWs have been organized in many new and existing areas; these are not listed in Table 5, but they can be found on the RCCW tab on the AWM ADVANCE site. Organizing an RCCW is truly a labor of love, as well as a great deal of work, and so these organizers deserve to be named and celebrated.

Many of the RCCWs so far have taken place at BIRS, and the AWM is very appreciative of its substantial support. In addition to BIRS, the research institutes AIM, ICERM, IMA, and IPAM have all hosted RCCWs, and MSRI has launched an annual summer program for groups of women to collaborate on research projects, some of which may have started at RCCWs. Microsoft Research has generously cosponsored all of the AWM Research Symposia, the WIN Research Collaboration Conferences, and the RCCWs hosted at ICERM, IMA, and IPAM. Travel funding for participants has been provided by grants from the Clay Institute, NSA, NSF, PIMS, and the Number Theory Foundation, among others.
Networks have also held their conferences at other research centers around the world, including Luminy (CIRM), the Lorentz Center, Nesin Village, Centre Henri Lebesgue, the Hausdorff Institute, the University of Leeds, Oaxaca (BIRS-CMO), and the Australian National University.

A significant portion of the funds from the AWM ADVANCE grant are devoted to participant expenses to attend these RCCWs or to speak at the AWM Workshops at JMM and SIAM and the special sessions for Research Networks at the biennial AWM Research Symposia.

### 2. AWM Research Symposia

To celebrate the fortieth anniversary of the AWM in 2011, Georgia Benkart, Kristin Lauter, and Jill Pipher organized a research symposium at Brown University/ICERM when Jill Pipher was AWM President and Founding Director of ICERM. This became the prototype for subsequent AWM Research Symposia, which are two-day weekend meetings, run on the model of the AMS sectional conferences, with high-profile plenary speakers and special sessions on focused research topics. Later supported by the AWM ADVANCE grant, the AWM Research Symposia have brought women mathematicians together to recognize their research achievements, build community, advance careers, and improve working conditions. Professional development activities include a panel on nonacademic careers, an exhibit hall, and networking opportunities. The subsequent AWM Research Symposia were held at Santa Clara University in 2013, the University of Maryland in 2015, UCLA/IPAM in 2017, Rice University in 2019, with the fiftieth anniversary symposium planned at the University of Minnesota/IMA, June 16–19, 2022.

#### 40 Years and Counting: The AWM’s Celebration of Women in Mathematics

In September 2011, drew an attendance of over 300 women and men and featured four invited talks, 18 sessions on a wide range of topics in pure and applied mathematics and mathematics education, and a total of 135 speakers. It received enormous help from the...
ICERM staff, funding from an NSF grant, and additional support from the AMS, Brown University, ICERM, the MAA, Microsoft Research, Pearson Education, SIAM, and the US Department of Energy. A day-long retreat at ICERM just prior to the meeting enabled the AWM Long-Range Planning Committee to reflect on the AWM’s accomplishments, review its programs, and strategically plan for the future.

AWM Research Symposium 2013 was organized by Hélène Barceló, Estelle Basor, Georgia Benkart, Ruth Charney, Frank Farris, and Jill Pipher at Santa Clara University in 2013. It featured plenary talks by three distinguished mathematicians: Inez Fung, Professor of Atmospheric Sciences at UC Berkeley and an expert on the mathematics of climate; the late Maryam Mirzakhani, 2014 Fields Medalist; and Lauren Williams, now the Dwight Parker Robinson Professor of Mathematics at Harvard University. There were invited and contributed sessions on a wide range of subjects in pure and applied mathematics, a poster session for graduate students, and a discussion of “The Imposter Syndrome” moderated by Hélène Barceló with panelists Ruth Charney, Brian Conrey, Jill Pipher, and Carol Wood.

AWM Research Symposium 2015, which took place at the University of Maryland in April 2015, was organized by Ruth Charney, Shelly Harvey, Kristin Lauter, Gail Letzter, Magnhild Lien, Konstantina Trivisa, and Talitha Washington. Many outstanding mathematicians volunteered to organize the 14 special sessions representing a wide swath of mathematics, and plenary lectures were given by Ingrid Daubechies, Maria Chudnovsky, Jill Pipher, and Katrin Wehrheim. First Lady Michelle Obama was invited as the banquet speaker, but was unable to attend.

The format of the 2015 symposium was similar to those at ICERM and Santa Clara, but also included an employment panel moderated by Gail Letzter with representatives from industry and government, prizes for outstanding post- ers, and presentation of the inaugural AWM Presidential Award. The Award was given to the founders of the EDGE (Enhancing Diversity in Graduate Education) Program, Sylvia Bozeman and Rhonda Hughes, in recognition of the EDGE Program’s strong record of supporting graduate students and building community among women from diverse backgrounds. Shirley Malcom, head of the Office of Opportunities in Science at AAAS, delivered an inspiring keynote address, followed by moving acceptance speeches from Sylvia Bozeman and Rhonda Hughes.
Program at Carleton College. IPAM hosted a welcoming event for students and the AWM Student Chapters, and the first Wikipedia edit-a-thon was led by Marie Vitulli and Ursula Whitcher to create Wikipedia pages for women in mathematics. Following a lunch presentation and discussion on establishing new AWM Research Networks by Magnhild Lien and Kristin Lauter, several new networks were formed. The Notices published the 2017 AMS Notices Sampler for the 2017 AWM Symposium.

AWM Research Symposium 2019, at Rice University April 6–7, 2019, attracted more than 350 attendees. Of the 20 special sessions in different research areas, 13 were organized by Research Networks supported by the ADVANCE grant. Plenary lectures were given by Susanne Brenner, Kristin Lauter, and Chelsea Walton, the banquet included a welcome from Rice Provost Marie Lynn Miranda, and a keynote address by a Rice undergraduate Mariam Manuel. The Wikipedia edit-a-thon, the career panel focusing on mathematicians in government and industry, and the lunch presentation on establishing and maintaining AWM Research Networks were carried over from 2017. The NSF AWM ADVANCE grant provided more than $70,000 in participant support for each of the 2017 and 2019 symposia, plus administrative funds and overhead support for the AWM.

The 2022 AWM Research Symposium celebrating the fiftieth anniversary of the AWM is to be hosted by the IMA (Institute for Mathematics and its Applications) in partnership with the University of Minnesota, June 16–19, 2022. The Research Symposia, while requiring many hands and significant funding to realize, have proved to be invaluable for building community among female mathematicians, showcasing women’s work in mathematics, and attracting attention, support, and sponsorship for the AWM mission. Proceedings volumes for the 2015, 2017, and 2019 symposia published in the AWM Springer Series highlight the events and the research.

3. AWM Springer Series

AWM launched the AWM Springer Series in 2014 to publish the proceedings of the Research Collaboration Conferences for Women, AWM Workshops, AWM Research Symposia, and other AWM events and panels. A list of the 23 volumes published to date is available on the series webpage. They include four volumes from the WIN conferences, several volumes each from WiSh (Women in Shape Modeling), WIMB (Women in Math Biology), and other networks, three volumes from the 2015, 2017, and 2019 AWM Research Symposia, one volume on mathematics education based on an AWM-AMS panel at the Joint Mathematics Meetings, a history of women in mathematics based on an AWM Contributed Papers session at MAA’s hundredth anniversary celebration at MathFest in 2015, and two volumes on Harmonic Analysis in honor of the late AWM President Cora Sadosky (1940–2010).

4. AWM Capitol Hill Visits

When the 2015 AWM Research Symposium was hosted at the University of Maryland, then AWM President Kristin Lauter invited all AWM Executive Committee (EC) members to join her in a day of visits to Capitol Hill on April 13, 2015, the day after the symposium. EC Member and Professor at Howard University Talitha Washington accepted Lauter’s invitation. They made appointments, introduced themselves and the AWM, and discussed STEM outreach funding and initiatives with Congresswoman Eddie Ber-nice Johnson, and with congressional staff in the offices of Senator Kirsten Gillibrand, Senator Patty Murray, and Congressman Paul Tonko. Representative Johnson asked for a list of women in mathematics who would be willing to speak at local events in her district in Dallas. They also discussed Representative Tonko’s bill, H.R.823 - Educating Tomorrow's Engineers Act of 2015, and President Obama’s initiative to increase the STEM workforce. In a follow-up phone meeting with the White House Council on Women and Girls, they proposed that the AWM Essay Contest Winner be invited to the annual White House Science Fair. These visits began the shaping of a broader advocacy agenda for the AWM that reflects the AWM’s mission of advancing women and girls in mathematics. In August 2015, when much of the AWM leadership returned to
Committee and agreed to serve as Chair of the P&A Committee and to formally establish the program with the help of Karen Saxe, who also joined the committee. Together they led the P&A to develop a more comprehensive policy agenda and legislative priorities, as well as a regular cadence of visits twice a year to coincide with the Conference Board of the Mathematical Sciences meetings in Washington, DC, in May and December. The legislative priorities formulated by the P&A Committee, published in the July/August 2016 issue of the AWM Newsletter, were:

1. Expand STEM educational opportunities.
2. Support research funding.
5. Create a welcoming environment in science and education, including policies to address sexual harassment and violence on university campuses.

In May 2016, AWM President Kristin Lauter was accompanied on the third AWM Hill visit by EC Members Talitha Washington and Talithia Williams to discuss the AWM legislative agenda with staff in the offices of Congresswomen Barbara Lee and Jackie Speier, and to support the bill “Computer Science for All” introduced by Representative Lee and several cosponsors. In Fall 2016, Representative Speier introduced H.R.6161 - Federal Funding Accountability for Sexual Harassers Act, and her office invited the AWM contingent to meet with her again during the December 2016 visit.

The December 2016 visit was the most exciting and successful visit to date. Beth Malmskog and Katherine Haymaker, faculty mentors for the AWM Student Chapter at Villanova, brought nine students from Philadelphia to join the AWM Hill Day on December 1. The AWM contingent

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**Figure 7.** Heading to the Hill: students gather with AWM leadership at the MAA’s MathFest, August 2015.

**Figure 8.** Lauter with Talithia Williams and Talitha Washington in DC.
attended a breakfast hosted by Pennsylvania Senator Bob Casey, visited more than 20 congressional offices in groups, and met several other Members of Congress on both sides of the aisle. The top legislative priority was the Women and Minorities in STEM Booster Act of 2016. The AWM groups also voiced support for the Computer Science for All Initiative, the INSPIRE Women Act, and the bill introduced by Representative Speier requiring that sexual harassment by Principal Investigators be reported to funding agencies, and that harassment reports be considered when awarding federal funding.

The visit was captured enthusiastically by Beth Malmskog in her AMS blogpost. There is evidence that the AWM’s Hill visits have had an impact through the legislation that the association has supported. Representative Eddie Bernice Johnson spoke about AWM support for one of her bills on the floor of the House. However, possibly the greater impact the program has had is on the women and men of the AWM and AWM Student Chapters who have participated in these visits, empowering and inspiring them to continue to make a difference by supporting each other and fighting for change. Some student participants even said that taking part in the day-long Hill visit had “changed their lives.” In 2017, Gail Letzter and Karen Saxe created a subcommittee in the P&A Portfolio, the Government Advocacy Committee, to run the Hill visits. The subcommittee is now chaired by Executive Committee member Michelle Snider and the visits continue.

Figure 9. On Capitol Hill: Villanova AWM Student Chapter with AWM leadership, December 2016.

Figure 10. AWM Student Chapter at Texas A&M University, 2017.

Figure 11. AWM Student Chapter at Florida Atlantic University, 2019.

5. AWM Student Chapters

AWM Student Chapters at colleges and universities are one of the association’s greatest assets. They provide a way to advance its mission by building community, supporting education and developing careers of students, and advocating for women in mathematics on college campuses. There are currently more than one hundred active AWM Student Chapters. In 2015, an annual webinar was begun for Student Chapter Presidents to meet with the AWM President and discuss ideas with each other. Newsletter Editor Anne Leggett resurrected a “Student Chapter corner” in the Newsletter to publish articles from the chapters on their activities, and online folders for chapters were created to share ideas and information. The AWM Awards Committee developed annual Student Chapter Awards in four categories: Community Outreach, Fundraising and Sustainability, Professional Development, and Scientific Excellence. Nominations are due by April 15 each year, and chapters can self-nominate for the awards, which are presented each summer at a reception at MathFest. Student Chapter members also are invited to attend the AWM Reception at the JMM to meet with Executive Committee members and network.
6. Social Media

When Jill Pipher was AWM President (2011–2013), the AWM launched a Facebook page. In January 2015, an ad hoc Media Committee was formed to improve communication with members, to reach and attract new members, to raise the profile and awareness of the AWM in the mathematics community and in the public, to draw media attention to AWM events, and to attract more corporate sponsors for the organization and its initiatives. That committee, chaired by Web Editor Adriana Salerno, included Anna Haensch, Marie Vitulli, Talitha Washington, Newsletter Editors Anne Leggett and Sarah Greenwald, Executive Director Magnhild Lien, and AWM President Kristin Lauter. One of the first actions of the new committee was to launch the AWM Twitter feed, @AWMmath. The 2015 AWM Research Symposium was live-blogged and tweeted by Salerno and Haensch. The AWM ADVANCE program also launched an active Twitter feed: @AdvAdvance. EC member Marie Vitulli later became AWM Media Coordinator and posted daily content to the AWM Facebook page, providing stimulating topics for discussion, interspersed with biographies of women in mathematics and updates on the activities of the AWM and women in the profession.

7. Awards

One of the biggest barriers to the advancement of women in mathematics and science is a lack of visibility and recognition for the vast talent, contributions, presence, and hard work of women in the field. It is therefore important to shine the light on those women and their contributions, by increasing the number of pictures of women mathematicians in publications and on social media and by seeing that women’s work is recognized through the prizes awarded by the major professional societies.

For this reason, the AWM has instituted its own prizes and recognitions and has strived to nominate women for awards conferred by other professional societies. The AWM Sadosky Prize in Analysis and the AWM Microsoft Research Prize in Algebra and Number Theory were established in 2012 and first awarded in 2014, and the AWM Joan and Joseph Birman Research Prize in Topology and Geometry was established in 2013 and first awarded in 2015. The AWM Dissertation Awards were developed by Rhonda Hughes in the Awards Committee and approved by the Executive Committee in January 2016. Announced annually at the AWM reception at the JMM, these four prizes call attention to the work of outstanding early-career women and the dissertation accomplishments of PhD students.

AWM Service Awards have been given each year since 2013, to acknowledge the AWM’s great reliance on the dedication of its volunteer members and, in particular, on the efforts of those who step up year after year for intensive immediate projects. The reason for instituting these awards was evident, but the AWM had recognized some outstanding contributions prior to their establishment. At the fortieth anniversary celebration at JMM 2011, for example, Bettye Anne Case, AWM’s longtime Meetings Coordinator, and Anne Leggett were honored for their exceptional service to the AWM. Anne has been the AWM Newsletter editor for over forty years and continues her stellar work in that essential role today.

8. AWM Scientific Advisory Committee

In order to ensure that outstanding work by women in mathematics is also recognized by other societies through prizes, fellow nominations, and named lectures, Past President Sylvia Wiegand chaired the Awards Committee and worked with President Ruth Charney to create the AWM Scientific Advisory Committee, approved by the Executive Committee in January 2015. The committee of six members serving staggered three-year terms was charged with generating names and procuring nominations for women to be recipients of the distinguished prizes, awards, and honors of organizations related to the mathematical sciences, including the AMS, MAA, SIAM, and AWM.

Charney chaired the committee for the first year and Wiegand for the next year, and the other inaugural members were Georgia Benkart, Suncica Canic, Barbara Keyfitz, and Susan Montgomery. The committee generated numerous nominations for highly deserving women, especially for AMS and SIAM Fellow recognition.

The Awards and Scientific Advisory Committees have advocated for more nominations of women for awards and recognition in the mathematical sciences; the percentages of women among recipients of awards and recognition would be rather low if the AWM did not advocate for women. For example, of the 12 women in the 2020 class of 52 AMS Fellows, at least six were bolstered by the AWM committee’s organized efforts to document their qualifications and support their selection.

9. AWM Fellows Program

Although the AWM Scientific Advisory Committee has helped to nominate many deserving women for awards, there remain so many talented women mathematicians and educators whose work is not sufficiently celebrated and encouraged. Consequently, when Sylvia Wiegand was chair of the Advisory Committee, she and Past President Rhonda Hughes proposed that an AWM Fellows Program be created to recognize those with sustained contributions to the AWM’s mission of supporting women and girls in mathematics. Joan Ferrini-Mundy chaired the Awards Committee in 2016, and the AWM Fellows Program was approved in January 2017. Carol Woodward chaired the next Awards Committee that defined the terms of the program. The inaugural class of fellows was celebrated at the AWM reception at the JMM in January 2018.
Conclusion

The AWM offers everyone in the mathematical community the opportunity to become involved and help advance women and girls in mathematics. Anyone can start or join an AWM Student Chapter or a Research Network for women, become a mentor, get a mentor, publish in the AWM Springer Series, or attend an AWM workshop at the JMM or the SIAM annual meeting! We hope to see you at the AWM Research Symposium in 2022*, and Happy Fiftieth Anniversary to the AWM!

References


*The AWM Research Symposium originally scheduled for June 2021 has been postponed until June 16–19, 2022. For up-to-date information see https://awm-math.org/meetings/awm-research-symposium/.
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American Women Mathematics PhDs of the 1940s
Margaret A. M. Murray

Introduction

Since 1993, I have been compiling and updating a database of information on 192 women awarded PhDs in mathematics by institutions of higher education in the United States and Canada during the years 1940–1959. Back in the 1990s, I compiled the list using several print sources (see [15, pp. 21–23] for details) and, in subsequent years, I’ve crosschecked, augmented, and updated it against a number of online sources (e.g., departmental histories, official university publications, public documents available on genealogy websites, online obituaries). While it’s difficult to be sure that the list of doctoral recipients is 100% complete, the total number for the US is reasonably close to that reported by the National Academy of Sciences (e.g., [9]). Since 2016, I’ve been gradually publishing information from the database at the Women Becoming Mathematicians website, [WomenBecomingMathematicians.net ([17])]. My work on both the database and the website is part of the larger project of clearly and accurately documenting the lives and careers of the first several generations of American women to earn PhDs in mathematics ([16]). The most substantial contribution to this work to date is that of Judy Green and Jeanne LaDuke who, in their book Pioneering Women in American Mathematics, provide detailed information on the first 228 American women who earned mathematics PhDs prior to 1940 ([7] and [8]). While my work does not approach the exhaustive detail of Green and LaDuke, it is my fervent hope that others will use these documentary resources as the foundation for future historical research.

In this article, my specific focus is on the American women mathematics PhDs of the 1940s. I begin by laying out some historical context for understanding this cohort of American women in mathematics. Next, I provide a list (current as of January 2021) of the 86 women I’ve been able to identify who earned mathematics PhDs in the US and Canada during the 1940s, and go on to describe some of their personal and professional characteristics. Finally, I discuss ongoing work on the Women Becoming Mathematicians website ([17]), which serves as a reference repository for basic background information on this generation of American women mathematics PhDs.

A brief comment on notation: when I introduce a particular woman mathematics PhD in the text, I enclose the name of the PhD-granting institution and the year the PhD was awarded in parentheses immediately following the woman’s name.

Historical Context

Across eras, civilizations, and cultures, it is clear that both women and men have carried out mathematical work. But in the recorded history of mathematics, there are very few accounts of women’s mathematical activity before the 19th century. The 19th century also marks the emergence of the research doctoral degree, beginning in Europe and then spreading to America and across the globe. In most of the emerging mathematical communities of the late 19th and early 20th centuries, the PhD in mathematics came to be viewed as a certification of accomplishment in research and as prerequisite for admission to the professional caste in research mathematics ([18], [15, pp. 1–3]).

So far as we know, Sonya Kovalevskaya was the first woman to earn a PhD in mathematics, awarded to her in absentia by the University of Berlin in 1874 ([13, p. 123]). In 1862, Yale University awarded the first US PhD in mathematics—to a man, J. H. Worrall ([18, p. 202]). Twenty years later, in 1882, Christine Ladd-Franklin became the first American woman to earn a PhD in mathematics (at the University of Göttingen) ([18, p. 172]).
first woman to earn a PhD in mathematics from a US institution, Johns Hopkins University—but, famously, Hopkins refused to actually award her the degree until 1926 ([7, p. 5]). Thus Columbia University was the first US institution to actually award a PhD in mathematics to a woman, Winifred Edgerton Merrill, in 1886; she was, in fact, the first American woman awarded a PhD in mathematics anywhere, and the first woman to be awarded a Columbia degree of any kind ([7, p. 10, pp. 246–247], [11]).

Over the three decades that followed (1890–1919), women earned an increasing share of US PhDs in mathematics, approaching 14% of the total in the decade immediately prior to the ratification of the 19th Amendment to the US Constitution (granting women the right to vote). Over the following two decades (1920–1939), women’s share of US mathematics PhDs leveled off but nevertheless remained fairly steady at just over 14% ([15, pp. 4–5]). But during the 1940s, the proportion of US mathematics doctorates earned by women began a precipitous decline; indeed, it was not until the 1980s that the proportion of US mathematics PhDs awarded to women attained and finally surpassed pre-World War II levels ([15, pp. 5–6], [7, pp. 113–115]).

Numerous social and political factors, both internal and external to the mathematical community, help to explain the puzzling decline and slow rebound of American women’s participation in doctoral-level mathematics from the 1940s to the 1980s. An account of these factors—including the postwar backlash against women’s work outside the home, retrenchment in the American mathematical community, and the resurgence of the women’s movement—makes for illuminating and cautionary reading, and gives the lie to the myth that continuous progress is possible for women in STEM ([15], [19], [20], [21]). Because they coincided with these years of tumultuous change in American society, the lives and careers of the women who earned mathematics PhDs in the 1940s (and 1950s) are a fascinating subject of study. And the women PhDs of the 1940s are of particular interest, having launched their careers in the transition between the relatively slow and steady growth in the American mathematical community prior to World War II and the period of explosive growth in that community—and in academia more generally—during the postwar period.

**The List**

Tables 1 and 2 below list the 86 American women mathematics PhDs of the 1940s. For each woman, I’ve provided the year in which her doctorate was awarded, the most complete version of her name that I’ve been able to identify, her years of birth and death, and the institution from which she earned the PhD. As of January 2021, five of the 86 women are still living; all are over the age of 95. One of these, Domina Eberle Spencer (MIT 1942), received her doctorate just shy of her 22nd birthday—the youngest degree recipient among the forties women—and has recently celebrated her 100th birthday. Among those who have died, three lived past age 100: Janet McDonald (Chicago 1943) and Sister M. Francis Borgia Stauder SSND (Notre Dame 1947) lived to 101, while Esther Seiden (California/Berkeley 1949) lived to 106. On the whole, this is a long-lived group, with an average lifespan well into the 80s.

A quick scan of the tables reveals several familiar names. Proceeding chronologically through the years 1940–1942, we first encounter Dorothy Maharam Stone (Bryn Mawr 1940)—a student of Anna Johnson Pell Wheeler (Chicago 1910)—well known for her research in measure theory and ergodic theory, with an Erdős number of 2. We also encounter Josephine Margaret Mitchell (Bryn Mawr 1942)—also a student of a well-known woman mathematician, Hilda Geiringer (Vienna 1917)—notable for her research in several complex variables. With her spouse, the mathematician Lowell I. Schoenfeld, Mitchell made a major bequest to the American Mathematical Society ([2]). And finally, we come to Alice Elizabeth Turner Schafer (Chicago 1942), who wrote a dissertation in projective differential geometry under Ernest P. Lane. Schafer held teaching positions at eight different colleges and universities, most notably Connecticut College and Wellesley College. But she is best known for her untiring advocacy on behalf of girls and women in mathematics, and for her indispensable role in establishing the Association for Women in Mathematics (AWM).

Continuing chronologically, we come to Euphemia Lofton Haynes (Catholic 1943), the first African-American woman to earn a PhD in mathematics. She earned her degree under Aubrey F. Landry, who supervised the theses of 28 students at Catholic University, 18 of whom were women ([7, p. 52]). Haynes devoted her career, both before and after the doctorate, to teaching in the public school system of Washington, DC, including several years at what is now the University of the District of Columbia (see [12]). Just 18 years earlier, Elbert Frank Cox had been the first African-American man to earn a mathematics PhD; working under the direction of William Lloyd Garrison Williams, he received his degree from Cornell in 1925 ([22]). Somewhat later in the 1940s, Evelyn Boyd Granville (Yale 1949) became the second African-American woman to earn a mathematics PhD. After completing a dissertation in complex analysis with Einar Hille, she went on to a wide-ranging career spanning academia, government, and industry. Shortly thereafter, Marjorie Lee Browne (Michigan 1950) became the third African-American woman to earn a mathematics PhD, working with George Rainich. Although Browne’s name does not appear in the forties list—her doctorate was awarded in 1950—she had completed all the requirements for the degree by the end of 1949.
Many other names in this list will be familiar to readers of the Notices. Mary Patricia Dolciani Halloran (Cornell 1947), known professionally as Mary Dolciani, was a prominent figure in mid-20th century American mathematics education. She earned her PhD in algebra and number theory under Burton W. Jones and went on to a long career at Hunter College. In 1974, Dr. Dolciani made a gift to the MAA to endow the Dolciani Mathematical Expositions series, now published by the AMS ([14]). Another endowment from the Mary P. Dolciani Halloran foundation funds the AMS Mary P. Dolciani Prize for Excellence in Research ([3]).

A doctoral student of the logician Alfred Tarski, Julia Bowman Robinson (California/Berkeley 1948) is well known as the first woman elected to the Mathematics section of the National Academy of Sciences; she earned this honor in 1976 as a result of her work in mathematical logic, specifically the resolution of Hilbert’s Tenth Problem. She was also the first woman elected President of the AMS. Jane Smiley Cronin Scanlon (Michigan 1949), a distinguished applied mathematician and student of Erich Rothe, spent much of her career on the faculty at Rutgers and was recently memorialized in the pages of the Notices ([11]). And Mary Ellen Rudin (Texas 1949), perhaps the best-known student of R. L. Moore, had a distinguished career in teaching (mainly at the University of Wisconsin), service to the profession, and research in general topology; her Erdős number is 1 ([4]).

**Group and Individual Characteristics**

While some characteristics of the women PhDs of the 1940s—both as a group and as individuals—emerge from examining the table, others require a deeper dive into the database. I highlight just a few of these characteristics here.

A total of 28 schools are represented among the degree-granting institutions. The top grantor of mathematics PhDs to women in the 1940s was the University of Illinois ([9]), followed by Catholic University of America ([8]), the University of Michigan ([8]), Radcliffe College ([7]), the University of Chicago ([6]), the University of California at Berkeley ([6]), and Cornell University ([4]). From 1902 to 1962, women could earn PhDs from Radcliffe College, the women’s coordinate college of Harvard University, but not from Harvard itself ([10]). In every way but name, however, these were Harvard PhDs, so for each such degree I have designated the degree-granting institution as Harvard/Radcliffe.

Fifteen of the 1940s PhDs listed are Roman Catholic sisters, more commonly referred to as nuns. (The letters that appear after their names signify the religious orders to which they belong; for example, the Missionary Sisters Servants of the Holy Spirit, also known as *Servae Spiritus Sanctae*, are denoted SSpS). Prior to 1940, a total of 18 Catholic sisters earned mathematics PhDs in the United States, 17 of whom received the degree during the years 1929–1939, as part of an ongoing effort to “upgrade the level of instruction in Catholic women’s colleges” ([7, p. 62]). This effort continued into the 1940s, and helps to explain the continued prominence of Catholic University as a grantor of mathematics PhDs to women during this decade. Because these Catholic sisters were typically already employed on college faculties before beginning graduate study, they earned their doctorates at a somewhat older-than-average age. And, as Green and LaDuke have observed, they generally did not have to seek out employment; their teaching assignments came directly from their religious orders ([7, p. 62]).

The list includes three mathematicians born in Canada. Jeanne Starrett LeCaine Agnew (Harvard/Radcliffe 1941) was born in Port Arthur, Ontario, received bachelor’s and master’s degrees from Queen’s University in Kingston, Ontario, and earned her PhD under the direction of George D. Birkhoff. During the war, she worked for the National Research Council of Canada (NRC); during the postwar years, she enjoyed a decades-long career at Oklahoma State University. Josephine Mitchell, mentioned previously, was born in Edmonton, Alberta, and earned her undergraduate degree at the University of Alberta before coming to Bryn Mawr. Upon receiving the PhD, Mitchell taught at nine different colleges and universities and worked in two private research labs before settling, at last, into a tenured full professorship at SUNY Buffalo. And Kathleen E. Butcher Whitehead (Michigan 1946), born in Shelburne, Ontario, earned her undergraduate degree at Queen’s before coming to the US for graduate study, first at Smith College and then at Michigan. She held a succession of faculty positions in New England, culminating in 26 years on the mathematics faculty at Tufts.

The list also includes just one PhD awarded by a Canadian university, that of Muriel Kennett Wales (Toronto 1941). She was born in Belfast but moved to Vancouver while still in infancy; she earned bachelor’s and master’s degrees at the University of British Columbia. Wales’ PhD appears to be both the fourth mathematics PhD awarded to a woman by a Canadian university and the fourth awarded by the University of Toronto—the first having been awarded to Cypra Cecilia Krieger in 1930. Like Jeanne Agnew, Wales worked for the NRC during the war years, first in Toronto and then in Montreal, but she appears to have withdrawn from mathematics after 1949. I’m indebted to Colm Mulcahy, who has a deep professional interest in the biographies of Irish mathematicians, for his assistance in tracking down the details of Wales’ life and work.

A large majority of the women who earned a mathematics PhD from US institutions in the 1940s enjoyed long careers in education, mainly at the postsecondary level. There were, however, some noteworthy exceptions. A few taught extensively at the secondary level, including Euphemia
Lofton Haynes, Sister Ingonda Maria von Mezynski SSpsS (Marquette 1944), who taught secondary school in both the US and in Germany, and Margaret Ellen Stump Matchett (Indiana 1946), a student of Emil Artin, who taught at the University of Chicago Laboratory School ([6]).

Some individuals devoted much of their careers to academic administration, most notably Jean Brosius Walton (Pennsylvania 1948), who wrote her dissertation in number theory with Hans Rademacher after serving as an assistant dean at Swarthmore. Upon graduation, Walton faced a choice: should she return to administration, or pursue a career in mathematics teaching and research? Ultimately, she chose administration, serving as dean of women, and later dean of students, at Pomona College ([15, p. 198]). In addition, some of the Catholic sisters were called upon to serve as administrators of the colleges that employed them, continuing a pattern observed by Green and LaDuke among the PhDs of the 1930s ([7, p. 63]). For example, Sister Rose Gertrude Calloway CSJ (Catholic 1948) served as dean of residential students, academic dean, and president of Mt. St. Mary’s College in California.

In a deviation from the pattern of pre-1940 PhDs, some of the 1940s women worked, at least for a time, entirely outside of teaching. Mary Dean Clement (Chicago 1941), a student of E. P. Lane, taught mathematics at Wells College, the University of Miami, Northwestern, and Chicago before joining the staff of the Chicago’s Institute for Air Weapons research, where she spent most of the rest of her career. Ruth Eileen O’Donnell Goodman (Pennsylvania 1944)—like Jean Walton a student of Rademacher—began her career teaching at Syracuse, Queens College, and Duquesne before moving on to the Westinghouse Labs. Madeline Mary Johnsen Alexander (Stanford 1946), who wrote a dissertation in probability under George Pólya, taught at Purdue and the University of Delaware before moving to industry, where she worked for North American Rockwell and TRW. Frances Renee Brand Bauer (Brown 1948), a student of the applied mathematician Willy Prager, worked first for the defense contractor Reeves Instrument Corporation and then moved to the research staff at the Courant Institute of NYU. Bauer’s career at Courant, where she collaborated with Paul Garabedian, spanned nearly half a century ([5]). And, during the 1950s and early 1960s, finding academia an inhospitable place for Black women, Evelyn Boyd Granville worked for the Army, IBM, and a number of private companies associated with the space program. She ultimately returned to teaching in 1967, first at Cal State Los Angeles, then at Texas College, and finally at the University of Texas at Tyler, from which she definitively retired in 1997.

To varying degrees, many of the forties women PhDs—especially those who remained in academia—pursued mathematical research after the doctorate. Among those already mentioned, Domina Spencer, Esther Seiden, Dorothy Maharam Stone, Josephine Mitchell, Julia Robinson, Jane Cronin Scanlon, Mary Ellen Rudin, and Frances Bauer had extensive research careers. To these could be added the names of Christine S. Williams Ayoub (Yale 1947), Yael Naim Dowker (Harvard/Radcliffe 1948), and more. Still others achieved a satisfying balance between teaching and scholarship that I’ve described elsewhere using the term scholar-teacher [15, p. 44]. Among those for whom this description seems especially apt are Marion Dell Wetzel (Northwestern 1943), Winifred Alice Asprey (Iowa 1945), Grace Elizabeth Bates (Illinois 1946), Mary Dolciani, Margaret Frances Willerding (St. Louis 1947), and Helen Kelsall Nickerson (Harvard/Radcliffe 1949). The individuals named are intended as examples only, and are in no way meant to constitute an exhaustive list.

In general, the lives and careers of these 86 American women PhDs reflect the social, economic, and political transitions of World War II and the postwar period. In my previous work ([15]), I’ve examined the lives of 17 of them in considerable depth and detail; profiles of many more are scattered throughout the literature and across the web. But in my view, all 86 of them are worthy of much deeper, more careful, and more systematic study.

The Women Becoming Mathematicians Website
As of early January 2021, the Women Becoming Mathematicians website ([17]) provides very basic biographical information on 86 women PhDs of the 1940s—as well as 106 women PhDs of the 1950s (who are worthy of detailed consideration in their own right). Each woman in the database has her own dedicated page, which currently includes the following basic information: her date and place of birth (and death, if applicable), her undergraduate and graduate institutions and degrees earned, the name of her doctoral adviser, and her place of primary employment. Beginning in spring 2021, I will supplement this with additional information from the database, which currently exists as two Word text files and two sortable Excel spreadsheets. The current plan is to provide each individual’s supplementary data as a pdf that can be viewed or downloaded from the webpage. While the data varies from person to person in its level of depth and detail, the sources of the information are all fully documented, and that source documentation will soon be readily available at the site.

I mean for the website to be a (hopefully permanent) repository for information from the database. In addition, I have a host of other documents—paper and electronic; text, image, and audio—which I plan to donate, sometime in the near future, to an appropriate repository. My goal is to make these items as accessible to scholars as possible. Ideally, I’d like these materials to reside among both the physical and the digital collections of the Archives of American Mathematics at the Dolph Briscoe Center for American History of the University of Texas at Austin.
Each year in my History of Mathematics classes, I meet a new group of students who know very little about the history of women in mathematics. Ultimately, my goal is to ensure that the history of American women in mathematics, at least, is not lost. The website, the database, and the accompanying materials constitute just one small but significant step toward meeting this goal.

References


Credits

Author photo is courtesy of Douglas Slauson.
Table 1. Women Mathematics PhDs from US & Canadian institutions, 1940–1944

<table>
<thead>
<tr>
<th>Year</th>
<th>Name</th>
<th>PhD-granting institution</th>
</tr>
</thead>
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<td>Sister Elisabeth Frisch OSB (1901–1993)</td>
<td>Catholic</td>
</tr>
<tr>
<td></td>
<td>Katharine Elizabeth Hazard (1915–1992)</td>
<td>Chicago</td>
</tr>
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<td>Sister Mary de Pazzi Rochford OSF (1897–1984)</td>
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<td></td>
<td>Sister Ingonda Maria von Mezynski SSpS (1911–2009)</td>
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Table 2. Women Mathematics PhDs from US & Canadian institutions, 1945–1949

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<th>Year</th>
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<td>Winifred Alice (“Tim”) Asprey (1917–2007)</td>
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<td>Corinne Rose Hattan (1903–1964)</td>
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<tr>
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<td>Margaret Ellen Stump Matchett (1918–2002)</td>
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<td></td>
<td>Kathryn Ann (“Kay”) Morgan (1922–2010)</td>
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<td>Irma Ruth Moses Reiner (1922–2014)</td>
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<td>Marie Anna Wurster (1918–2010)</td>
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<td>1947</td>
<td>Christine S. Williams Ayoub (1922– )</td>
<td>Yale</td>
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<td>Miriam Amalia Lipschutz-Yevick (1924–2018)</td>
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<td>Evelyn Agnes Fix (1904–1965)</td>
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<td>Jane Smiley Cronin Scanlon (1922–2018)</td>
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<td>Evelyn Boyd Granville (1924– )</td>
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<td>Maria Alice Weber Steinberg (1919–2013)</td>
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All Girls All Math

Lindsay Augustyn, W. James Lewis, and Judy Walker

All Girls All Math is a summer math camp for high school students held each year at the University of Nebraska–Lincoln (UNL). Now in its 24th year, the camp offers high school students with a strong interest in mathematics an opportunity to learn mathematics not studied in their schools while immersed in a community of other young students who share their interest in mathematics. The program has proven to be an outstanding experience, not only for the participants but also for the UNL students who are employed as tutors and coordinators and for alumnae who return to teach the camp.

The Beginning

When coauthor Judy Walker joined the UNL mathematics department in 1996 as a new Assistant Professor, she was one of only three women in the department. The others were Wendy Hines, who was also an Assistant Professor, and Professor Sylvia Wiegand, a future President of the Association for Women in Mathematics. Eager to contribute to making mathematics a more attractive profession for women, Walker and Hines began discussing possible activities that would encourage high school girls to consider mathematics as a college major. Two ideas were central in developing the All Girls All Math summer camp: introducing new mathematical concepts at the high school age and meeting new peers who also love mathematics. They wanted to help participants learn mathematics that was outside of the usual high school curriculum—the camp’s content should be accessible to high school students without being either remedial or accelerated—and, ideally, it would be connected to their own research. They also had the idea that girls would benefit from an opportunity to meet other young girls who shared their love of mathematics, learning this new mathematics in an environment free of the social tensions that are often present in their own classrooms.

To make the proposed All Girls All Math summer camp a reality, Walker and Hines needed staff and financial support. They applied for and received a grant from the Mathematical Association of America’s (MAA) Tensor program. Additional funding was provided by UNL’s College of Arts and Sciences and Department of Mathematics. Staff support was provided by UNL’s Center for Science, Mathematics and Computer Education (CSMCE).

Forty Nebraska high school students applied, and 14 highly talented girls were selected to attend the weeklong camp, which was held in July 1997. Walker taught a course on number theory and cryptology, and Hines taught a course on fractals and chaos. The campers lived in a university residence hall, ate dormitory food, studied mathematics, and made friends with girls like themselves who enjoyed mathematics. For many campers, this experience was their first time being part of a community of peers who shared their interest in mathematics—a welcome relief from the sense of isolation they often felt in their own high schools’ math courses.

Eager to build on the success of the first camp, Walker and Hines proposed having two camps in Summer 1998 and offered All Girls All Math as a national program, reserving about half of the spots for Nebraska students. Advertising nationally, applications increased significantly, and 34 young women had the opportunity to participate in one of two camps held that summer.

UNL has now hosted 35 camps over 24 years. In total, about 780 high school girls have participated in All Girls All Math. The first nine camps averaged 14 students per camp, while over the past decade the program averaged 26 students per camp until 2020, when the COVID-19 pandemic required a new approach. A handful of participants have come from countries outside of the US.
The Camp and its Curriculum

The All Girls All Math (AGAM) summer camp starts on a Sunday afternoon as campers arrive throughout the day, check into the residence hall and with camp coordinators, spend time getting to know one another, and play games. Sunday evening, campers are welcomed by the chair or another leader from the Department of Mathematics at UNL. To motivate the students to quickly engage in problem solving and recognize the benefit of working in groups, a few problems are posed, and the participants are offered prizes for their solutions. The activity supports team building as the students form problem-solving groups and share the prizes if they win.

Campers participate in the cryptography course created by Walker for three hours each morning. Typically, there is a one-hour lecture followed by two hours where campers work together on problem sets in groups of two or three. The course begins with a historical discussion of cryptography as they explore the Spartan Scytale and the Caesar cipher and, more generally, substitution ciphers. The participants study modular arithmetic, greatest common divisors, and solve linear congruence equations. They learn to use Maple to do computations and engage in a discussion about exponentiation modulo \( n \) and the idea of “fast exponentiation.” Much of this is done by hand so they understand it is feasible before they begin using Maple. The group moves on to Euler’s theorem and a discussion of RSA public key cryptography. Students get excited by the idea that all of the mathematics they have studied has such a practical application. The campers recognize that encryption using small primes is not secure, but that using Maple, it is easy to find relatively large primes, while factoring a large number into a product of primes remains hard. The campers have fun using public keys to send encrypted email to one another. The course ends with a scavenger hunt where the clues are given in code.

Feedback during the camp’s early years resulted in replacing the chaos course developed by Hines with a series of minicourses (stand-alone three-hour courses). In the afternoons, the minicourses offer campers the opportunity to sample different areas of mathematics. Over time, quite a few minicourses have been offered, with topics such as aerodynamics, knot theory, fractal geometry, Boolean networks in biology, and the mathematics of the game of Set. Feedback from campers enables the organizers to refine and improve the minicourses that are most successful and retire those that do not work well in the camp setting.

Recreational activities, such as tours of the volleyball complex in the Devaney Sports Center and rock climbing at the UNL Outdoor Adventures Center, are held in the evenings to help build camaraderie [3].

The Case for Supporting Women in Mathematics

Most mathematicians strongly support the idea that for the mathematics profession to be healthy, it must succeed in developing mathematical talent among all groups. In particular, mathematics will benefit if more women earn undergraduate and graduate degrees in mathematics. However, as reported in the September 2020 issue of the AMS Notices, only 29% of the doctoral recipients in 2017–2018 were women [2]. But that’s actually down from the high of 32% in 2014. Remove the statistics and biostatistics data, and one learns that in mathematics and applied mathematics, women earn only 24% of the PhDs. The results are even more disappointing when restricted to US citizens: only 21.6% of 746 PhDs awarded in 2017–2018 to US students went to women.

Walker and Hines recognized that a program like they envisioned could contribute to attracting high school girls to mathematics majors, or more generally to STEM (science, technology, engineering, and mathematics) majors in college. In the mid-1990s, research findings began to show that girls achieve more in an all-women environment. The probability that a woman college student will obtain an advanced degree is positively associated with the percentage of faculty at her undergraduate institution who are women [6]. Girls-only extracurricular programs, such as summer camps, have been shown to favorably influence a number of mathematics-related outcomes for participants, such as greater STEM participation [5, 8].

Research also shows more self-confidence in the pursuit of engineering majors among gender-segregated educational settings. While career aspirations are similar for graduates of single-sex and coeducational schools, a divide occurs when it comes to engineering [7]. Single-sex school alumnae are more likely than their coeducated peers to state that they plan to become engineers. In 10 independent schools that were studied, single-sex alumnae were three times more likely than women graduates of coeducational schools to report that they intended to pursue a career in engineering [7].

Krings [4] found a correlation between the number of women who earned master’s degrees in mathematics and the percentage of women at that university, whether faculty, administrators, or fellow students. This finding suggests that high school girls are more likely to pursue mathematics if they have more women role models and meet more women mathematics students like themselves [3].

Benefits to Current and Former UNL Students

While the obvious benefit of AGAM is to the high school students who participate in the program, there is also a clear benefit to UNL undergraduates, graduate students, and PhD alumnae.

After a few years, Walker and Hines realized that they could not sustain the program by being its instructors every
summer, and decided instead to hire women who had earned their PhD at UNL and were now faculty members at other institutions. To date, at least 25 alumnae have had the opportunity to return to Lincoln and teach the camp.

Katie Johnson, Associate Professor of mathematics at Florida Gulf Coast University (FGCU), who earned her PhD from UNL in 2012, was a camp instructor in 2013 and 2017. In her current position, she helps organize GEMS (Girls in Engineering, Math, and Science), a weekend program for middle school students in which they complete hands-on STEM activities. She said her experience at UNL influenced her work at FGCU by encouraging her to not shy away from difficult topics that are not part of the usual K–12 curriculum, emphasizing the importance of such experiences for middle school students, and demonstrating that when done well, these programs can be lots of fun for everyone involved.

Erica Johnson, Associate Professor of mathematics at St. John Fisher College, has helped teach the codes course at least seven times since graduating from UNL in 1998 and writes: “It has been a personal and professional pleasure to have been involved with AGAM through the years. Working with curious students is its own reward; equally rewarding was being part of an experience that brings young women together and creates a space where they are encouraged to love math and science.”

To support students as they engage in problem solving, AGAM hires UNL women graduate students to lead problem-solving sessions. In addition, the graduate students live in the dorms with the campers, serve as chaperones, and, quite naturally, serve as mentors for the girls. To date, more than 20 UNL graduate students have participated in the program.

While the camp is organized by a talented staff in the CSMCE with expertise in event planning, each year an undergraduate is hired to serve as the camp coordinator.

Amy (Been) Bennett, now a postdoctoral fellow in the UNL Department of Mathematics, was involved with AGAM as an undergraduate coordinator during three of her four years as a mathematics major at UNL, from 2010 to 2012. Bennett remembers how the students arrived feeling a bit intimidated, but left as part of a vibrant group of problem solvers—and friends.

In the early years, the undergraduate coordinators were traditionally mathematics majors. With the recent expansion of summer REUs and mathematics internships, the CSMCE has turned its attention to recruiting secondary education majors to serve as the undergraduate coordinators. The immersive weeklong camp gives these aspiring high school teachers the experience of how to teach and lead high-school-aged students, as well as the opportunity to act as a mentor and role model to them. The impact of the camp on the undergraduate coordinator can be just as valuable as on the participants.

Lessons Learned from Alumnae

Keeping in touch with AGAM alumnae can be a challenge, as the initial address the CSMCE has for the participants is their parent’s address. As they transition to college, graduate school, and work, the campers move, and it is hard to keep track of them. The program surveyed alumnae in 2003, 2011, and 2019.

In 2019 an online survey was sent to 392 of the alumnae from 1999 to 2017 for whom the CSMCE had what was believed to be active email addresses. Responses were received from 191 former campers (a 49% response rate). At the time, 28% were still in high school, 47% were undergraduates, 5% were in graduate school, and 20% were no longer in school. Among the responses, 120 (63%) commented on how the camp had made a positive impact on their lives.

“AGAM gave me confidence in my mathematical and problem-solving skills and showed me that not all mathematicians are men. The math club at my high school was composed predominantly of boys, particularly among the highest achievers,” said Carolyn Brown Kramer, Assistant Professor of Practice in the Department of Psychology at UNL who participated in the camp in 1998. “It also expanded my social network in a positive way during the challenging high school years and gave me a sense of independence to be away from home for a week.”

Six responders explained how AGAM launched their interest in an engineering career or major. The exposure AGAM gives to graduate education and to meeting women who have earned master’s and doctoral degrees provides role models to the campers. Twenty-three of the 191 alumnae have either earned a graduate degree—including two PhDs, one MD, and two JDs—or were enrolled in graduate school at the time of the survey.

The program in 1998 inspired one Nebraska student to pursue becoming a secondary mathematics teacher. Jill Edgren, now a math teacher in Wood River, Nebraska, made lasting connections with Nebraska faculty, including Hines, and later earned a master’s degree from UNL and was selected as a Nebraska Noyce Master Teaching Fellow as part of the National Science Foundation’s Robert Noyce Teacher Scholarship Program.

“AGAM truly took me to a four-year undergraduate program, as Dr. Cheryl Miner encouraged me to pursue more than an associate degree,” said Edgren, who went on to become president of the Nebraska affiliate of NCTM. “I went into math education and saw her promoting the Math in the Middle program, and I felt inspired to apply. Ultimately, I earned a master’s degree. This degree and a repeated experience with Dr. Wendy Hines, since we first met through AGAM, opened the door for me to begin to teach dual-credit courses.”

Of the 118 responders who provided major and minor information, STEM majors and minors collectively make
up 80% of the alumnae. A 2012 camper reported seeing a fellow participant in college, and they still keep in touch occasionally now that they are out of school.

Whether residential or online, fostering tight-knit friendships and preserving a sense of community are high priorities in the program’s mission. In fact, 36% of the 191 respondents said they still stay in contact with another participant.

A camper from 2017 described not only the close friendship she made during camp with a group of girls but also how they remained in contact during college visits. “I had a group of myself and four other girls, one of which was my roommate, and we did everything together. We still have group chats on Snapchat and texts that are active, we all have Streaks on Snapchat, and we still comment on every photo someone posts on Instagram. One of my friendships in particular is with [one of the girls from Los Angeles, and I’m from Michigan]. That’s a huge distance, yet she still remembered me when she came to Michigan for a college visit. It was very special, and unique, to have that friend from across the country be able to see you again, and I wouldn’t have gotten that if it wasn’t for this program.”

For 2019, the most recent in-person camp, participants mentioned, in an anonymous survey after the camp ended, that the best parts of camp were: being surrounded by people who enjoy math; learning one topic that continued all week, while also being introduced to a new topic each day; making new friends; helping one another out when they had questions; and feeling more connected and less out of place in the mathematical world.

Adapting to an Online Camp in 2020

Today, the benefits of this successful program are sustained with the support of current UNL mathematics Professors Yu Jin and Mikil Foss. As we received applications for the 2020 camp, the nation and world were impacted by the COVID-19 pandemic. The decision was made to refund all registration fees, to offer an automatic acceptance to the 2021 camp, and to provide an online camp to students who were still interested in participating virtually.

One of UNL’s graduate students, who would receive her PhD in August 2020, was hired to teach the course. The undergraduate coordinator—a secondary education in science major—with the help of an undergraduate math major, incorporated effective and engaging virtual activities throughout the weeklong schedule that would mitigate the loss of in-person interaction. The campers took part in solving a virtual escape room and participated in online workshops with Lincoln’s Duncan Aviation, the University of Nebraska State Museum, and the National Museum of Mathematics in New York City.

Fourteen of the 15 online campers who completed the 2020 evaluation agreed that they felt they belonged during the virtual experience. There was strong agreement that the woman-focused atmosphere was a valuable aspect of the program.

“Before the camp, I doubted my commitment to math,” said a 2020 online camper. “I also didn’t really want to pursue math as a major. After the camp, I realized that I do love math and not having a clear, perfect goal right now is OK.”

Read more about the online transition AGAM made in 2020 in the December 2020 issue of AMS Notices in the Early Career Good Ideas section [1].

After the successful iteration of the camp in a virtual setting in 2020, the program can more easily extend its reach to students globally. One participant in 2020 joined on Zoom from Brazil. In the future, the camp will be offered virtually, until a safe return to traveling and hosting in-person gatherings can occur.

Summary

Twenty-three years after the first camp, AGAM has grown to be a nationally renowned summer program that fosters lasting connections and talented future mathematicians. AGAM has acted as a springboard to give participants the confidence to major or pursue a career in mathematics or another STEM field. Alumnae have gone on to choose careers in a variety of fields, including architecture, engineering, teaching, finance, medicine, and the law. Participants also continue to share with us how they cross paths along their mathematical journeys, using the closed Facebook group for AGAM alumnae and responding to surveys.

Like many mathematicians who have created programs for students at the K–12, undergraduate, or graduate level, the AGAM organizers have learned that it is easier to create than sustain a program. Much of the challenge comes down to finding sufficient resources, including money, faculty, and staff support.

In addition to local support, AGAM has been supported by the National Security Agency (NSA), the MAA (five awards), and the AMS. The AMS has provided AGAM’s most consistent support. Starting in 2000, AGAM received 19 awards in 21 years from AMS’s Epsilon program averaging $5,000 per award. Equally important, the renewed AMS support provides evidence of the program receiving national recognition.

More recently, the University of Nebraska Foundation has received donations from the department’s alumni as well as AGAM alumnae. The program hopes to develop this into a dependable source of support. The 2020 camp also received support from Nebraska’s NSF EPSCoR grant.

The cost per camper of an in-person experience is approximately $1,200, not including the cost of staff support. In addition to extramural support, the program relies on registration fees. Initially, the registration fee was $125 for Nebraska students and $250 for out-of-state students. As costs increased and funding decreased, the program instituted a voluntary, tiered registration fee system where
a family could choose their registration fee ($1,000 or $500 for out-of-state students and $1,000, $500, $350 for Nebraska students) based on the family's ability to pay. To keep the program accessible for low-income students, scholarships are offered to cover the cost of the registration fee as well as travel scholarships for students traveling from outside Nebraska. This support was made possible by the AMS, which in recent years has specified that half of its support should go toward student scholarships, and by a University Foundation donor.

With the addition of virtual events, getting creative in how mathematics is presented to adolescents can benefit how math is perceived and positively influence the lives of young students.

In the words of former AGAM instructor Dr. Katie Johnson, "Play and creativity in math are critical to increase interest in mathematics."

References
Giovanni Battista Guccia: Pioneer of International Cooperation in Mathematics

Reviewed by Karen Hunger Parshall

Giovanni Battista Guccia (1855–1914), Sicilian nobleman and mathematician, was a man with a vision. In 1884, he founded and bankrolled the Circolo Matematico di Palermo in an effort to correct what he described to Luigi Cremona as “the miserable state of abandonment of the studies of mathematics in Palermo” (p. 94), his hometown. Four years later, he had succeeded in turning what had been a pamphlet detailing the society’s activities into a new mathematical research journal, the *Rendiconti del Circolo Matematico di Palermo*. And, by at least 1904, he had conceived the idea of transforming the Circolo into an international mathematical society that would, in his words, “spread and disseminate mathematical production worldwide, making use of the progress achieved by modern civilization in the field of international relations” (p. 158). In 1914, no less a mathematician than Edmund Landau was in Palermo to fête the society’s thirtieth anniversary. As he described it, the Circolo had “only a minority of its members from the city where it is situated, but … reunites almost a thousand mathematicians from all countries of the world and, among these the greatest and most illustrious scholars from Italy, Germany, England, France, the United States, Hungary, and from all nations where our science is cultivated” (p. vi).

Guccia’s life and, in particular, his conception and animation of the Circolo and its *Rendiconti* shape the story that Benedetto Bongiorno and Guillermo Curbera tell in their book. Bongiorno, former Vice President of the Circolo and a member of the Department of Mathematics and Computer Science of the Università degli Studi di Palermo, and Curbera, curator of the Archives of the International Mathematical Union and a member of the Department of Mathematical Analysis at the Universidad de Sevilla, began their collaborative project with questions that may seem simple: What was Guccia’s family background? What was the source of his personal wealth? Why did he pour his private resources into a mathematical society and its publication? The answers to these questions uncovered, they detail Guccia’s “circumstances, his projects and achievements, and his place in the development of an international community of mathematics” (p. x).

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more euphonious ‘Thrinacis.’ Its shape is defined by three capes: Peloritis, which with Caenys and Columna Rhégino-rum forms the strait; Pachynus, which lies out towards the east and is washed by the Sicilian Sea, thus facing towards the Peloponnesus and the sea-passage to Crete; and third, Lilybaeum, the cape that is next to Libya, thus facing at the same time towards Libya and the winter sunset. As for the sides which are marked off by the three capes …” (p. 2). Here at the beginning, and in numerous places throughout the text, the reader is left to wonder at the relevance to the life and times of Guccia of information presented. There is an unfortunate tendency both to over-quote and to return to the distant past in an effort to set the historical stage.\footnote{Much later in the book (p. 154), and without recalling what was revealed on p. 2, the reader does learn that the trinacria was incorporated in the Circolo’s logo thus symbolizing its Sicilian roots.}

Still, for those readers unfamiliar with Sicily, there is much here to learn, such as that “[a]t the end of the fifteenth century, the Spanish crowns of Aragon and Castile joined, and as a result Sicily became one of the many possessions of the King of Spain, who named viceroys to rule the island. Spaniards brought new crops from America (such as tomatoes, corn, tobacco, and also the prickly pear cactus), as well as the Inquisition, causing the expulsion of Jews from Sicily in 1492” (p. 5). A genealogy of the noble Tomasi family—which traces back on Sicily to the sixteenth century—is also included since one later member of that family, Giulio Fabrizio Tomasi, was Guccia’s uncle, had scientific interests (in astronomy), and may “possibly” (p. 27) have influenced his nephew. Although this influence is only speculated, a section on the development of astronomy in Palermo follows that opens with descriptions first of the Ptolemaic and then of the Copernican models before moving to the founding of the astronomical observatory of Palermo by Giuseppe Piazzzi in 1820. Only after all of this general scene-setting does the reader get to the authors’ original and painstaking genealogical work on the Guccia family, which not only situates the mathematician within the Sicilian aristocracy but also pinpoints the link between the Guccia and Tomasi families.

The narrative next moves to Guccia’s “Formative Years” (p. 35) but, here again, periodically stalls. There are overly detailed historical interludes, this time sections on the educational system of the Italian Risorgimento, that is, the period from roughly 1815 through the unification of the independent Italian states in 1861, as well as on the history of the university and mathematics in Sicily and on Italian science and mathematics during the Risorgimento. While these topics merit mention and at least a sense of them is important to the story, judicious editing—with the question “what does the reader actually need to know to understand Guccia and his life?” in mind—would have significantly pared them down and greatly improved the book’s flow and focus. The section on the university and mathematics in Sicily, for example, goes back to what the authors describe as a “mandatory” beginning “with Archimedes, known as ‘the greatest mathematician of Antiquity’” (p. 44), while that on Italian science and mathematics, ostensibly in the Risorgimento, nevertheless reminds the reader at some length that “[t]he Renaissance produced in Italy not only beautiful art but also deep mathematical activity” (p. 55) and then lists the names of Italian mathematical scientists beginning with that of Luca Pacioli (1447–1517) and running through to Girolamo Cardano (1501–1576), Galileo Galilei (1564–1642), and Evangelista Torricelli (1608–1647), among others. Despite these diversions, the reader comes away with a reasonably clear sense of Guccia’s education, first in engineering at the Technical Institute of Palermo but, after 1875, in pure mathematics at the University of Rome.

As a twenty-year-old student, Guccia had attended—perhaps, the authors again speculate, owing to Tomasi’s influence—a meeting of the Società Italiana per il Progresso delle Scienze that was held in Palermo in 1875. On that occasion, Guccia met Rome’s Luigi Cremona and subsequently resolved to abandon engineering for pure mathematics. By 1880, the young Palermitan had defended a thesis “On a Class of Surfaces Representable Point by Point on a Plane”—directly in line with his advisor’s work in algebraic geometry (the authors provide some mathematical details here and elsewhere in their book)—and had returned to Palermo after a postdoctoral summer and fall that had taken him to Paris, Reims and a meeting of the Association Française pour l’Avancement des Sciences, and finally London. In all of these places, he met and interacted with some of the late nineteenth century’s leading mathematicians. These connections to the international mathematical community—as well as new ones that he made in Scandinavia and elsewhere throughout the remainder of his career—strongly influenced his subsequent thoughts and actions.

Next followed what the authors term a period of maturation” (p. 90) from 1881 to 1883 during which Guccia worked on his mathematics, tended to his family’s business of supplying water to the city of Palermo, and continued to travel both for his health and for mathematics. The twenty-eight-year-old mathematician also “conceived the idea of founding a mathematical society in Palermo, with the aim of having a place where the [city’s] mathematicians … could meet and discuss current research problems” (pp. 95–96). As the authors comment, “[i]t is remarkable that the mathematicians of the University of Palermo followed” (p. 96) the young mathematician’s lead, especially since, at that moment, he was still an independent scholar. Yet, follow they did.

The Circolo Matematico di Palermo was founded in 1884, and one year later, the group, which had
started out with twenty-seven members, had increased to thirty-four. Regular meetings were held in Guccia’s palazzo: a library had been started; and Guccia had written almost a dozen research papers. As he told Cremona in March 1885, “[t]he Circolo thrives and the mathematical awakening achieved in Palermo is remarkable” (p. 103). By 1888, Guccia had also transformed the Circolo’s Rendiconti into a true research journal complete with an editorial board that consisted of five mathematicians from Palermo and fifteen from elsewhere in Italy. The year 1888 also marked the admission of foreigners—among them, the Swedish mathematician and founder-editor of Acta Mathematica, Gösta Mittag-Leffler—into the Circolo’s membership.

If, as the authors characterize it, this represented the end of the “first stage” (p. 85) of Guccia’s organizational efforts, the “second stage” (p. 127) from roughly 1889 to 1908 involved his “most ambitious” goal, that of “developing the Circolo into an international association of mathematicians” (p. 127). Indeed, at the time of its thirtieth anniversary, three in four of the Circolo’s members were foreign, with leading mathematicians like William Young and G. H. Hardy from Great Britain, Henri Poincaré and Camille Jordan from France, Felix Klein and David Hilbert from Germany, Teiji Takagi from Japan, Wacław Sierpinski from Poland, and Eliakim Hastings Moore and Oswald Veblen from the United States lending their support to Guccia’s venture.

This period also witnessed Guccia’s successful application, in 1889, for the chair of higher geometry at the University of Palermo and much political intrigue. Mathematicians outside of Palermo tried to thwart Guccia’s plans in favor of an agenda to found an Italian—as opposed to a Sicilian—mathematical society. They also attempted to scuttle his efforts to have the quadrennial International Congresses of Mathematicians (ICMs) (begun in 1897)—and not a society associated with one particular country—serve as the venue for the internationalization of mathematics.

In the end, the Circolo coexisted with both the Unione Matematica Italiana (founded in 1922 by Luigi Bianchi, Vito Volterra, and Salvatore Pincherle) and with the ICMs. As evidenced by the encomiums delivered in 1914, the Rendiconti was perceived as perhaps Guccia’s most fundamental organizational contribution. In Vito Volterra’s view, Guccia had given “to his journal an international imprint, thus placing it on secure and solid grounds. Due to esteem and sympathy, he acquired for the Rendiconti works by mathematicians from all over the world. With assiduous, persevering and ingenious work, with fine discernment, he was able to choose among authors, among schools, among directions, so that the Circolo has published works that have made history in the scientific world” (p. 203). Moreover, Guccia was personally credited with having supported mathematicians—both those just starting out and those well established—at critical moments in their careers by carefully editing their papers and publishing them with dispatch. A series of eleven color reproductions from the archives clearly reveal the acuity of Guccia’s editorial eye and provide a rare glimpse into the editorial process.

Guccia died just months after the Circolo’s thirtieth birthday. His passing—as well as the outbreak of World War I followed by Mussolini’s rise—severely taxed the mathematical society. When the Allied bombardment of Sicily destroyed large swaths of Palermo, the printing house that Guccia had set up in 1893 for the publication of the Rendiconti was one of the casualties. Still, “[a]fter World War II, the Circolo Matematico di Palermo was reconstructed and the Rendiconti commenced the publication of its second series” (p. 231). Guccia’s vision thus, at least in part, continues to be realized today.

The English-reading public owes Bongiorno and Curbera a debt of gratitude for making this story accessible to them, even if the authors have retold much of the story that Aldo Brigaglia and Guido Masotto presented in Italian in 1982. It is unfortunate, however, that their efforts in English were not more thoroughly supported by their editors at Springer, since linguistic infelicities, although not ultimately detrimental to an understanding of the intended meaning, riddle the book’s otherwise beautifully produced and lavishly illustrated pages. This lack of effective editing would seem to be an increasing problem with texts by non-native English speakers and suggests that publishing companies should be more mindful of their responsibilities as publishers and editors.

Karen Hunger Parshall

Credits
Book cover is courtesy of Springer International Publishing.
Photo of the author is courtesy of Brian Parsons.

2Aldo Brigaglia and Guido Masotto, II Circolo Matematico di Palermo (Bari: Edizioni Dedalo, 1982). As the authors of the present book see it that earlier work, although “fundamental and pioneering” (p. ix), drew rather exclusively from the archive of the Circolo and the Rendiconti, which limited its authors’ ability to uncover more “about the person who dedicated his life both to the Circolo and the Rendiconti,” (p. ix), namely, Guccia.
Secrets of the Surface
The Mathematical Vision of Maryam Mirzakhani
by George Csicsery

Secrets of the Surface is a glorious documentary, suitable for a general audience, about the life and achievements of Maryam Mirzakhani (1977–2017). Her brief life was full of firsts: Mirzakhani was the first female Iranian to earn a gold medal (and a perfect score!) at the International Mathematical Olympiad (IMO) and the first female Fields Medalist. Secrets of the Surface paints a vivid and detailed picture of a true superstar, starting with her childhood and adolescence in Iran before moving on to her doctoral work at Harvard, her professorship at Stanford, and her Fields Medal.

Friends and family, mentors and collaborators all contribute to the story, which is equal parts touching, enthralling, and informative. Mirzakhani herself narrates a surprising amount of the movie (these passages come from a 2014 recording produced by the Simons Foundation). Computer-generated illustrations and animations make Mirzakhani’s work come to life in a manner that is understandable by a wide audience. The filmmaker, George Csicsery, is well known in the mathematical community for his documentaries about famous mathematicians, such as Shiing-shen Chern, Paul Erdős, Ron Graham, Paul Halmos, Julia Robinson, and Yitang Zhang. The Mathematical Sciences Research Institute (MSRI) supported the making of the film, which can be streamed through Vimeo or purchased in DVD or BluRay format.

Secrets of the Surface is a touching and inspirational tale that is perfect for undergraduate mathematics majors. The filmmaker encourages public screenings and provides the e-mail address geocsi@zalafilms.com for those interested in arranging such screenings. A fourteen-page discussion guide is available in PDF format on the filmmaker’s website.

x+y
A Mathematician’s Manifesto for Rethinking Gender
by Eugenia Cheng

Eugenia Cheng, author of How to Bake Pi (2015), Beyond Infinity (2018), and The Art of Logic (2019), has written another engaging mathematically inspired book for the popular audience. “The problem I am going to address in this book,” Cheng declares, “is the divisiveness of arguments around gender equality.” What is the point of bringing mathematics into a discussion of gender? Most obviously, mathematical thinking brings precision and clarity to definitions and arguments. Throughout x+y, Cheng points out the many flawed arguments and misuses of logic and statistics that permeate contemporary discussions of gender. “Math gives us a framework for making justifications and also for evaluating them,” she explains, “so it gives us a way of assessing the value of any particular opinion. This is why math can be relevant to all sorts of things that don’t appear to be obviously ‘mathematical’.”

Although there are a few biographical sketches of important and influential mathematical figures, such as Emmy Noether and Maryam Mirzakhani, a dash of elementary algebra, and a couple of graphs, this thought-provoking book is aimed for a mass-market audience and assumes no knowledge of advanced mathematics. Nevertheless, Cheng’s own research field, category theory, informs her approach to gender. She focuses on relationships between individuals, often within a power structure, as opposed to set membership. “The categorically inspired approach will show us ways in which we can treat men and women the same if they relate to others in the same way,” Cheng tells us, “it means that we can find the types of behavior that are important or beneficial, find people who exhibit those behaviors, and treat those people as ‘the same’. This provides a more nuanced solution, although it is also more difficult than simply paying women the same because ‘men and women are the same’. But it is crucially less divisive.”
Hyperbolic Knot Theory
by Jessica S. Purcell

Whether for teaching or studying, it is nice when a topic comes with a wealth of concrete examples upon which to illustrate the ideas and tools. In the case of hyperbolic geometry of 3-manifolds, knot complements in the 3-sphere play this role. There is, however, another direction to the relationship. Knot theory has a longer history than 3-dimensional hyperbolic geometry, dating back to Tait’s work in the 1870s, and continues to be a widely studied and researched topic in its own right. Work of Riley and Thurston in the 1970s and work of Mochow-Prasad and Gordon-Luecke in the 1980s established that the hyperbolic structure of a knot complement is a knot invariant, thus showing that hyperbolic geometry is one of several essential tools to study knots.

Purcell deftly adopts both perspectives: knot theory as a source of illustrative examples and as a featured topic. She starts by exhibiting the complement of the figure-8 knot in the three-sphere as the gluing of two ideal tetrahedra, setting up the tools to identify its hyperbolic structure, and compute its volume. The rest of Part 1 contains a short introduction to the tools of hyperbolic geometry in dimensions 2 and 3, presenting hyperbolic manifolds using triangulations and the gluing construction, and as quotients of hyperbolic space by the action of a discrete isometry group. Part 1 also includes a discussion of complete hyperbolic structures and hyperbolic Dehn fillings. Part 2 deals with the hyperbolic geometry of special families of knots such as twist knots, 2-bridge knots, and alternating knots where tools are abundant and edifying. Part 3 focuses on knot invariants coming from hyperbolic geometry which have been important in building the still growing “census” of 3-manifolds and their properties.

The prerequisites for this book are a basic background in fundamental groups and covering spaces and some experience with differential topology and Riemannian geometry. With many exercises, illustrations, and useful references, the book is designed for an interactive course, but could also be used for self-study or as a reference that brings its readers quickly up to speed on a wide range of current research topics in low-dimensional hyperbolic geometry.

Hochschild Cohomology for Algebras
by Sarah J. Witherspoon

Hochschild cohomology was developed alongside group cohomology in the 1940s as a natural extension of Poincare’s topological homology and cohomology theories from the end of the 19th century. Both are important tools in algebraic topology and representation theory, and continue to be active areas of research, but compared with group theory, there are fewer textbooks on Hochschild cohomology. Part of the reason may be that it has become a broad subject with many branches coming from different applications, including algebraic geometry, category theory, and K-theory.

In this book, Witherspoon presents Hochschild cohomology with a focus on its uses in studying algebras and representation theory. She introduces Hochschild cohomology both in its original form and as a Gerstenhaber algebra, with an associative product and a nonassociative Lie bracket. She then explores examples coming from different kinds of settings such as smooth commutative algebras, Koszul algebras, and quivers, and presents the Hochschild-Kostant-Rosenberg Theorem. The heart of the book is devoted to topics like algebraic deformation theory, properties of the Gerstenhaber bracket, infinity algebras, and support varieties for finite-dimensional algebras and Hopf algebras.

The book is intended for advanced graduate students with a strong background in algebra. There are many exercises and a helpful appendix giving a quick refresher on the basics of homological algebra. The book will also be useful as a reference for algebraists interested in using tools from cohomology theory.
Mathematics has historically been used as a vital tool for providing insight and understanding on the mechanisms of the spread, control, and mitigation of emerging and re-emerging infectious diseases, dating back to the pioneering works of Daniel Bernoulli (on modeling the potential impact of a smallpox vaccine) in the 1760s and the compartmental modeling frameworks of the likes of William Kermack, Anderson McKendrick, and Sir Ronald Ross in the early 1900s. This lecture will address some of the mathematical techniques and theories used to formulate, parametrize, and analyze mathematical models for the transmission dynamics and control of infectious diseases.

In this public lecture, Gumel will emphasize discussion on infectious diseases that continue to inflict major public health and socio-economic challenges to humankind, including the ongoing novel 2019 coronavirus pandemic.
A vast array of physical phenomena, ranging from the propagation of waves to the location of quantum particles, is dictated by the behavior of Laplace eigenfunctions, i.e., solutions to the Helmholtz equation

$$-\Delta \phi_\lambda = \lambda \phi_\lambda.$$ 

In quantum mechanics, $|\phi_\lambda(x)|^2$ describes the probability density of finding a free quantum particle of energy $\lambda$ at the point $x$. It is then natural to ask how large can $\phi_\lambda(x)$ be?

Starting in the 1950s, Avakumovich, Levitan, and Hörmander proved that, for a smooth compact Riemannian manifold $M$, there is a constant $C$ such that

$$\max_{x \in M} |\phi_\lambda(x)| \leq C\lambda^{(n-1)/4}, \quad (*)$$

where $n$ is the dimension of $M$. This bound is sharp, with the zonal harmonics on the round sphere saturating it (these are eigenfunctions that have sharp peaks near the north and south poles). However, for most manifolds, the eigenfunctions are expected to be much smaller than the bound in $(*)$. More importantly, for a fixed manifold and most points $x \in M$, the value $\phi_\lambda(x)$ is not expected to saturate the bound.

In this talk we will discuss how the growth of $\phi_\lambda(x)$ responds to the long-time behavior of the geodesics that run through $x$.

To study this problem, we developed a framework in which the eigenfunction $\phi_\lambda$ is decomposed as a sum of what we call geodesic beams near the point $x$. In broad terms, a geodesic beam is a piece of the eigenfunction that has been localized to a geodesic that runs through $x$. This localization is accomplished using semiclassical analysis and is done in such a way that each geodesic beam is (locally) an approximate solution to the Helmholtz equation.

In this talk, we present the geodesic beam techniques and explain how to use them to obtain quantitative improvements on standard estimates such as $(*).$ For example, we will see that

$$|\phi_\lambda(x)| \leq C\lambda^{(n-1)/4}/\sqrt{\log \lambda},$$

whenever the point $x$ is not maximally self-conjugate at $1/\log \lambda$ scales.

Remarkably, this framework allows for the treatment of several other problems related to eigenfunction concentration, including $L^p$ norms, averages over submanifolds, and both pointwise and integrated Weyl laws. One consequence of this method is that if $M$ is any nontrivial product manifold with Laplace eigenvalues $0 = \lambda_0 \leq \lambda_1 \leq \lambda_2 \leq \cdots$, then the eigenvalue counting function, $N(\lambda) = \#\{j : \lambda_j \leq \lambda\}$, satisfies the Weyl law

$$N(\lambda) = \frac{1}{(2\pi)^n} \text{vol}_R(B) \text{vol}(M) \lambda^n + O\left(\lambda^{n-1} \frac{1}{\log \lambda}\right),$$

as $\lambda \to \infty$, where $B$ is the unit ball in $\mathbb{R}^n$. 

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Time-Inconsistent Problems

Jiongmin Yong

Consider the following initial value problem of an ordinary differential equation:

\[ \begin{align*}
    \dot{X}(s) &= f(s,X(s),u(s)), & s & \in [t, T], \\
    X(t) &= x.
\end{align*} \tag{1} \]

In this differential equation, \( u(\cdot) \), the control, is selected from some class \( \mathcal{U}[t, T] \) of measurable functions with values in a metric space \( \mathcal{U} \), and the solution \( X(\cdot) \) of (1) is called the state process. In applications, \( X(s) \) could be the total wealth from some stocks, tonnes of fish in a large lake, or the location of a moving object at time \( s \), and \( u(\cdot) \) could be a trading strategy, a harvest rate, or a driving force. Different selections of controls \( u(\cdot) \) lead to different state processes \( X(\cdot) \). To measure the performance of the control toward a goal, one may introduce the following cost functional:

\[ J(t, x; u(\cdot)) = \int_t^T e^{-\delta(s-t)}g(X(s), u(s))ds. \tag{2} \]

In this integral, \( g \) is the running cost rate and \( e^{-\delta(s-t)} \) is the exponential discounting (with discount rate \( \delta > 0 \)). To find a control that minimizes the cost functional, a classical optimal control problem can be stated as follows.

Problem (C). For a given initial pair \( (t, x) \in [0, T] \times \mathbb{R}^n \), find \( \bar{u}(\cdot) \in \mathcal{U}[t, T] \) such that

\[ J(t, x; \bar{u}(\cdot)) = \inf_{u(\cdot) \in \mathcal{U}[t, T]} J(t, x; u(\cdot)) \equiv V(t, x). \]

In the above, \( \bar{u}(\cdot) \) is an optimal control and \( V(\cdot, \cdot) \) is defined as the value function. Bellman’s principle of optimality can be stated as: for any \( (t, x) \in [0, T] \times \mathbb{R}^n \) and \( \tau \in (t, T] \),

\[ V(t, x) = \inf_{u(\cdot) \in \mathcal{U}[t, \tau]} \left\{ \int_t^\tau e^{-\delta(s-t)}g(X(s), u(s))ds + e^{-\delta(\tau-t)}V(\tau, X(\tau)) \right\}. \]

This principle gives: if \( \bar{u}(\cdot) \) is an optimal control at \( (t, x) \) with \( \bar{X}(\cdot) \) being the corresponding optimal state process, then for any \( \tau \in (t, T] \),

\[ V(t, x) = J(t, x; \bar{u}(\cdot)) = \int_t^\tau e^{-\delta(s-t)}g(X(s), \bar{u}(s))ds + e^{-\delta(\tau-t)}J(\tau, X(\tau); \bar{u}(\cdot)) \geq V(t, x). \]

This implies

\[ V(\tau, \bar{X}(\tau)) = J(\tau, \bar{X}(\tau); \bar{u}(\cdot)_{|[\tau, T]}), \]

which means the restriction \( \bar{u}(\cdot)_{|[\tau, T]} \) of an optimal control \( \bar{u}(\cdot) \) selected for the initial pair \( (t, x) \) on \( [t, T] \) is an optimal control for the initial pair \( (\tau, \bar{X}(\tau)) \) on \( [\tau, T] \). In other words, an optimal control \( \bar{u}(\cdot) \) determined at time \( t \) will stay optimal thereafter. Such a phenomenon is called the time-consistency of the control problem. There are two major reasons causing time-inconsistency: time-preferences and risk-preferences. To elaborate a bit more, normal people usually are not 100% rational and they often over-weigh the immediate satisfaction level (the utility) or regard the immediate time period more precious. Here is an example: if you are invited to referee a paper, would you review the paper immediately or would you wait until the associate editor sends you a reminder? Time-preferences play a role here. Different people could have different opinions on upcoming uncertain events. An easy example to illustrate risk-preferences is the opinion on whether to buy a risky stock.

Mathematically, time-preferences can be described by discounting, and risk-preferences can be described by subjective probability. It is possible to present such problems in a stochastic setting. But for simplicity, let us continue with the deterministic case. Problem (C) (with exponential discounting) is time-consistent, which represents a situation in which the controller (the person who is controlling the system) is rational. Now if the controller is not 100% rational, then the discounting might not be exponential. A typical nonexponential discounting is hyperbolic discounting, which is illustrated as

\[ J(t, x; u(\cdot)) = \int_t^T \frac{1}{1 + a(s-t)}g(X(s), u(s))ds \tag{3} \]

for some \( a > 0 \). With such a cost functional, the corresponding optimal control problem will be time-inconsistent, i.e., if \( \bar{u}(\cdot) \) is an optimal control for \( (t, x) \) defined on \( [t, T] \) with optimal state process \( \bar{X}(\cdot) \), then there...
will be a later time \( \tau \in (t, T] \) such that the optimal control \( \hat{u}(\cdot) \) for \((\tau, \hat{X}(\tau))\) defined on \([\tau, T]\) does not satisfy the following:

\[
\hat{u}(s) = \bar{u}(s), \quad \text{a.e. } s \in [\tau, T].
\]

For time-inconsistent optimal control problems, it is not wise to find optimal controls. Instead, one should look for equilibrium strategies. The idea is to regard the problem as a multiperson differential game in which today's self plays with future selves. To make the future reasonably satisfactory, today's self should be willing to sacrifice some immediate satisfaction. Saving for retirement is such an example. Under proper conditions, such equilibrium strategies can be constructed.

Jiongmin Yong

Credits

Photo of Jiongmin Yong is courtesy of Jiongmin Yong.
Linear biopolymers such as RNA, DNA, and proteins often need to fold stably or transiently in order to function [1–4]. Changes to the configuration of these folded biopolymeric chains may lead to changes in function, loss of function, or dysfunction (i.e., toxic effects). For instance, the misfolding of proteins has been implicated in many diseases including some cancers and neurodegenerative diseases such as Parkinson’s and Alzheimer’s disease. Sophisticated biomolecular machines known as “chaperones” help to fold biomolecules into their “correct” conformation. Intriguingly, tens of thousands of proteins are helped by a small number of chaperones, suggesting that chaperones sense some generic properties of their client proteins and are not sensitive to all molecular details. It is possible that chaperones detect certain generic chemical properties (e.g., stickiness or “hydrophobicity”), some generic geometric properties (e.g., diameter of the folded parts), or they may sense topological properties of the protein chains. Chemical and geometric properties have been extensively studied, yet the chaperone puzzle has not been resolved. The topology of folded molecular chains is a less well-understood aspect of their structure. Being able to describe, experimentally measure, and represent the topology of a molecular chain could provide additional information potentially useful in understanding biology and disease. In recent decades, we observed advances in both describing molecular fold topology and experimentally characterizing it: knot theory has been applied to classify biomolecules and their folding [5]; optical tweezers, a technology recognized by the 2018 Physics Nobel Prize, has enabled grabbing and pulling of a knotted molecular chain via its ends. The optical tweezers technology also enables monitoring chaperone-assisted protein folding in real time [6,7]. However, there is a key mismatch between the two approaches: optical tweezers typically target the molecular ends, measure end-to-end distance, and characterize the molecular folds by breaking intrachain interactions (i.e., the so-called “contacts” that represent constraints that keep two points on the chain in close physical proximity; represented by red dots in Figure 1(a), while knot theory ignores chain ends and intrachain contacts and focuses on entanglement. Molecular knot analysis often starts with connecting the chain ends through reasonable (albeit debatable) protocols. Many physical constraints on movement are ignored. In particular, knot isotopy allows a full range of flexibility and movement, but actual biomolecules can physically achieve only a subset of them. Furthermore, most biomolecules, including more than 97% of structurally identified proteins, are unknotted according to knot theory. Thus, describing the topology of a folded linear molecular chain requires new approaches [8].

My coworkers and I addressed this problem by introducing “circuit topology” [8]. It starts with the recognition that molecules ranging from proteins, to RNA, and complete chromosomes, are all folded linear chains held
Keystones

contacts. In contrast, knot theory utilizes a “top-down” approach where a chain is globally approximated as a knot, which may be split into smaller tangles.

We are extending the original notion of circuit topology to include the additional complexity of 3D fold conformations seen in natural and engineered molecular chains [9]. Here, (generalized) circuit topology refers to the arrangement of contacts within a folded chain, where contacts represent constraints of various origins, not limited to direct chemical bonds as described above. Physical entanglements, sometimes introduced by knotting, may also keep chain segments in “contact” (Figure 1(c)). Similar to contacts shown in Figure 1(b), physical entanglements can be demonstrably categorized using the language of contacts. In contrast, knot theory utilizes a “top-down” approach where a chain is globally approximated as a knot, which may be split into smaller tangles.

Figure 1. (a) Dual trap optical tweezers setup. Two laser beams grab the chain ends that are immobilized on the surface of two spherical particles. The folded chain can then be subjected to mechanical force. (b) Possible arrangements of two pairwise contacts, as described in the circuit topology framework. (c) Two examples in which chain entanglements introduce physical constraints when the chain ends are subjected to force.

Figure 2. Unfolding pathways of a chain with two contacts folded into series and parallel topology. One can readily extend the chain from its ends to make the total length of the four chains identical. The bending stiffness of the chains is ignored in these examples, for simplicity. Please see [8] for further information.

A Circuit topology utilizes a “bottom-up” approach, ignores entanglement, and focuses on intrachain contacts. The most obvious intrachain contacts in proteins are hydrogen bonds, disulfide bonds, residue-residue contacts, and secondary structures (beta-beta, alpha-alpha, and alpha-beta) depending on how much we zoom into the structure. Similarly, different types of intrachain contacts are seen in RNAs (e.g., base pairing in SARS-CoV-2 RNA pseudoknot) and genomic DNA (contacts as identified by HiC technology). The arrangement of these contacts along the chain was called circuit topology by analogy with the arrangement of elements in electrical circuits. To identify the arrangements, we choose two contacts and ignore the rest. For two contacts, three arrangements are possible: they are either in parallel (P), in series (S), or cross (X) each other (with respect to the “end-to-end axis”; see Figure 1(b)). This step allows one to represent complex topologies by a collection of simple relations, which are tabulated in the form of a topology matrix \( T_{ij} \) (circuit topology relation between contacts \( i \) and \( j \)). This simple definition of contact arrangement is based on physical insights: chain loops formed in series are independent while parallel loops are not and may facilitate the formation of each other (the so-called zipping effect). The patterns within the topology matrix can be readily quantified and related to experimental observations (e.g., folding rate). Another advantage is that the formal structure of this approach allows one to formalize topological constraints that folded chains must obey [8]. For example, the \( P \) relation is transitive (i.e., for three contacts \( A, B, \) and \( C \), if \( A \) is parallel to \( B \) and \( B \) is parallel to \( C \), then \( A \) is parallel to \( C \)) while the \( S \) and \( X \) relations are not. Such relations allow for the development of inference algorithms that help in extracting structural information from optical tweezers data. To summarize, circuit topology utilizes a “bottom-up” approach, ignores entanglement, and focuses on intrachain contacts. In contrast, knot theory utilizes a “top-down” approach where a chain is globally approximated as a knot, which may be split into smaller tangles.
circuit topology [9]. These extensions allow one to study both knotted and unknotted molecular chains all using one topological framework. Circuit topology thus provides a generic quantitative topological framework that can be readily used to study the physics of folded polymers.

We demonstrated the power of this approach by studying folding and unfolding dynamics of polymers [10]. For example, we showed (theoretically and through molecular simulations) that the circuit topology is a folding rate predictor [11]. In particular, the frequency of $R$ relations within the topology matrix correlates with the rate. One can show that when geometric measures such as contact order or chain length fail, the circuit topology of a folded chain predicts the folding rate. Interestingly, using simple chain models and circuit topology analysis, one can see how chaperones may guide folding towards the desired topology [12,13]. Chaperones constrain (protein) chains and thereby enhance the formation likelihood of certain contact arrangements. Chaperones can even introduce transient contacts within the chain by bridging two parts of the polymer, and thus can directly alter the apparent topology of the intermediate fold. This is not only important from a biological point of view, but also opens up new avenues in molecular engineering [14]. In the field of molecular engineering, a major gap in our knowledge is how to synthesize folded molecular chains, such as easy-to-fold DNA or protein origami. This is challenging on multiple levels, including designing the topology, guiding the folding process towards the desired topology (increasing the yield of the reaction), and carrying out necessary purification and characterization approaches [10,14]. Our proof-of-concept studies show that circuit topology analysis can be readily applied to address these challenges.

Circuit topology rules place constraints on unfolding transitions, allowing one to interpret optical tweezers data [8]. By grabbing the ends of a folded (unknotted) molecular chain, one can measure the end-to-end distance of the folded conformation $L_F$ (see Figure 2(a)). Obviously, a fully unfolded conformation of the chain will have an end-to-end distance $L_{U1}$ larger than $L_F$. Upon pulling the ends of the molecule, we will observe a sequence of lengths in between $L_F$ and $L_{U1}$. Not all values between $L_F$ and $L_{U1}$ are possible and often the sequence of events is not unique due to the stochastic nature of contact disruption. Furthermore, at each unfolding step, contacts belonging to the “shortest end-to-end path” will sense the mechanical force applied to the chain ends and thus break. In the special case shown in Figure 2(a), only one unfolding pathway is possible and two length changes will be experimentally observed (corresponding to breaking two contacts in steps 1 and 2). In this case, the process is deterministic. In a series conformation shown in Figure 2(b), two possibilities are available for the first unfolding step. Note that simultaneous disruption of two contacts is unlikely. In a case with a complex topology, many length transitions are possible. Allowed and forbidden transitions can be readily extracted from circuit topology of the chain. For example, in Figure 2(a), one cannot break loop $L_2$ before $L_1$ by pulling the ends of the molecule. This is trivial when we see the conformation in Figure 2(a). However, in an experimental setting one cannot “see” the contacts and in many cases the structure of the protein is not known. In this situation, circuit topology may allow one to extract structure (number and arrangement of contacts) from a series of measured lengths.

The use of circuit topology in molecular structure analysis leads to many open mathematical questions, including: What symmetries can be identified in the topology matrices? What is an appropriate metric for the comparative analysis of two distinct topologies? How do we account for uncertainties caused by a lack of resolution or noise in the experimental data? Answering these questions may require combinatorics, topology, statistical methods, machine learning techniques, and advanced concepts from programming and algorithms.

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References


Credits
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Alireza Mashaghi
AMS Prizes & Awards

Leroy P. Steele Prize for Lifetime Achievement

About this Prize
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2. Analysis/Probability (2026)
3. Algebra/Number Theory (2021)
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The Steele Prize for Mathematical Exposition is awarded for a book or substantial survey or expository research paper. The amount of this prize is US$5,000.

Next Prize: January 2022

Nomination Deadline: March 31, 2021

Nomination Procedure: https://www.ams.org/steele-prize
Nominations can be submitted between February 1 and March 31. Nominations for the Steele Prize for Seminal Contribution to Research should include a letter of nomination, a complete bibliographic citation for the work being nominated, and a brief citation to be used in the event that the nomination is successful.

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An Interview with Boris Hasselblatt

Rachel Crowell

This is an edited version of an interview with Boris Hasselblatt, whose initial two-year term as AMS Secretary began on February 1, 2021. Hasselblatt is a professor at Tufts University. The interview was conducted in fall 2020 by freelance writer Rachel Crowell.

Figure 1. AMS Secretary Boris Hasselblatt.

Notices: When and how did you first discover your passion for mathematics and its community?

Hasselblatt: It was actually a little bit late, because I started out as a physics student. Gradually, over time, I realized that mathematics was what I loved about being a physicist. So I didn’t make a change to being a mathematician until sometime in graduate school after I came to the US. But the fun started well before then. I studied in Germany (originally in Berlin), where you matriculate in a department. I was a physics student in a physics department and I was something like a year or two into the overall program and taking math classes beyond what was required. I started really having fun taking those, including by working with others. I enjoyed a community of others studying and working on homework together. Mathematics as a community happened in my life even more once I experienced a mathematics graduate program for the first time.

Notices: Your research is “in the modern theory of dynamical systems with an emphasis on hyperbolic phenomena and on geometrically motivated systems.” What excites you the most about your research currently?

Hasselblatt: One thing is the collaboration with others. In fact, there was a phase transition in my life, which was that my first papers during and after my doctoral studies were entirely solo papers. I worked on a project together with Amie Wilkinson, who’s now at the University of Chicago, and that flipped my mode of operation entirely. I have never, since that moment, done a solo research paper. I have really enjoyed the excitement of working on mathematics together. I like the fun of interacting with another person, but also that the mathematics gets more interesting because ideas come not just from me and out of nowhere, but rather from someone else, from a different perspective, from a different background. Virtually all my papers I’ve done with others are more interesting than the papers I wrote alone. So that’s the excitement. These collaborations are geographically distributed and have involved people with an amazing range of backgrounds. This brought with it a lot more learning than projects done by myself have produced, because I need to absorb these other ideas and
I've been an AMS member for a long time, since my graduate school days and continuously ever since. I've appreciated what the Society does. I have been involved in mathematics in ways that go beyond doing mathematics, like as department chair, organizing conferences, and the like; and in university administration as associate provost for a while. This seemed like an opportunity to bring these skills to bear, but in a new context where I could play a role in leadership or administration in a way that supports something larger than myself, a leading organization that is strong and well run, that is serving the mathematics community.

Another attraction is simply that this is a new challenge. I am learning now, and I will learn even faster once the job really begins. And then hopefully, I will be able to make a contribution to how the AMS can serve the community and science as well.

**Notices:** What are some of your biggest priorities as the new Secretary?

**Hasselblatt:** That's a good question. I'll be careful in answering it, because the Secretary, of course, is not meant to be at the forefront of policy or making changes. It is a role that is more focused on keeping the trains running, having some leadership in the scientific program. It is also an elected role. So I will try to be careful where I weigh in and how I do. The hope is that, in the end, she turns out to have been right.

**Notices:** What are a few key ideas or questions you have been exploring lately?

**Hasselblatt:** I'll talk about a project which I've done with my most prolific collaborator: by far, I published the most joint papers altogether with Patrick Foulon. They have covered a few different subjects, but the common theme is contact flows, and we just finished the second paper on the construction of new examples of these. He had thought up a surgery construction, which means cutting something up and sewing it back together, just like what surgery is supposed to be, to produce new examples of flows that I might describe as fake mechanical flows. Technically, these are called contact flows. One starts with free particle motion, which is also called the geodesic flow, and does this cutting and gluing, and the effect is that the resulting flow is definitely no longer of that same mechanical nature, but it retains the contact structure. That was a novelty of our initial paper on the subject. And then already quite some time ago, and in a significant way, we got into studying these a little bit more deeply. We looked into how the resulting flow is more complicated, in the sense of entropy or the typical exponential complexity. And it was interesting in two ways, one of which was that we more deeply understood these, and we are producing interesting new phenomena. And the other thing was that when Patrick Foulon gave a talk about this, Anne Vaugon came on board; she is, as well, a French mathematician. And this completely transformed the project and made it much broader by bringing in an enormous depth of contact topology, which was a subject about which I previously heard, but which I never really understood. And this produced something much more interesting and much broader. So the underlying ideas are that of producing new dynamical systems from old ones through surgery, and in this case, the novelty specifically that they retain this contact structure, which was new, and that that now turns out to, surprisingly, connect to contact topology, whose origins overlap with those of dynamical systems. We may have contributed to creating a substantial overlap between these two fields by this piece of work.

**Notices:** Why did you decide to apply for the position as Secretary of the American Mathematical Society?

**Hasselblatt:** It was several things coming together. One, I have to say, was encouragement by people to apply. Bryna Kra in particular, who has been quite involved with the AMS, encouraged me to apply, because she thought I might actually be good at this job. The more I learn about the job, the more daunting it seems. But the hope is that, in the end, she turns out to have been right.

Figure 2. Boris Hasselblatt.

I've been an AMS member for a long time, since my graduate school days and continuously ever since. I've appreciated what the Society does. I have been involved in mathematics in ways that go beyond doing mathematics, like as department chair, organizing conferences, and the like; and in university administration as associate provost for a while. This seemed like an opportunity to bring these skills to bear, but in a new context where I could play a role in leadership or administration in a way that supports something larger than myself, a leading organization that is strong and well run, that is serving the mathematics community.

Another attraction is simply that this is a new challenge. I am learning now, and I will learn even faster once the job really begins. And then hopefully, I will be able to make a contribution to how the AMS can serve the community and science as well.

**Notices:** What are some of your biggest priorities as the new Secretary?

**Hasselblatt:** That's a good question. I'll be careful in answering it, because the Secretary, of course, is not meant to be at the forefront of policy or making changes. It is a role that is much more focused on keeping the trains running, having some leadership in the scientific program. It is also an elected role. So I will try to be careful where I weigh in and how I do. I am not coming in with change priorities, but together with the Executive Director I am hoping to maintain and strengthen the way a lot of the mechanics of the AMS work. I'm really proud of what the AMS and previous Secretaries have done. And one big part of the Secretary's job that may be most suited to thinking about where I see making a difference are the Joint Mathematics Meetings and the sectional meetings. The meetings program of the AMS is important for our community, and it is well supported by the meetings department of the AMS. But it is the Secretariat, which means the Secretary and the
Associate Secretaries, in crucial ways, that have a lot to do with how these meetings go, both whether a given meeting runs well, but also how the meetings are envisioned. And I aim to make sure that the Joint Mathematics Meetings retain the breadth they had when they were joint with other mathematics societies as co-organizers. The Associate Secretaries and I look forward to supporting broader Joint Mathematics Meetings and sectional meetings. This includes diversity in every dimension and is part of increasing the diversity and inclusivity of these meetings, of the American Mathematical Society, of the profession, and of mathematics. And this is already an urgent priority for the AMS. The Joint Mathematics Meetings will be run by the American Mathematical Society alone. I am glad that we already have structures in place to host broader meetings with inspired programming that will make these meetings as successful as they were while they were run jointly with the MAA and have the same breadth. In fact, more breadth, offerings of broader interest, and a lot of vibrancy are coming in the future.

Notices: What are you most looking forward to about this position?

Hasselblatt: Initially, some of the learning because there’s still a lot of the detail that I will be learning. But fundamentally, what makes this job interesting is that the American Mathematical Society is an organization that’s well run in the sense of being really rich in highly competent and engaged volunteers, and that the operation in Providence is really solid and has a lot of talent. When working with the committees of volunteers, as well, I have these two sets of people, staff and volunteers, that form the operation of the AMS. I am grateful to be working with smart people, committed to a common cause, pulling in the same direction. Supporting the development of ideas, contributing maybe in a small way, and helping the whole enterprise succeed. The Secretary’s role to a great extent as well is to prepare business meetings and support the communication between the various committees and the Council or Board of Trustees to whom they report. So facilitating these interactions between entities as well as among individuals, on committees and across committees, to let us all lead the AMS into the future. This combination of creativity working together in a well-run organization is going to be rewarding.

Notices: What does acceptance of this position mean to you?

Hasselblatt: It’s a life-changing challenge and a chance to contribute. It may have helped when I applied for the job that I didn’t fully understand the challenges or how big the job is. But now that I’m learning, I am looking forward to the opportunities these challenges bring: to learn, grow, and build while working in a community and a team of excellent staff. It is huge that I am being so well mentored and supported by Carla Savage, the current Secretary, who will stay onboard to hold my hand for about half a year after I take on this challenge. The inspiring leadership of Catherine Roberts and the brilliance of Steven Ferrucci and Laura Byrum put me in a position to succeed. Together with strong staff and Associate Secretaries who so generously volunteer their time, I look forward to enjoying every year and hope that things will run even better and grow more exciting every year, in terms of what the AMS does, how effective the AMS is, how interesting the offerings are that the AMS has for the mathematical community, and how much difference we make altogether.

Notices: Is there anything that you would like to say about the significance of your predecessor’s contributions as Secretary?

Hasselblatt: They are pervasive and run deep, and I will be in a much better position to say that some time into my job because I will only fully appreciate just how much Carla Savage has done when I have to do it myself. I already appreciate the extent to which she understands, supports, and advances the AMS in a myriad of small details, in the big picture, and in her incredible wisdom, which shows in the way she interacts with people and the various parts of the institution, as well as in the gentle ways in which guidance is provided when needed. She supports the AMS committees, the Council, the Board of Trustees, and the President. She keeps in mind how the whole organism works to make sure that all parts of it are as efficient and as effective as possible by being connected to each other and advancing their goals in concert with the overall priorities of the Society. I think this is just a really small way in which one can describe what Carla has done. A better appraisal will come from those who have worked closely with her for a longer time than I have. And from me once I can more fully appreciate what the job is and how well she’s done it as I try to keep up with the example that she has been setting.

Notices: What are a few things you enjoy when you’re not doing math?

Hasselblatt: One of them is music. I have been singing since my high school days. And in the last two decades, I’ve done it at Trinity Church in Boston, where there is a choir that is better run than any one I ever joined before. It’s music done with a purpose, both at high quality and with an important service to community in ministry. Some aspects of travel are fun. This has involved sabbaticals and travels I have undertaken just with my wife. Some trips were to exotic locales, others local, such as fall foliage trips in New England.
Music is something that I find resonates; maybe you think of the trope that mathematics and music go together. But the deeper thing is that music, like mathematics, involves creativity but within a set of rules—although maybe that most importantly applies to composers or solo performers. For me, it has provided a community in which the pursuit of excellence and purpose is exercised together with others. In a good choir, every one member takes responsibility for the whole group and yet can also feel secure in being supported by the group as music is happening. That reflects many ways in which I had previously been enjoying my engagement with my university and working and living in my department. There is a collegial environment in which people pull their weight but can also trust others to do so. And that is the kind of life I look forward to in my engagement with the AMS as Secretary: there are going to be smart people who are very engaged, and you are pulling in the same direction taking responsibility, but you can likewise count on others to share that responsibility. That explains to some extent things that resonate with the way I look forward to embracing this new assignment.

Credits
Figures are courtesy of the Trustees of Tufts College and Momo Shinzawa.
This is an edited version of an interview with Douglas Ulmer, whose initial two-year term as AMS Treasurer began on February 1, 2021. Ulmer is a professor at the University of Arizona. The interview was conducted in fall 2020 by freelance writer Rachel Crowell.

**Notices:** When and how did you first discover your passion for mathematics and its community?

**Ulmer:** Already in elementary school, I enjoyed mathematics and seemed to be good at it. That continued through high school, so math was on my short list of options for a college major (some of the other options being comparative literature, philosophy, and music). My first real experiences of a mathematics community came later in graduate school. I’m happy that many departments are now taking more of an interest in building community among their undergraduate majors, and I think this will help inspire more students to persist in the degree and perhaps even go further in mathematics.

**Notices:** You work in algebraic geometry and number theory. What excites you about working in those fields?

**Ulmer:** I have always enjoyed the precision and structure of algebra, and I’ve also always enjoyed geometry (I’m a bit of a visual thinker), so their combination as algebraic geometry has been a passion since I first learned a bit about it in college. Most of my work in number theory has been in a subfield called “arithmetic algebraic geometry” which recasts number-theoretical questions in ways that make them amenable to geometric (or geometry-inspired) solutions. The other cool thing about number theory is that it brings tools from absolutely every corner of mathematics. It’s fun to see that and sometimes to use unexpected bits of math yourself.

**Notices:** What’s an example of a time that you used unexpected bits of math?

**Ulmer:** I have recently had occasion to use Fourier series and a hard theorem about real analytic functions in papers about number-theoretic questions.

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Ulmer: Whenever you have an algebraic curve (the solution set of a two-variable polynomial equation) you get a fancier gadget called its Jacobian. The Jacobian is usually of dimension greater than one (so in some sense is more complicated than your original curve), but it has extra structures (like a group law) that are powerful tools for studying the original curve. I’ve been thinking about some of those extra structures (the $p$-torsion subscheme of the Jacobian) for curves over fields of characteristic $p$ and also how these Jacobians break up into direct sums (the decomposition into simple factors up to isogeny) for curves over the complex numbers with extra symmetries.

Ulmer: There are more women studying mathematics and more women in faculty positions (although still not nearly the same representation in senior positions). There is greater geographic dispersion of excellent mathematicians. There are more and deeper interactions with areas of science beyond physics, and people seem to be more relaxed about “pure” versus “applied.” A larger fraction of university mathematicians appreciate that fostering success among the students we have (versus the students we wish we had) is key to the future of the profession and the future of science.

Notices: What do you think is driving the greater geographic dispersion of excellent mathematicians?

Ulmer: There are more universities with research ambitions outside the traditional centers, so more expectations for faculty research and more well-trained PhDs to do this research.

Notices: What do you think is driving the change in perspective you mentioned regarding pure and applied math?

Ulmer: We’ve seen more applications of mathematics that were previously thought to be absolutely pure, for example, applications of number theory to cryptography, applications of algebraic geometry to information transmission and other aspects of computer science, and applications of combinatorics to biology.

Notices: Can you expand on the point you made earlier about more mathematicians appreciating “that fostering success among the students we have (versus the students we wish we had) is key to the future of the profession and the future of science”? What do you mean?

Ulmer: There are at least two aspects to this. On one hand, there has been significant democratization of higher education. This is great, but it means that we have to be ready to help students from a much broader range of backgrounds succeed. And on the other, there is a much bigger demand for mathematically sophisticated workers, so we can’t afford the old “look to the right, look to the left, one of you won’t be here next term” approach that fails a large fraction of students.

Notices: In what ways have you seen public perceptions of math change?

Ulmer: There is greater recognition among the public that the mathematical sciences have a big impact (potentially positive or negative) on our quality of life via things like secure communications, data science, and pandemic modeling.
Ulmer: What are you most looking forward to about this position?

Ulmer: I was looking forward to regular visits to Providence (I was a graduate student at Brown University), Ann Arbor, and Chicago, but that is not in the cards for the moment! I also look forward to working with the volunteer leaders and staff of AMS, a very interesting, accomplished, and dedicated group of people.

Ulmer: What first inspired you to get involved with leadership in the AMS?

Ulmer: I have benefited a lot from the AMS over my career, especially from the program of meetings, the journals, and the policy committees. It seems fitting to try to help keep the organization healthy and thriving for the next generation.

Ulmer: Have you held other leadership positions with the AMS?

Ulmer: No, but I have held other leadership roles outside of the AMS, like Southwestern Center cofounder and director, director of graduate studies, and department head/chair.

Ulmer: What does acceptance of this position mean to you?

Ulmer: I’m honored! It’s an opportunity to help an organization with a long and distinguished history and a bright future. Of course, I hope to have some fun at the same time.

Ulmer: How has the AMS been faring financially during the COVID-19 pandemic?

Ulmer: Pretty well overall. Revenues have been reduced, especially due to meetings being moved online, but expenses have also been reduced. The staff in Providence have done a great job of adapting to these challenges and moving the whole operation to the internet and it looks like we will have a small positive bottom line for 2020. No doubt 2021 will be challenging, but I think we are well prepared.

Ulmer: What steps do you think can be taken to ensure its continued success throughout the rest of the pandemic and into the economic recovery period?

Ulmer: I think the main challenges for the AMS were there before the pandemic and will be there, perhaps reinforced, after: we need to remain relevant to potential members, adapt to new challenges in publishing, and make sure our endowment is wisely managed.

Ulmer: What would you say to young mathematicians about the importance of investing—with membership dues and their time—in professional organizations such as the AMS during a time when many folks are stretched thin with responsibilities and financial pressures?

Ulmer: First, I’d say that if you’ve decided that mathematics is your life’s work, then what better place to invest some energy and effort than the main organization that advocates for the field?

Regarding our current difficulties, I acknowledge that they are daunting, but we have faced serious challenges before (energy crises, wars and terrorist attacks, financial crashes...), we face many now (the pandemic, racism and inequality, climate change), and there will be others we haven’t even imagined yet. All that said, mathematics endures as one of the most significant areas of human accomplishments and one of the most important areas of continuing investigation. It’s worth doing, and it’s worth advocating for, so the AMS is worth supporting.

Ulmer: In "A Word from Jane Hawkins" published in April 2020, Hawkins mentioned that “the AMS has not been earning enough to completely cover its costs, but luckily money is added in annually from the interest on the endowment, and that makes up the difference." Are you concerned about that?
Ulmer: I would come at this from a different angle. Some of the activities of the AMS, like books, journals, and MathSciNet, “pay for themselves” (i.e., generate revenue that covers their costs). Others, like meetings, Math Research Communities, and lobbying for research support, do not. We’re fortunate to have a healthy endowment, and the income from it allows the AMS to offer more programs and services than it would be if it were supported solely by the publishing activities.

The AMS publishes financial accounts in the Report of the Treasurer (the last was in the December 2020 Notices), so anyone interested can see the details there. In very round numbers, the AMS currently spends about $30M per year on its activities, and about $6M of that comes from income on the endowment.

Notices: For how long can the interest on the endowment make up for this?

Ulmer: With proper management, the endowment will provide similar levels of support in perpetuity. (In this regard, it is like a university endowment, albeit a lot smaller!) If, as we hope and expect, donors continue to support the AMS with further donations, we could expect a larger contribution to the operations of the AMS in the future.

Notices: What do you see as some of your biggest priorities as the new Treasurer?

Ulmer: I mentioned that income from the endowment is a major resource for the AMS. I’d like to foster a more disciplined approach to deciding what we invest in and when, why and how we change those allocations. I’m not suggesting a wholesale change in policies, but rather making our investments a bit more resilient to financial crises and adapted to the increasingly important role they play in funding our activities.

Notices: What are some of the other things you want to accomplish?

Ulmer: Another really important issue for the Treasurer and the other trustees is to adapt the AMS to the needs of its current and future membership. Many professional societies are facing reduced interest in membership and although the AMS is doing pretty well in this regard, we can’t be complacent.

Finally, the nature of publishing is changing (think open access journals, arXiv, and e-books and the vast amount of information available via a quick Google search). Adapting to and even leading change in these areas will be important.
brought together previous studies and related them to the analytical tools used in harmonic analysis. This discovery later led to Meyer’s demonstration that waves can form mutually independent sets of mathematical objects called orthogonal bases. His work inspired Daubechies to construct orthogonal wavelets with compact support, and later biorthogonal wavelets, which revolutionized the field of engineering. Both worked on the development of wavelet packages, which allow improved adaptation to the particularities of a signal or image.

“A second revolution in data and signal processing techniques came in the first decade of the twenty-first century with the development of the theories of compressed sensing or compressive sampling and matrix completion, fruit of the collaboration between Terence Tao and Emmanuel Candès. This work enables the efficient reconstruction of scattered data based on very few measurements. One of the core issues in medical imaging and, in general, in all areas of signal processing is how to reconstruct a signal from partial, noisy measurements. Advanced reconstruction techniques, such as compressed sensing and matrix completion, enable the number of required samples to be reduced, which in medical imaging means being able to examine the patient faster. … The compressed sensing technique has contributed significantly to signal processing by enabling the compressed version of a signal to be reconstructed using a small number of linear measurements. This means a lower sampling frequency, less data, less use of storage resources, decreased speed requirements for analog-to-digital converters, and less time required for data transmission. These mathematical theories, developed by Meyer, Daubechies, Tao, and Candès, highlight the unifying and cross-cutting role of mathematics in different scientific and engineering disciplines, with practical solutions applicable in multiple fields, as well as constituting an example of the usefulness of work in pure mathematics.”

Yves Meyer earned his PhD from the University of Strasbourg in 1966 under Jean-Pierre Kahane. He served as professor of mathematics at Paris-Sud University (1966–1980), the Ecole Polytechnique (1980–1986), and Paris-Dauphine University (1986–1995). He was a senior researcher at CNRS from 1995–1999, then joined the faculty at Ecole Normale Supérieure de Cachan. He has been professor emeritus at Cachan since 2004. Meyer is a member of the
French Academy of Science, the American Academy of Arts and Sciences (honorary), and the US National Academy of Sciences and is an Inaugural Fellow of the AMS. His honors include the Salem Prize (1970) and the Gauss Prize (2010). He was awarded the Abel Prize in 2017.

Ingrid Daubechies received her PhD in 1980 from the Free University of Brussels, where she was employed until 1987. She spent much of 1986 visiting the Courant Institute of Mathematical Sciences. In 1987 she joined the AT&T Bell Laboratories Mathematical Research Center, serving until 1994, while concurrently holding a professorship in the Department of Mathematics of Rutgers University. She became a professor in the Department of Mathematics at Princeton University in 1994 and was director of the program in applied and computational mathematics from 1997 to 2001. She joined Duke University in 2011 and currently holds the James B. Duke Chair. Among her honors and awards are the AMS Leroy P. Steele Prize for Mathematical Exposition (1994), the Satter Prize in Mathematics (1977), a MacArthur Fellowship (1992), the National Academy of Sciences Medal in Mathematics (2000), the Steele Prize for Seminal Contribution to Research (2011), the Benjam Franklin Medal in Electrical Engineering (2011), the BBVA Foundation Frontiers of Knowledge Award in the Basic Sciences (with David Mumford, 2012), the Benter Prize in Applied Mathematics (2018), and the Fudan-Zhongzhi Science Award (2018). She is the first woman awarded the Nermers Prize in Mathematics (2012). She is a member of the American Academy of Arts and Sciences, the US National Academy of Sciences, the Royal Netherlands Academy of Arts and Sciences, the American Philosophical Society, the London Mathematical Society, and the Paris Academy of Sciences. She is a Fellow of the Association for Women in Mathematics and an Inaugural Fellow of the AMS.

Terence Tao earned his PhD in mathematics from Princeton University in 1996. He joined the faculty of the University of California at Los Angeles, where he is currently full professor. He was awarded a Fields Medal in 2006. Among his many honors and awards are the Salem Prize (2000), the Bôcher Prize (2002), the Ostrowski Prize (with Ben Green, 2005), the Levi L. Conant Prize (2005), a MacArthur Fellowship (2006), the SASTRA Ramanujan Prize (2006), the Alan T. Waterman Award (2008), the Nermers Prize (2010), the Polya Prize (with Emmanuel Candès, 2010), and the Breakthrough Prize in Mathematics (2014). He is a member of the Australian Academy of Science, the US National Academy of Sciences, and the American Academy of Arts and Sciences and an Inaugural Fellow of the AMS.

Emmanuel Candès earned his PhD in statistics from Stanford University in 1998. He has held positions as professor of applied and computational mathematics and Ronald and Maxine Linde Professor at the California Institute of Technology. In 2009, he joined the faculty at Stanford University, where he is currently Barnum-Simon Professor of Mathematics and Statistics, Professor of Electrical Engineering, and codirector of the Data Science Institute. His honors include the Alan T. Waterman Award (2006), the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing (2005), the SIAM 2010 George Pólya Prize (with Terence Tao, 2010), the Collatz Prize (2011), the Lagrange Prize in Continuous Optimization (2012), the Dannie Heineman Prize (2013), and the George David Birkhoff Prize (2015). He was awarded a MacArthur Genius Grant in 2017. He is a member of the US National Academy of Sciences and the American Academy of Arts and Sciences and a Fellow of the AMS.

The Princess of Asturias Award for Technical and Scientific Research recognizes “the work of fostering and advancing research in the field of mathematics, astronomy and astrophysics, physics, chemistry, life sciences, medical sciences, earth and space sciences or technological sciences, including those disciplines corresponding to each of these fields, as well as their related technologies.” The Princess of Asturias Foundation is a nonprofit private institution whose essential aims are to contribute to extolling and promoting those scientific, cultural, and humanistic values that form part of the universal heritage of humanity and consolidate the existing links between the Principality of Asturias and the title traditionally held by the heirs to the Crown of Spain. The prize carries a cash award of 50,000 euros (approximately US$60,600).

—From a Princess of Asturias Foundation announcement

**Sato Awarded Operator Algebra Prize**

Yasuhiko Sato of Kyushu University has been awarded the sixth Operator Algebra Prize for his outstanding contributions to the classification theory of amenable C*-algebras and group actions on them. The prize consists of a cash award of approximately US$3,000, a prize certificate, and a medal.

The Operator Algebra Prize is awarded every four years to a person under forty years of age, either of Japanese nationality or principally based in a Japanese institution, for outstanding contributions to operator algebra theory and related areas.

—Yasuyuki Kawahigashi, Chair Operator Algebra Prize Committee
Chatterjee Awarded Infosys Prize

Sourav Chatterjee of Stanford University has been awarded the Infosys Prize in Mathematics "for his groundbreaking work in probability and statistical physics. Professor Chatterjee’s collaborative work has played a critical role in areas such as the emerging body of work on large deviations for random graphs." He received his PhD in statistics in 2005 from Stanford University under the direction of Persi Diaconis. He was a member of the faculty at the University of California, Berkeley, from 2006 to 2011 and at the Courant Institute of Mathematical Sciences, New York University, from 2009 to 2013. He joined the faculty at Stanford in 2013. In 2010 he was awarded the Rollo Davidson Prize, and in 2012 he was the first recipient of the Doeblin Prize in Probability. He was awarded the Line and Michel Loève International Prize in Probability in 2013. He is a Fellow of the Institute of Mathematical Statistics. Chatterjee tells the Notices: "I like playing chess, reading Wikipedia, listening to the music of Rabindranath Tagore (in Bengali, which is my native tongue), and going on long drives with my family (which has become hard due to Covid!)." The prize is awarded by Infosys Science Foundation "to honor outstanding achievements of contemporary researchers and scientists." It carries a cash award of US$100,000.

—from an Infosys Science Foundation announcement

2020 Fudan–Zhongzhi Science Award

Three physicists whose work involves the mathematical sciences have been awarded the 2020 Fudan–Zhongzhi Science Award. Sir Michael V. Berry of the University of Bristol discovered the geometric phase that is widely known as the Berry phase in basic quantum mechanics, as well as the Berry connection and curvature. Charles L. Kane of the University of Pennsylvania has made pioneering contributions to the field of topological insulators. Qi-Kun Xue of Tsinghua University reported experimental observation of the quantum anomalous Hall effect. The award is given in recognition of scientists who have made fundamental and groundbreaking achievements in physics, mathematics, and biomedicine. The prize, divided among recipients, carries a cash award of US$442,000.

—from a Fudan University announcement

2020 Australian Mathematical Society Awards

The Australian Mathematical Society has announced several awards for 2020.

Luke Bennetts of the University of Adelaide was honored with the Australian Mathematical Society Medal for his work on “challenging mathematical problems applied to geophysical problems, in particular wave–ice interaction and catastrophic ice-shelf disintegration in polar regions.” He received his PhD in applied mathematics from the University of Reading in 2007. His honors include the Christopher Heyde Medal of the Australian Academy of Science (2016), a Simons Fellowship (2017), and a Humboldt Fellowship (2020–2021). Bennetts tells the Notices: “Outside of work, I enjoy hiking in the Adelaide hills with my two young sons, and I’m a keen rock climber.”

Nalini Joshi of the University of Sydney and Ole Warnaar of the University of Queensland were awarded George Szekeres Medals. Joshi is “a world leader in the
theory and applications of differential equations, contributing mathematical results that have impact in fields as diverse as particle physics, quantum mechanics, large prime-number distributions, and wireless communications.” Her significant contributions to the mathematics community include leadership, gender equity, and promotion of mathematics. She received her PhD from Princeton University under Martin Kruskal. She is a Fellow of the Australian Academy of Science and a recipient of the Eureka Award for Outstanding Mentor of Young Researchers (2018). Joshi tells the Notices: “I am an avid reader, and have been since I was a child in Burma, where I was born.” Warnaar “is a leading expert in special functions, partition theory, and algebraic combinatorics.” According to the prize citation, he and his collaborators “were the first to extend the celebrated Rogers–Ramanujan identities to infinite families of affine Kac–Moody algebras. He developed a beautiful theory of partial theta functions, was one of the pioneers in the field of elliptic hypergeometric functions, and is one of the leading experts on the Selberg integral and its applications.” He received his PhD from the University of Amsterdam. He is the current president of the Australian Mathematical Society. He tells the Notices: “I am a judo coach, and keen hiker and rock climber. During quieter moments I like to read or watch art house movies at the local theatre with my wife Celestien.” The Gavin Brown Best Paper Prize was awarded to John Bamberg, Michael Giudici, and Gordon F. Royle, of the University of Western Australia, for “Every flock generalized quadrangle has a hemisystem,” Bulletin of the London Mathematical Society 42 (2010).

Norman Do of Monash University received the Australian Mathematical Society Award for Teaching Excellence. His “approach to teaching combines a remarkably enthusiastic lecturing style, interactive approaches to tutorials, and initiatives targeted at students across the spectrum of mathematical proficiency.”

—From Australian Mathematical Society announcements

ICMI Freudenthal and Klein Awards Given

The International Commission on Mathematical Instruction (ICMI) announced the awarding of its 2019 Hans Freudenthal and Felix Klein Medals. Gert Schubring of Bielefeld University and the Universidade Federal do Rio de Janeiro was selected the recipient of the Freudenthal Medal “in recognition of his outstanding contribution to research on the history of mathematics education.” The Freudenthal Medal honors “innovative, consistent, highly influential and still ongoing programs of research in mathematics education.” Tommy Dreyfus of Tel Aviv University was honored with the Klein Medal for his contribution to research “as well as his leading role in shaping and consolidating the research community and in fostering communication between researchers.” The Klein Medal honors “the most meritorious members of the mathematics education community.”

—From an ICMI announcement

AAAS Fellows Elected

The American Association for the Advancement of Science has elected its class of Fellows for 2020.

The new Fellows of the Section on Mathematics are:
- Harold P. Boas, Texas A&M University
- Leslie Hogben, Iowa State University and American Institute of Mathematics
- Kristin Lauter, Microsoft Research
- Paul K. Newton, University of Southern California
- Esmond G. Ng, Lawrence Berkeley National Laboratory
- Karen Hunger Parshall, University of Virginia
- Malgorzata Peszynska, Oregon State University
- Jack Xin, University of California, Irvine

The new Fellows of the Section on Statistics are:
- Sudipto Banerjee, University of California, Los Angeles
- David L. Banks, Duke University
- Deborah J. Donnell, Fred Hutchinson Cancer Research Center
- Timothy C. Hesterberg, Google, Inc.
- Qi Long, University of Pennsylvania
- Ying Lu, Stanford University School of Medicine
- Richard L. Smith, University of North Carolina at Chapel Hill
- Elizabeth A. Stuart, Johns Hopkins Bloomberg School of Public Health

—From an AAAS announcement
2020 NSF CAREER Awards

The National Science Foundation (NSF) has named a number of recipients in 2020 of Faculty Early Career Development (CAREER) Awards. The awards support early-career faculty members who have the potential to serve as academic role models in research and education and to lead advances in the mission of their departments or organizations. Following are the names, institutions, and proposal titles of the awardees selected by the NSF Division of Mathematical Sciences (DMS) for 2020.

- **Arash Amini**, University of California, Los Angeles: High-dimensional statistical models for unsupervised learning
- **David Anderson**, Ohio State University: Equivariant and infinite-dimensional combinatorial algebraic geometry
- **David Ayala**, Montana State University: Factorization homology and quantum topology
- **Jennifer Balakrishnan**, Boston University: New directions in $p$-adic heights and rational points on curves
- **Pierre Bellec**, Rutgers University: Post-differentiation inference
- **Jeffrey Calder**, University of Minnesota, Twin Cities: Harnessing the continuum for big data: Partial differential equations, calculus of variations, and machine learning
- **Roger Casals Gutiérrez**, University of California, Davis: Legendrian and contact topology in higher dimensions
- **Jesse Chan**, William Marsh Rice University: Taileored entropy stable discretizations of nonlinear conservation laws
- **Nicolas Charon**, Johns Hopkins University: Shape analysis in submanifold spaces: New directions for theory and algorithms
- **Tristan Collins**, Massachusetts Institute of Technology: Differential equations, algebraic geometry, and string theory
- **Jeffrey Danciger**, University of Texas, Austin: Lowly homogeneous geometric manifolds and their moduli spaces
- **Peng Ding**, University of California, Berkeley: The design-based perspective of causal inference in complex experiments
- **Tarek Elgindi**, Duke University: Formation of small scales and dissipation in incompressible fluids
- **Yang Feng**, New York University: Statistical inference of network and relational data
- **James Freitag**, University of Illinois, Chicago: Applied model theory
- **Shirshendu Ganguly**, University of California, Berkeley: Various geometric aspects of Kardar–Parisi–Zhang universality: Fractal dimensions, noise sensitivity, line ensembles, and large deviations
- **Elizabeth Gross**, University of Hawaii: Identifiability and inference for phylogenetic networks using applied algebraic geometry
- **Daniel Halpern-Leistner**, Cornell University: Moduli spaces and derived categories
- **Hao Huang**, Emory University: Algebraic methods in extremal combinatorics
- **Paata Ivanisvili**, North Carolina State University: Discrete structures and orthogonal systems
- **Hao Jia**, University of Minnesota, Twin Cities: New mechanisms for stability, regularity and long-time dynamics of partial differential equations
- **Zheng Ke**, Harvard University: Learning probabilistic factor models
- **Daniel Krashen**, Rutgers University: The arithmetic of fields and the complexity of algebraic structures
- **Sean Lawley**, University of Utah: How diffusion, dimension, geometry, and redundancy affect cellular dynamics
- **Mona Merling**, University of Pennsylvania: Applications of equivariant homotopy theory to manifolds
- **François Monard**, University of California, Santa Cruz: Integral geometry: Theory, implementations, and applications
- **Naveen Naidu Narisetty**, University of Illinois, Urbana-Champaign: Flexible and efficient exploration of the Bayesian framework for high-dimensional modeling
- **Yang Ning**, Cornell University: High-dimensional $M$-estimation under nonstandard conditions
- **Sung-Jin Oh**, University of California, Berkeley: Dynamics of nonlinear dispersive partial differential equations
- **Jose Perea**, Michigan State University: Machine learning, mapping spaces, and obstruction theoretic methods in topological data analysis
- **Aaditya Ramdas**, Carnegie-Mellon University: Online multiple hypothesis testing: A comprehensive treatment
- **Eric Riedl**, University of Notre Dame: Hyperbolicity properties of hypersurfaces
- **Veronika Rockova**, University of Chicago: Statistical inference for Bayesian machine learning
- **Bharath Sriperumbudur**, Pennsylvania State University: Statistical learning, inference, and approximation with reproducing kernels
Rhodes Scholars 2020

The Rhodes Trust has announced the names of the American scholars chosen as Rhodes Scholars for 2020. Following are the names and brief biographies of the scholars whose work involves the mathematical sciences.

Garima P. Desai of Fremont, California, graduated in May 2020 from the University of California, Santa Cruz, with a double major in environmental studies and economics. She currently works as a transportation planner in Oakland, California. While at UC-Santa Cruz, she worked as a research assistant on issues related to housing and transportation. She is passionate about using economics as a tool to solve pressing climate issues. At Oxford, she plans to pursue an MSc in economics for development and an MSc in environmental change and management.

Nicolas J. W. Fishman of Washington, DC, is a senior at Stanford University completing majors in computer science and sociology. He is passionate about developing technologies that will expand autonomy. He has conducted independent research at Harvard University, Stanford University, Northwestern University, and the National Human Genome Research Institute, as well as Data for Progress. He is active in get-out-the-vote work around the American elections and in the Young Democratic Socialists of America. At Oxford, he will pursue an MSc in statistical science and an MSc in history of science, medicine, and technology.

Samuel E. Patterson of Marietta, Georgia, is a senior at the University of Maryland, Baltimore County, where he will receive a BS in mathematics, a BS in statistics, and a BA in economics. He has done summer research in economics and education at Harvard University and in business at the University of Chicago. An accomplished musician, he is the music director of a community organization, plays upright and electric jazz bass, and volunteered to teach the basics of computer programming to middle school students. His deep work in economics through an equity lens has focused on the importance of transportation infrastructure to improve economic opportunity. He intends to do the MSc in nature, society, and environmental governance at Oxford.

Evan C. Walker of Rowlett, Texas, is in her final year at the US Military Academy, where she majors in operations research with focuses in statistics and linear algebra. Her thesis analyzes the demographics of promotion and attrition among US Army Field Grade Officers. She is a regimental commander, served as the chief liaison between survivors of sexual harassment or assault and on-campus medical professionals, and is president of an initiative to mentor minority cadets. She is also captain of the nationally ranked and gender-integrated Army Boxing Team and last year placed second nationally in her weight class. She plans to do the MSc in sociology and the MSc in statistical science at Oxford.

—From a Rhodes Trust announcement

Credits

Photo of Ingrid Daubechies is courtesy of Les Todd: Duke Photography.
Photo of Terence Tao is courtesy of Reed Hutchinson/UCLA.
Photo of Emmanuel Candès is courtesy of John D. and Catherine T. MacArthur Foundation.
Photo of Sourav Chatterjee is courtesy of Rod Searcey.
Photo of Luke Bennetts is courtesy of Randy Larcombe.
Photo of Nalini Joshi is courtesy of Annie Fenwicke.
Photo of Ole Warnaar is courtesy of Hung Vu, University of Queensland.

Omer Tamuz, California Institute of Technology: Probability on groups and semigroups of probabilities
Nam Trang, University of North Texas: Current and future developments of the core model induction
Cynthia Vinzant, North Carolina State University: Determinantal, hyperbolic, and log-concave polynomials in theory and applications
Lutz Warnke, Georgia Tech Research Corporation: Understanding the evolution of random graphs with complex dependencies: Phase transition and beyond
Emily Witt, University of Kansas Center for Research: New frontiers for Frobenius, singularity theory, differential operators, and local cohomology
Jesse Wolfson, University of California, Irvine: Resolvent degree, Hilbert’s 13th problem, and geometry
Chuan Xue, University of Minnesota, Twin Cities: Multiscale modeling of axonal cytoskeleton dynamics and axonal transport
Haizhao Yang, Purdue University: Deep-learning-based scientific computing: Mathematical theory and algorithms
Anru Zhang, University of Wisconsin, Madison: Inference for high-dimensional structures via subspace learning: Statistics, computation, and beyond
Xin Zhou, University of California, Santa Barbara: New development in geometric variational theory
Andrew Zimmer, Louisiana State University: Intrinsic and extrinsic conditions in several complex variables

—NSF announcements
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Fellows of the AMS

The Fellows of the American Mathematical Society program recognizes members who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics.

AMS members may be nominated for this honor during the nomination period, which opened on February 1, 2021, and runs through March 31, 2021. Selection of the new class of AMS Fellows is managed by the AMS Selection Committee, composed of twelve members of the AMS who are also Fellows. Those selected are subsequently invited to become Fellows, and the new class of Fellows is publicly announced each year on November 1.

Learn more about the qualifications and the process for submitting a nomination at http://www.ams.org/ams-fellows.

—AMS Programs Department

Deaths of AMS Members

Edward Fadell, of Wisconsin, died on January 1, 2018. Born on March 8, 1926, he was a member of the Society for 67 years.

Sufian Y. Hussein, of Salem, Oregon, died on December 30, 2017. Born on October 15, 1929, he was a member of the Society for 63 years.

Eleonor Killam, of Amherst, Massachusetts, died on January 1, 2018. Born on May 18, 1933, she was a member of the Society for 61 years.

Jingliang, of Ontario, Canada, died on January 8, 2018. Born on January 3, 1940, he was a member of the Society for 34 years.

J. E. Nymann, of Georgetown, Texas, died on January 1, 2018. Born on November 24, 1938, he was a member of the Society for 53 years.

Hans H. Storrer, of Switzerland, died on December 26, 2017. Born on November 25, 1939, he was a member of the Society for 50 years.

Buck Ware, of Chico, California, died on January 3, 2018. Born on January 4, 1940, he was a member of the Society for 47 years.

—AMS Programs Department

China Exchange Program

Applications are still being accepted on MathPrograms for the China Exchange Program until March 31, 2021. The purpose of these grants is to support research travel of mathematicians at institutions in the United States or Canada on visits to research colleagues in China and to support research travel of (generally, less senior) mathematicians from China to the United States or Canada for visits with US or Canadian research colleagues at US or Canadian institutions. For more information, see http://www.ams.org/china-exchange.

—AMS Programs Department

Virtual Mathematical Congress of the Americas (MCA 2021) Grants

With funding from the National Science Foundation (NSF), the AMS is administering the selection, award, and reimbursement process of a grant program to provide support for US-based mathematicians to attend the Mathematical Congress of the Americas. This year, MCA 2021, scheduled for July 19–24, 2021, is being held virtually. Priority will be given to early-career mathematicians whose participation in MCA 2021 is not supported with other funding. Funding will cover the MCA registration fee plus a stipend of up to $250. This stipend is intended to assist in offsetting conference-related expenses that will allow the awardee a more focused meeting participation, which otherwise would not be possible. For more information, see http://www.ams.org/mca. Applications are being accepted now through April 30, 2021, on MathPrograms. All information currently available about the MCA 2021 program and organization is located on the MCA 2021 website: www.mca2021.org/en. For questions about the travel grant program, contact the AMS Programs Department at mca-info@ams.org.

—AMS Programs Department
Mathematics Opportunities

Listings for upcoming mathematics opportunities to appear in Notices may be submitted to notices@ams.org.

Early Career Opportunity
2022 Breakthrough Prize, New Horizons Prizes, and Mirzakhani Prize

The 2022 Breakthrough Prize in Mathematics will be awarded to an individual who has made outstanding contributions to the field of mathematics. The Prize carries a cash award of US$3 million.

Nominations are also open for the New Horizons in Mathematics Prizes of US$100,000 for early-career researchers who have already produced important work in the fields of mathematics and physics.

The Maryam Mirzakhani New Frontiers Prize is awarded to early-career women mathematicians. This prize carries a cash award of US$50,000, which may be shared among one or more mathematicians.

The deadline for nominations for all prizes is April 1, 2021. For more information and a nomination form, see https://breakthroughprize.org.

—From Breakthrough Prize Foundation announcements

Early Career Opportunity
AMS-Simons Travel Grants

The AMS-Simons Travel Grants are administered by the AMS with support from the Simons Foundation. Each grant provides an early-career mathematician with $2,500 funding for two years to be used for research-related travel expenses. Applicants must be located in the United States (or be US citizens employed outside the US) and must have completed the PhD within the last four years. The department of the awardee will also receive a small amount of funding to help enhance its research environment.

The application period for 2021 opened February 1, 2021, and runs through 11:59 pm Eastern time on March 31, 2021. Up to seventy awardees will be chosen from the set of applicants, and the earliest date for supported travel would be July 1, 2021. To learn more about eligibility and how to apply, visit https://www.ams.org/AMS-SimonsTG

—AMS Programs Department

Early Career Opportunity
NSF CAREER Awards

The National Science Foundation (NSF) Faculty Early Career Development (CAREER) Program supports early-career faculty members who have the potential to serve as academic role models in research and education and to lead advances in the missions of their departments or organizations. Activities pursued by early-career faculty members should build a firm foundation for a lifetime of leadership in integrating education and research. The deadline for proposals is July 26, 2021. See the website https://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503214.

—NSF announcement

Early Career Opportunity
Project NExT 2021–2022

MAA Project NExT (New Experiences in Teaching) is a year-long professional development program of the Mathematical Association of America (MAA) for new or recent PhDs in the mathematical sciences. The program is designed to connect new faculty members with master teachers and leaders in the mathematics community and to address the three main aspects of an academic career: teaching, research, and service. We welcome and encourage applications from new(ish) and recent PhDs in postdoctoral, tenure-track, and visiting positions. We particularly

The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at www.nsf.gov/dms and for the Directorate of Education and Human Resources at www.nsf.gov/dir/index.jsp?org=ehr. To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at www.nsf.gov/mps/dms/about.jsp

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Encourage applicants from underrepresented groups (including women and minorities). Applications for the 2021 cohort are due April 15, 2021. For applications and further information, see www.maa.org/programs-and-communities/professional-development/project-next. The AMS is a sponsor of MAA Project NExT.

—MAA announcement

Early Career Opportunity

CAS–TWAS Scholarship

The Chinese Academy of Sciences (CAS) and the World Academy of Sciences (TWAS) offers the CAS–TWAS Chinese Scholarship to scholars to pursue a doctoral degree in China. The 2021–2022 Program is now accepting applications from international scholars for the 2021 academic session. The deadline date for applications is March 31, 2021. See the website https://ascholarship.com/cas-twas-scholarship-2020-cas-twas-presidents-fellowship.

—From a CAS–TWAS announcement
Classified Advertising

Employment Opportunities

CHINA

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at the Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

Please send your resume to mathjobs@tju.edu.cn.

For more information, please visit cam.tju.edu.cn or contact Ms. Erica Liu at mathjobs@tju.edu.cn, telephone: 86-22-2740-6039.

The Notices Classified Advertising section is devoted to listings of current employment opportunities. The publisher reserves the right to reject any listing not in keeping with the Society's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any information therein. Advertisers are neither screened nor recommended by the publisher. The publisher is not responsible for agreements or transactions executed in part or in full based on classified advertisements.

The 2021 rate is $3.65 per word. Advertisements will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted. There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads. Upcoming deadlines for classified advertising are as follows: May 2021—February 24, 2021; June/July 2021—April 26, 2021; August 2021—May 28, 2021; September 2021—June 28, 2021; October 2021—July 23, 2021; November 2021—August 24, 2021; December 2021—September 24, 2021. US laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. Advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws.

Submission: Send email to classads@ams.org.
NEW BOOKS

New Books Offered by the AMS

Algebra and Algebraic Geometry

**Thinking Algebraically**
An Introduction to Abstract Algebra
Thomas Q. Sibley, St. John’s University, Collegeville, MN

*Thinking Algebraically* presents the insights of abstract algebra in a welcoming and accessible way. It succeeds in combining the advantages of rings-first and groups-first approaches while avoiding the disadvantages. After an historical overview, the first chapter studies familiar examples and elementary properties of groups and rings simultaneously to motivate the modern understanding of algebra. The text builds intuition for abstract algebra starting from high school algebra. In addition to the standard number systems, polynomials, vectors, and matrices, the first chapter introduces modular arithmetic and dihedral groups. The second chapter builds on these basic examples and properties, enabling students to learn structural ideas common to rings and groups: isomorphism, homomorphism, and direct product. The third chapter investigates introductory group theory. Later chapters delve more deeply into groups, rings, and fields, including Galois theory, and they also introduce other topics, such as lattices. The exposition is clear and conversational throughout.

The book has numerous exercises in each section as well as supplemental exercises and projects for each chapter. Many examples and well over 100 figures provide support for learning. Short biographies introduce the mathematicians who proved many of the results. The book presents a pathway to algebraic thinking in a semester- or year-long algebra course.

AMS/MAA Textbooks, Volume 65

General Interest

**Common Sense Mathematics**
Second Edition
Ethan D. Bolker, University of Massachusetts Boston, MA, and Maura B. Mast, Fordham University, Bronx, NY

Ten years from now, what do you want or expect your students to remember from your course? We realized that in ten years what matters will be how students approach a problem using the tools they carry with them—common sense and common knowledge—not the particular mathematics we chose for the curriculum. Using our text, students work regularly with real data in moderately complex everyday contexts, using mathematics as a tool and common sense as a guide. The focus is on problems suggested by the news of the day and topics that matter to students, like inflation, credit card debt, and loans. We use search engines, calculators, and spreadsheet programs as tools to reduce drudgery, explore patterns, and get information. Technology is an integral part of today’s world—this text helps students use it thoughtfully and wisely.

This second edition contains revised chapters and additional sections, updated examples and exercises, and complete rewrites of critical material based on feedback from students and teachers who have used this text. Our focus remains the same: to help students to think carefully—and critically—about numerical information in everyday contexts.
New in Contemporary Mathematics
Algebra and Algebraic Geometry

Advances in Representation Theory of Algebras

Ibrahim Assem, University of Sherbrooke, Quebec, Canada, Christof Geiß, Universidad Nacional Autónoma de México, México, and Sonia Trepode, Universidad Nacional de Mar del Plata, Buenos Aires, Argentina, Editors

The Seventh ARTA (“Advances in Representation Theory of Algebras VII”) conference took place at the Instituto de Matemáticas of the Universidad Nacional Autónoma de México, in Mexico City, from the 24th to the 28th of September 2018, in honour of José Antonio de la Peña’s 60th birthday.

Papers in this volume cover topics Professor de la Peña worked on, such as covering theory, tame algebras, and the use of quadratic forms in representation theory. Also included are papers on the categorical approach to representations of algebras and relations to Lie theory, Cohen–Macaulay modules, quantum groups and other algebraic structures.

Contemporary Mathematics, Volume 761

Analysis

Linear Dynamical Systems on Hilbert Spaces: Typical Properties and Explicit Examples

S. Grivaux, Université de Lille, Villeneuve d’Ascq, France, É. Matheron, Université d’Artois, Lens, France, and Q. Menet, Université d’Artois, Lens, France

Memoirs of the American Mathematical Society, Volume 269, Number 1315

Differential Equations

Resolvent, Heat Kernel, and Torsion Under Degeneration to Fibered Cusps

Pierre Albin, University of Illinois at Urbana-Champaign, Illinois, Frédéric Rochon, Université du Québec à Montréal, Québec, Canada, and David Sher, University of Michigan, Ann Arbor, Michigan

Memoirs of the American Mathematical Society, Volume 269, Number 1312
NEW BOOKS

Geometry and Topology

**Differential Function Spectra, the Differential Becker-Gottlieb Transfer, and Applications to Differential Algebraic K-Theory**

Ulrich Bunke, Universität Regensburg, Germany, and David Gepner, Purdue University, Lafayette, Indiana

Memoirs of the American Mathematical Society, Volume 269, Number 1316

[bookstore.ams.org/memo-269-1316](http://bookstore.ams.org/memo-269-1316)

**Local Boundedness, Maximum Principles, and Continuity of Solutions to Infinitely Degenerate Elliptic Equations with Rough Coefficients**

Lyudmila Korobenko, Reed College, Portland, Oregon, Cristian Rios, University of Calgary, Alberta, Canada, Eric Sawyer, McMaster University, Hamilton, Ontario, Canada, and Ruipeng Shen, Tianjin University, People's Republic of China

Memoirs of the American Mathematical Society, Volume 269, Number 1311

[bookstore.ams.org/memo-269-1311](http://bookstore.ams.org/memo-269-1311)

Mathematical Physics

**The 2D Compressible Euler Equations in Bounded Impermeable Domains with Corners**

Paul Godin, Université Libre de Bruxelles, Belgium

This item will also be of interest to those working in differential equations.

Memoirs of the American Mathematical Society, Volume 269, Number 1313

[bookstore.ams.org/memo-269-1313](http://bookstore.ams.org/memo-269-1313)

New AMS-Distributed Publications

Analysis

**A Second Course in Analysis**

M. Ram Murty, Queen's University, Kingston, ON, Canada

This book represents material suitable for a two-semester graduate course in analysis. Based on courses given by the author since 2007, it is targeted towards graduate students preparing for
a research career in mathematics. After reviewing what a typical student would have learned in single variable and multivariable calculus courses, the first chapter presents familiar material from a mature mathematical perspective. Subsequent chapters cover measure theory, Fourier transforms, and complex analysis, and the book concludes with an introduction to algebraic topology that shows the symbiosis between algebra and analysis.

A publication of Hindustan Book Agency; distributed within the Americas by the American Mathematical Society. Maximum discount of 20% for all commercial channels.

Hindustan Book Agency

Differential Equations

The Spectrum of a Schrödinger Operator in a Wire-Like Domain with a Purely Imaginary Degenerate Potential in the Semiclassical Limit
Y. Almog, Louisiana State University, Baton Rouge, LA, and B. Helffer, Université Paris-Sud, Orsay, France

Consider a two-dimensional domain shaped like a wire, not necessarily of uniform cross section. Let $V$ denote an electric potential driven by a voltage drop between the conducting surfaces of the wire. The authors consider the operator $A_h = -h^2 \Delta + iV$ in the semi-classical limit $h \to 0$ and obtain both the asymptotic behavior of the left margin of the spectrum, as well as resolvent estimates on the left side of this margin. They extend here previous results obtained for potentials for which the set where the current ($\nabla V$) is normal to the boundary is discrete, in contrast with the present case where $V$ is constant along the conducting surfaces.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the US, Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Mémoires de la Société Mathématique de France, Number 166

bookstore.ams.org/smfmem-166

A publication of Hindustan Book Agency; distributed within the Americas by the American Mathematical Society. Maximum discount of 20% for all commercial channels.
Meetings & Conferences of the AMS
March Table of Contents

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Paid meeting registration is required to submit an abstract to a sectional meeting.

Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at www.ams.org/meetings.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to https://www.ams.org/meetings/meetings-general for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of \LaTeX{} is necessary to submit an electronic form, although those who use \LaTeX{} may submit abstracts with such coding and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX{}. Visit www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Associate Secretaries of the AMS

Central Section: Georgia Benkart, University of Wisconsin–Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403; email: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: lapidus@math.ucr.edu; telephone: 951-827-5910.

Meetings in this Issue

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See www.ams.org/meetings for the most up-to-date information on the meetings and conferences that we offer.

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. Please see our full Policy on a Welcoming Environment at https://www.ams.org/welcoming-environment-policy.
Meetings & Conferences of the AMS

IMPORTANT information regarding meetings programs: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See https://www.ams.org/meetings.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL.

New: Sectional Meetings Require Registration to Submit Abstracts. In an effort to spread the cost of the sectional meetings more equitably among all who attend and hence help keep registration fees low, starting with the 2020 fall sectional meetings, you must be registered for a sectional meeting in order to submit an abstract for that meeting. You will be prompted to register on the Abstracts Submission Page. In the event that your abstract is not accepted or you have to cancel your participation in the program due to unforeseen circumstances, your registration fee will be reimbursed.

Spring Southeastern Virtual Sectional Meeting

Now meeting virtually, EST (hosted by the American Mathematical Society)

**March 13–14, 2021**
Saturday – Sunday

**Meeting #1164**
Southeastern Section
Associate secretary: Brian D. Boe

Program first available on AMS website: January 28, 2021
Issue of Abstracts: Volume 42, Issue 2

**Deadlines**
For organizers: Expired
For abstracts: Expired

*The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.*

**Invited Addresses**
- **Yaiza Canzani**, University of North Carolina-Chapel Hill, *Eigenfunction concentration via geodesic beams*.

**Special Sessions**
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Advanced Topics in Graph Theory and Combinatorics, **Songling Shan**, Illinois State University.
Advances in Computational Dynamics, **Jorge I. Gonzalez**, Georgia Institute of Technology, **Andrey Shilnikov**, Georgia State University, and **J.D. Mireles James**, Florida Atlantic University.
Celestial Mechanics and Applied Astrodynamics, **Bhanu Kumar** and **Molei Tao**, Georgia Institute of Technology.
Commutative Algebra and its Interaction with Algebraic Geometry and Combinatorics, **Justin Chen**, Georgia Institute of Technology, and **Youngsu Kim**, California State University, San Bernardino.
Differential Graded Methods in Commutative Algebra, Saeed Nasseh, Georgia Southern University, and Adela Vraciu, University of South Carolina, Columbia.

Functional Differential Equations, Theory and Applications, Joan Gimeno, University of Rome Tor Vergata, and Rachel Kuske and Jiaqi Yang, Georgia Institute of Technology.

Graphs in Data Science, Nicolas Fraiman, University of North Carolina, Chapel Hill, and Soledad Villar, Johns Hopkins University.

Groups, Geometry, and Topology, Tara Brendle and Maxime Fortier-Bourque, University of Glasgow, and Dan Margalit and Yvon Verberne, Georgia Institute of Technology.

Integrable Nonlocal Systems, Matthew Russo, Florida State University.

Optimization and Real Algebraic Geometry, Saugata Basu and Ali Mohammad Nezhad, Purdue University.

Recent Developments on Analysis and Computation for Inverse Problems for PDEs, Dinh-Liem Nguyen, Kansas State University, and Loc Nguyen and Khoa Vo, University of North Carolina at Charlotte.

Stochastic Control and Related Topics, Andrzej Swiech, Georgia Institute of Technology, and Jiongmin Yong, University of Central Florida.

Superalgebras, Quantum Groups, and Related Topics, Jonas Hartwig and Dwight A. Williams, II, Iowa State University.

Topology and Geometry of 3- and 4-Manifolds, Siddhi Krishna, Georgia Institute of Technology and Columbia University, Miriam Kuzbary, Georgia Institute of Technology, Beibei Liu, Max Planck Institute for Mathematics and Georgia Institute of Technology, and JungHwan Park, Georgia Institute of Technology.

Tropical Geometry, F1-connections and Matroids, Kalina Mincheva, Tulane University, and Jaiung Jun, SUNY at New Paltz.

Spring Eastern Virtual Sectional Meeting
Now meeting virtually, EDT (hosted by the American Mathematical Society)

March 20–21, 2021
Saturday – Sunday

Meeting #1165
Eastern Section
Associate secretary: Steven H. Weintraub

Program first available on AMS website: January 28, 2021
Issue of Abstracts: Volume 42, Issue 2

Deadlines
For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see https://www.ams.org sectional.html.

Invited Addresses
Abba Gumel, Arizona State University, Mathematics of Infectious Diseases (Einstein Public Lecture in Mathematics).

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Geometry in Dynamics, Nguyen-Bac Dang, Stony Brook University, and Nicole Looper, Rohini Ramadas, and Joseph H. Silverman, Brown University.

Applications and Asymptotic Properties of Discrete Dynamical Systems: A Session in Honor of the Retirement of Orlando Merino, Elliott Bertrand, Sacred Heart University, Zachary Kudlak, United States Coast Guard Academy, Mustafa Kulenovic, University of Rhode Island, and David McArdle, University of Connecticut.

Applied Combinatorics, Carina Curto, Pennsylvania State University, and Pedro Felzenszwalb and Caroline Klivans, Brown University.

Commutative Algebra, Laura Ghezzi, Department of Mathematics, New York City College of Technology-CUNY, Saeed Nasseh, Georgia Southern University, and Oana Veliche, Northeastern University.


Fractional Calculus and Fractional Differential/Difference Equations, Lyubomir Boyadjiev, City University of New York, Pavel Dubovski, Stevens Institute of Technology, and Mark Edelman, Yeshiva University and New York University.
MEETINGS & CONFERENCES

Gauge Theory, Geometry, and Low-Dimensional Topology, Paul Feehan, Rutgers University, and Daniel Ruberman, Brandeis University.

Geometric and Functional Inequalities and Nonlinear Partial Differential Equations, Joshua Flynn, University of Connecticut, Nguyen Lam, Memorial University of Newfoundland Grenfell Campus, Junwang Li, Brown University, and Guozhen Lu, University of Connecticut.

Hopf Algebras, Tensor Categories, and Related Homological Methods, Pablo S. Ocal, Texas A&M University, and Julia Plavnik, Indiana University Bloomington.

Metric Techniques in Analysis, Vasileios Chousionis and Sean Li, University of Connecticut.

Mirror Symmetry and Enumerative Geometry, Mandy Cheung, Harvard University, and Siu-Cheong Lau and Yu-Shen Lin, Boston University.

Moduli of Curves, Hilbert Schemes, and Tropical Geometry, Ignacio Barros, Northeastern University, Noah Ginsiracusa, Bentley University, and Rob Silversmith, Northeastern University.

New Applications and Methods in Financial Mathematics, Gu Wang, Worcester Polytechnic Institute, and Bin Zou, University of Connecticut.

Nonlinear Wave Equations, General Relativity, and Connections to Fluid Dynamics, Stefanos Aretakis, University of Toronto, Aynur Bulut, Louisiana State University, and Sung-Jin Oh, University of California, Berkeley.

Probability and Combinatorics, Zhongyang Li, University of Connecticut, and Mei Yin, University of Denver.

Recent Advances in Schubert Calculus and Related Topics, Cristian Lenart and Changlong Zhong, State University of New York at Albany.

Recent Developments in Automorphic Representations, Spencer Leslie, Duke University, and Tian An Wong, University of Michigan-Dearborn.

Recent Developments in Differential Geometry, Megan Kerr, Wellesley College, and Catherine Searle, Wichita State University.

Stochastic Analysis, Parisa Fatheddin and Aurel Stan, Ohio State University, Marion.

Spring Central Virtual Sectional Meeting

Now meeting virtually, CDT (hosted by the American Mathematical Society)

April 17–18, 2021
Saturday – Sunday

Meeting #1166
Central Section
Associate secretary: Georgia Benkart

Program first available on AMS website: To be announced
Issue of Abstracts: Volume 42, Issue 2

Deadlines
For organizers: Expired
For abstracts: February 16, 2021

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Invited Addresses

Fabrice Baudoin, University of Connecticut, Title to be announced.
Malabika Pramanik, University of British Columbia and BIRS, Title to be announced.
Maksym Radziwill, Caltech, Title to be announced.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Commutative Algebra (Code: SS 7A), Ayah Almousa, Cornell University, and Sean Sather-Wagstaff, Clemson University.

Graph Theory and Applications (Code: SS 9A), Katherine F. Benson, University of Wisconsin-Stout, Christine A. Kelley, University of Nebraska-Lincoln, and JD Nir, University of Manitoba.

Interactions between Representation Theory, Poisson Geometry, and Noncommutative Algebra (Code: SS 5A), Jason Gaddis, Miami University, Padmini Veerapaneni, Tennessee Technological University, and Xingting Wang, Howard University.

Legendrian Knots and Surfaces (Code: SS 3A), Honghao Gao, Michigan State University, and Dan Rutherford, Ball State University.
Nonsmooth Analysis and Geometry (Code: SS 1A), Luca Capogna, Worcester Polytechnic Institute, and Gareth Speight and Nageswari Shanmugalingam, University of Cincinnati.

Numerical Linear Algebra (Code: SS 8A), Lothar Reichel, Kent State University, Jianlin Xia, Purdue University, and Qiang Ye, University of Kentucky.

Probabilistic and Diffusion Methods in Analysis and Geometry (Code: SS 4A), Rodrigo Bañuelos and Jing Wang, Purdue University, and Ju-Yi Yen, University of Cincinnati.

Recent Progress in Analytic Number Theory (Code: SS 6A), Seungki Kim, University of Cincinnati, Xiannan Li, Kansas State University, and Xuancheng Fernando Shao, University of Kentucky.

Sharp Estimates in Harmonic Analysis (Code: SS 2A), Kabe Moen, University of Alabama, Leonid Slavin, University of Cincinnati, and Alex Stokolos, Georgia Southern University.

Spring Western Virtual Sectional Meeting

Now meeting virtually, PDT (hosted by the American Mathematical Society)

May 1–2, 2021
Saturday – Sunday

Meeting #1167
Western Section
Associate secretary: Michel L. Lapidus

Program first available on AMS website: February 25, 2021
Issue of Abstracts: Volume 42, Issue 2

Deadlines
For organizers: Expired
For abstracts: March 9, 2021

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Functional Analysis and Operator Theory (Code: SS 9A), Michel L. Lapidus, University of California, Riverside, Marat V. Markin, California State University, Fresno, and Igor Nikolaev, St. John’s University.

Algebraic and Combinatorial Aspects of Polytopes (Code: SS 14A), Federico Ardila, San Francisco State University and Los Andes University, Laura Escobar, Washington University in St. Louis, and Raul Penaguiao, University of Zurich.

Algebraic K-theory, Mathematical Physics, and Perfectoid Spaces (Code: SS 12A), Shanna Dobson, California State University, Los Angeles.

Analysis, Combinatorics, and Geometry of Fractals (Code: SS 4A), Kyle Hambrook, San Jose State University, and Chun-Kit Lai, San Francisco State University.

Categorical and Combinatorial Methods in Representation Theory, and Related Topics (Code: SS 19A), Mee Seong Im, U.S. Naval Academy, Bach Nguyen, Xavier University of Louisiana, and Arik Wilbert, University of Georgia.

Commutative Algebra (Code: SS 21A), Juliette Bruce, Mathematical Sciences Research Institute, Berkeley, Monica Lewis, University of Michigan, and Sean Sather-Wagstaff, Clemson.

Connections between homotopical algebra and geometry (Code: SS 6A), Ryan Grady, Montana State University, and Chris Rogers, University of Nevada, Reno.

Diagrammatic and Combinatorial Methods in Representation Theory (Code: SS 17A), Robert Muth, Washington & Jefferson College, Nick Davidson, Reed College, Peter Tingley, Loyola University Chicago, and Tianyuan Xu, University of Colorado Boulder.

Differential Geometry and Geometric PDE (Code: SS 1A), Alfonso Agnew, Nicholas Brubaker, Thomas Murphy, Shoo Seto, and Bogdan Suceava, California State University, Fullerton.

Geodesics in Hyperbolic 2- and 3-Manifolds (Code: SS 10A), Maria Trnkova, University of California, Davis, and Andrew Yarmola, Princeton University.

Geometric Analysis (Code: SS 22A), Ovidiu Munteanu, University of Connecticut, and David Bao, San Francisco State University.

Geometric and Categorical Methods in Representation Theory (Code: SS 16A), Ana Balibanu, Harvard University, Daniele Rosso, Indiana University Northwest, and Jonathan Wang, Massachusetts Institute of Technology.

How do Industry Professionals Use Big Data? (Code: SS 15A), Luella Fu, San Francisco State University.
Localization and delocalization in ergodic quantum systems (Code: SS 2A), Ilya Kachkovskiy, Michigan State University, and Wencai Liu and Rodrigo Matos, Texas A&M University.

Nonlinear PDEs and fluid dynamics (Code: SS 13A), Igor Kukavica, Juhi Jang, and Wojciech Ozanski, University of Southern California.

Quivers, Tensors, and Their Applications (Code: SS 20A), Visu Makam, Institute for Advanced Study, Francesca Gandini, Kalamazoo College, and Alana Huszar and Robert Cochrane, University of Michigan.

Regularity Theory for Linear and Nonlinear PDEs (Code: SS 11A), Zongyuan Li, Rutgers University, Weinan Wang, University of Arizona, and Xueying Yu, Massachusetts Institute of Technology.

Research in Mathematics by Early Career Graduate Students (Code: SS 8A), Michael Bishop, Marat V. Markin, and Khang Tran, California State University, Fresno.

Social Change through Mathematics and Education (Code: SS 18A), Federico Ardila and Shandy Hauk, San Francisco State University, Ashia Wilson, Massachusetts Institute of Technology, and Robin Wilson, California State Polytechnic University, Pomona.

Topological Perspectives in Graph Theory, Classical and Recent (Code: SS 3A), Jonathan L. Gross, Columbia University, Timothy Sun, San Francisco State University, and Thomas W. Tucker, Colgate University.

Women in Commutative Algebra - One hundred years of Idealtheorie in Ringbereichen (Code: SS 7A), Eloïsa Grifo and Alessandra Costantini, University of California, Riverside.

**Grenoble, France**

Université de Grenoble-Alpes

**July 5–9, 2021**

Monday – Friday

**Meeting #1168**

Associate secretary: Michel L. Lapidus

Program first available on AMS website: Not applicable

Issue of Abstracts: Not applicable

**Deadlines**

For organizers: Expired

For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see [https://www.ams.org/internmtgs.html](https://www.ams.org/internmtgs.html).

**Special Sessions**

**Advances in Functional Analysis and Operator Theory**, Marat V. Markin, California State University, Fresno, USA, Igor Nikolaev, St. John’s University, USA, Jean Renault, Universite d’Orleans, France, and Carsten Trunk, Technische Universität Ilmenau, Germany.

**Algebraic Geometry (Associated with Plenary Speaker Claire Voisin)**, Radu Laza, Stony Brook University, USA, Catriona Maclean, Grenoble, France, and Claire Voisin, Paris, France.

**Automorphic Forms, Moduli Spaces, and Representation Theory (Associated with Plenary Speaker Vincent Lafforgue)**, Jean-Francois Dat, Sorbonne Université, France, and Bao-Chau Ngo, University of Chicago, USA.

**Classical and Quantum Fields on Lorentzian Manifolds**, Dietrich Häfner, Université Grenoble Alpes, France, and Andras Vasy, Stanford University, USA.

**Combinatorial and Computational Aspects in Topology**, Eric Samperton, University of Illinois, USA, Saul Schleimer, University of Warwick, United Kingdom, and Greg McShane, Université Grenoble-Alpes, France.

**Contact Geometry**, David E. Blair, Michigan State University, USA, Gianluca Bande, Università degli Studi di Cagliari, Italy, and Eric Loubeau, Université de Bretagne Occidentale, France.

**Deformation of Artinian algebras and Jordan type**, Anthony Iarrobino, Northeastern University, USA, Pedro Macias Marques, Universidade de Evora, Portugal, Maria Evelina Rossi, Università degli Studi di Genova, Italy, and Jean Valles, Universite de Pau et des Pays de l’Adour, France.

**Deformation Spaces of Geometric Structures**, Sara Maloni, University of Virginia, USA, Andrea Seppi, Université Grenoble Alpes, France, and Nicolas Tholozan, École Normale Supérieure de Paris, France.

**Derived Categories and Rationality**, Matthew Ballard, University of South Carolina, USA, Emanuele Macrì, Université Paris-Saclay, France, and Patrick McFaddin, Fordham University, USA.

**Differential Geometry in the Tradition of Élie Cartan (1869 - 1959)**, Vincent Borelli, Université Claude Bernard, Bogdan Suceavă, California State University, Fullerton, USA, Mihaela B. Vajiac, Chapman University, USA, Joeri Van der Veken,
KIJ Leuven, Belgium, Marina Ville, Université de Tours, France, and Luc Vrancken, Université Polytechnique Hauts-de-France, Valenciennes, France.

Drinfeld Modules, Modular Varieties and Arithmetic Applications, Tuan Ngo Dac, CNRS Université Claude Bernard Lyon 1, France, Matthew Papanikolas, Texas A&M University, USA, Mihran Papikian, Pennsylvania State University, USA, and Federico Pellarin, Université Jean Monnet, France.

Fractal Geometry in Pure and Applied Mathematics, Hafedh Herichi, Santa Monica College, USA, Maria Rosaria Lancia, Sapienza Università di Roma, Italy, Therese-Marie Landry, University of California, Riverside, USA, Anna Rozanova-Pierrat, CentraleSupélec, Universite Paris-Saclay, France, and Steffen Winter, Karlsruhe Institute of Technology, Germany.

Functional Equations and Their Interactions, Guy Casale, IRMAR, Université de Rennes 1, France, Thomas Dreyfus, IRMA, Université de Strasbourg, France, Charlotte Hardouin, IRMAR, Université de Toulouse 3, France, Joel Nagloo, CUNY, New York, USA, Julien Roques, Institut Camille Jordan, Université de Lyon 1, France, and Michael Singer, North Carolina State University, Raleigh, USA.

Graph and Matroid Polynomials: Towards a Comparative Theory, Emeric Gioan, LIRMM, France, Johann A. Makowsky, Israel Institute of Technology-ITT, Israel, and James Oxley, Louisiana State University, USA.

Groups and Topological Dynamics, Nicolas Matte Bon, University of Lyon, France, Constantine Medynets, United States Naval Academy, USA, Volodymyr Nekrashevych, Texas A&M University, USA, and Dmytro Savchuk, University of South Florida, USA.

Group Theory, Algorithms and Applications, Indira Chatterji, Université de Nice, France, Francois Dahmani and Martin Deraux, Institut Fourier, Université Grenoble, Alpes, France, and Delaram Kahrobaei, CUNY and NYU, USA.

History of Mathematics Beyond Case-Studies, Catherine Goldstein, CNRS, IMJ-PRG, France, and Jemma Lorenat, Pitzer College, USA.

Integrability, Geometry, and Mathematical Physics, Luen-Chau Li, Pennsylvania State University, USA, and Serge Parmentier, Université Claude Bernard Lyon 1, France.

Inverse Problems, Hanna Makaruk, Los Alamos National Laboratory (LANL), USA, Robert Owczarek, University of New Mexico, Albuquerque and Los Alamos, USA, Tomasz Lipniacki, Polish Academy of Sciences, Poland, and Piotr Stachura, Warsaw University of Life Sciences-SGGW, Poland.

Low-Dimensional Topology, Paul Kirk, University Bloomington, USA, Christine Lescop, CNRS, Institut Fourier, Université Grenoble Alpes, France, and Jean-Baptiste Meilhan, Institut Fourier, Université Grenoble, Alpes, France.

Mathematical Challenges in Complex Quantum Systems (Associated with Plenary Speaker Simone Warzel), Alain Joye, Institut Fourier, Université Grenoble Alpes, France, Jeffrey Schenker, Michigan State University, USA, Nicolas Rougerie, Université Grenoble-Alpes and CNRS, France, and Simone Warzel, Zentrum Mathematik, TUM München, Germany.

Mathematical Knowledge Management in the Digital Age of Science, Patrick Ion, University of Michigan, Ann Arbor, USA, Thierry Bouche, Université Grenoble-Alpes, France, and Stephen Watt, University of Waterloo, Canada.

Mathematical Physics of Gravity, Geometry, QFTs, Feynman and Stochastic Integrals, Quantum/Classical Number Theory, Algebra, and Topology, Michael Maroun, AMS-MRC Boston, USA, and Pierre Vanhove, EMS/SMF CEA Paris Saclay, France.

Modular Representation Theory, Pramod N. Achar, Louisiana State University, USA, Simon Riche, Université Clermont Auvergne, France, and Britta Spath, Bergische Universität Wuppertal, Germany.

Percolation and Loop Models (Associated with Plenary Speaker Hugo Duminil-Copin), Ioan Manolescu, University of Fribourg, Switzerland.

Quantitative Geometry of Transportation Metrics, Florent Baudier, Texas A&M University, USA, Dario Cordero-Erasquin, Sorbonne Université, France, Alexandros Eskenazis, University of Cambridge, United Kingdom, and Eva Pernecka, Czech Technical University in Prague, Czech Republic.

Recent Advances in Diffeology and their Applications, Jean-Pierre Magnot, Université d’Angers, France, and Jordan Watts, Central Michigan University, USA.

Rough Path and Malliavin Calculus, Fabrice Baudoin, University of Connecticut, USA, Antoine Lejay, University of Lorraine, France, and Cheng Ouyang, University of Illinois at Chicago, USA.

Spectral Optimization, Richard S. Laugesen, University of Illinois at Urbana Champaign, USA, Enea Parini, Aix Marseille University, France, and Emmanuel Russ, Grenoble Alpes University, France.

Statistical Learning (Associated with Plenary speaker Peter Bühlmann), Christophe Giraud, Paris Saclay University, France, Cun-Hui Zhang, Rutgers University, USA, and Peter Bühlmann, ETH Zürich, Switzerland.

Sub-Riemannian Geometry and Interactions, Luca Rizzi, CNRS, Institut Fourier, Grenoble, France, and Fabrice Baudoin, University of Connecticut, USA.
Mathematical Congress of the Americas 2021
The third Mathematical Congress of the Americas (MCA)

**July 19–24, 2021**
Monday – Saturday

**Meeting #1169**
Associate secretary: Steven H. Weintraub
Program first available on AMS website: To be announced

Due to current uncertainty about the possibility of traveling this year, and in order to make planning ahead possible, the Steering Committee of the MCA has decided to make MCA2021 a fully online event.

**Buffalo, New York**
*University at Buffalo (SUNY)*

**September 18–19, 2021**
Saturday – Sunday

**Meeting #1170**
Eastern Section
Associate secretary: Steven H. Weintraub

Program first available on AMS website: August 5, 2021
Issue of Abstracts: Volume 42, Issue 3

**Deadlines**
For organizers: February 18, 2021
For abstracts: July 27, 2021

*The scientific information listed below may be dated. For the latest information, see [https://www.ams.org/sectional.html](https://www.ams.org/sectional.html).*

**Invited Addresses**
- Kirstin Eisentraeger, Pennsylvania State University, *Title to be announced*.
- Jason Manning, Cornell University, *Title to be announced*.
- Jennifer Mueller, Colorado State University, *Title to be announced*.

**Omaha, Nebraska**
*Creighton University*

**October 9–10, 2021**
Saturday – Sunday

**Meeting #1171**
Central Section
Associate secretary: Georgia Benkart

Program first available on AMS website: August 19, 2021
Issue of Abstracts: Volume 42, Issue 3

**Deadlines**
For organizers: March 9, 2021
For abstracts: August 10, 2021

*The scientific information listed below may be dated. For the latest information, see [https://www.ams.org/sectional.html](https://www.ams.org/sectional.html).*

**Invited Addresses**
- Daniel Erman, University of Wisconsin-Madison, *Title to be announced*.
- Jasmine Foo, University of Minnesota-Twin Cities, *Title to be announced*.
- Kay Kirkpatrick, University of Illinois Urbana-Champaign, *Title to be announced*. 
Albuquerque, New Mexico
University of New Mexico

October 23–24, 2021
Saturday – Sunday
Meeting #1172
Western Section
Associate secretary: Michel L. Lapidus

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Dissipative Systems and Their Applications (Code: SS 3A), Mingji Zhang and Bixiang Wang, New Mexico Institute of Mining and Technology.
Inverse Problems: In Memory of Professor Zbigniew Oziewicz (Code: SS 1A), Hanna Makaruk, Los Alamos National Laboratory, and Robert Owczarek, University of New Mexico.
Recent Advances in Studies of Electrodiffusion Phenomena (Code: SS 2A), Weishi Liu, University of Kansas, Hamid Mofidi, University of Iowa, and Mingji Zhang, New Mexico Institute of Mining and Technology.

Mobile, Alabama
University of South Alabama

November 20–21, 2021
Saturday – Sunday
Meeting #1173
Southeastern Section
Associate secretary: Brian D. Boe

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Invited Addresses
Sara Del Valle, Los Alamos National Laboratory, Title to be announced.

Seattle, Washington
Washington State Convention Center and the Sheraton Seattle Hotel

January 5–8, 2022
Wednesday – Saturday
Meeting #1174
Associate secretary: Georgia Benkart
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced
MEETINGS & CONFERENCES

Charlottesville, Virginia
University of Virginia

March 11–13, 2022
Friday – Sunday

Meeting #1175
Southeastern Section
Associate secretary: Brian D. Boe

Program first available on AMS website: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: August 12, 2021
For abstracts: January 18, 2022

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Invited Addresses

Moon Duchin, Tufts University, Title to be announced (Einstein Public Lecture in Mathematics).
Laura A Miller, University of North Carolina at Chapel Hill, Title to be announced.
Betsy Stovall, University of Wisconsin-Madison, Title to be announced.
Yusu Wang, The Ohio State University, Title to be announced.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Difference, Differential, Fractional Differential and Dynamic Equations with Applications (Code: SS 6A), Muhammad Islam and Yousef Raffoul, University of Dayton.
Advances in Infectious Disease Modeling: From Cells to Populations (Code: SS 5A), Lauren Childs, Stanca Ciup, and Omar Saucedo, Virginia Tech.
Advances in Operator Algebras (Code: SS 11A), Ben Hayes and David Sherman, University of Virginia.
Algebraic Groups: Arithmetic and Geometry (Code: SS 1A), Raman Parimala, Emory University, Andrei Rapinchuk, University of Virginia, and Igor Rapinchuk, Michigan State University.
Celebrating Diversity in Mathematics (Code: SS 7A), Lauren Childs, Virginia Tech, Sara Maloni, University of Virginia, and Rebecca R.G., George Mason University.
Combinatorial Methods in Geometric Group Theory (Code: SS 19A), Tarik Aougab, Haverford College, Marrissa Loving, Georgia Institute of Technology, and Priyam Patel, University of Utah.
Commutative Algebra (Code: SS 2A), Eloisa Grifo, University of California, Riverside, and Sean Sather-Wagstaff, Clemson University.
Homotopy Theory (Code: SS 10A), Julie Bergner and Nick Kuhn, University of Virginia.
Integrable Probability (Code: SS 14A), Leonid Petrov, University of Virginia, and Axel Saenz, Tulane University.
Knots and Links in Low-Dimensional Topology (Code: SS 13A), Thomas Mark, University of Virginia, and Allison Moore, University of California Davis.
Knot Theory and its Applications (Code: SS 20A), Hugh Howards and Jason Parsley, Wake Forrest University, and Eric Rawdon, St Thomas University.
Mathematical String Theory (Code: SS 8A), Ilarion Melnikov, James Madison University, Eric Sharpe, Virginia Tech, and Diana Vaman, University of Virginia.
Probabilistic Methods in Geometry and Analysis (Code: SS 12A), Fabrice Baudoin and Li Chen, University of Connecticut.
Recent Advances in Graph Theory and Combinatorics (Code: SS 17A), Neal Bushaw, Virginia Commonwealth University, and Martin Rolek and Gexin Yu, College of William and Mary.
Recent Advances in Harmonic Analysis (Code: SS 3A), Amalia Culiuc, Amherst College, Yen Do, University of Virginia, and Eyvindur Ari Palsson, Virginia Tech.
Recent Advances in Mathematical Biology (Code: SS 23A), Junping Shi, College of William & Mary, Xisheng Shuai, University of Central Florida, and Yixiang Wu, Middle Tennessee State University.
Recent Progress on Singular and Oscillatory Integrals (Code: SS 4A), Betsy Stovall and Joris Roos, University of Wisconsin-Madison.
MEETINGS & CONFERENCES

Representation Theory of Algebraic Groups and Quantum Groups: A Tribute to the Work of Cline, Parshall and Scott (CPS) (Code: SS 18A), Chun-Ju Lai and Daniel K. Nakano, University of Georgia, and Weiqiang Wang, University of Virginia.

Representation Theory of Algebras and Related Combinatorics (Code: SS 24A), Markus Schmidmeier, Florida Atlantic University, and Khrystyna Serhiyenko, University of Kentucky.

Tensors and Complexity (Code: SS 16A), Visu Makam, Institute for Advanced Study, and Rafael Oliveira, University of Waterloo.

topics in Convexity and Probability (Code: SS 22A), Steven Hoehner, Longwood University, and Mark Meckes and Elizabeth Werner, Case Western Reserve University.

Trends in Teichmüller Theory (Code: SS 21A), Thomas Koberda and Sara Maloni, University of Virginia, and Giuseppe Martone, University of Michigan.

Vertex Algebras and Geometry (Code: SS 9A), Marco Aldi, Virginia Commonwealth University, Michael Penn, Randolph College, and Nicola Tarasca and Juan Villarreal, Virginia Commonwealth University.

Youth and Enthusiasm in Arithmetic Geometry and Number Theory (Code: SS 15A), Evangelia Gazaki and Ken Ono, University of Virginia.

Medford, Massachusetts
Tufts University

March 19–20, 2022  Saturday – Sunday

Meeting #1176
Eastern Section
Associate secretary: Steven H. Weintraub

Program first available on AMS website: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: August 24, 2021
For abstracts: January 18, 2022

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Invited Addresses

Daniela De Silva, Barnard College, Columbia University. Title to be announced.
Enrique R. Pujals, Graduate Center, CUNY, Title to be announced.
Christopher T Woodward, Rutgers University, New Brunswick, Title to be announced.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Analysis on Homogeneous Spaces (Code: SS 7A), Jens Christensen, Colgate University, Matthew Dawson, CIMAT, Mérida, México, and Fulton Gonzalez, Tufts University.

Automorphisms of Riemann Surfaces, Subgroups of Mapping Class Groups and Related Topics (Code: SS 3A), S. Allen Broughton, Rose-Hulman Institute of Technology, Jen Paulhus, Grinnell College, and Aaron Wootton, University of Portland.

Equivariant Cohomology (Code: SS 4A), Jeffrey D. Carlson, The Fields Institute, and Loring Tu, Tufts University.

Homological Methods in Commutative Algebra (Code: SS 6A), Janet Striuli, Fairfield University and National Science Foundation, and Oana Veliche, Northeastern University.

Inverse Problems and Their Applications (Code: SS 1A), Youssef Qranfal, Wentworth Institute of Technology.


Mathematics of Data Science (Code: SS 2A), Vasileios Maroulas, University of Tennessee Knoxville, and James M. Murphy, Tufts University.

Symmetries of Polytopes, Maps, and Graphs (Code: SS 5A), Gabe Cunningham, University of Massachusetts Boston, and Mark Mixer, Wentworth Institute of Technology.
MEETINGS & CONFERENCES

West Lafayette, Indiana
Purdue University

March 26–27, 2022
Saturday – Sunday

Meeting #1177
Central Section
Associate secretary: Georgia Benkart

Program first available on AMS website: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: August 31, 2021
For abstracts: January 25, 2022

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Invited Addresses

Christine Berkesch, University of Minnesota, Title to be announced.
Matthew Hedden, Michigan State University, Title to be announced.
Brian Street, University of Wisconsin, Title to be announced.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Analysis and Probability in Sub-Riemannian Geometry (Code: SS 5A), Jeremy Tyson, University of Illinois at Urbana-Champaign, and Jing Wang, Purdue University.
A Women in Analysis Research Network Event (Code: SS 4A), Donatella Danielli-Garafalo, Purdue University, and Irina Mitrea, Temple University.
Combinatorial Algebra and Geometry (Code: SS 6A), Christine Berkesch, University of Minnesota, and Laura Matusevich and Aleksandra Sobieska, Texas A&M University.
Harmonic Analysis (Code: SS 2A), Shaoming Guo and Brian Street, University of Wisconsin-Madison.
Integrability, Symmetry and Physics (Code: SS 8A), E. Birgit Kaufmann and Oleksandr Tsymbaliuk, Purdue University.
Quantum Algebra and Quantum Topology (Code: SS 1A), Shawn Cui, Purdue University, Julia Plavnik, Indiana University, and Tian Yang, Texas A&M University.
Recent Developments in Commutative Algebra (Code: SS 7A), Jennifer Kenkel, University of Michigan, and Linquan Ma and Uli Walther, Purdue University.
The Interface of Harmonic Analysis and Analytic Number Theory (Code: SS 3A), Theresa Anderson, Purdue University, Robert Lemke Oliver, Tufts University, and Eyvindur Palsson, Virginia Tech University.

Denver, Colorado
University of Denver

May 14–15, 2022
Saturday – Sunday

Meeting #1178
Western Section
Associate secretary: Michel L. Lapidus

Program first available on AMS website: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: October 12, 2021
For abstracts: March 15, 2022
El Paso, Texas
University of Texas at El Paso

September 17–18, 2022
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Chattanooga, Tennessee
University of Tennessee at Chattanooga

October 15–16, 2022
Saturday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see [https://www.ams.org/sectional.html](https://www.ams.org/sectional.html).

Invited Addresses

Giulia Saccà, Columbia University, Title To Be Announced.
Chad Topaz, Williams College, Title To Be Announced.
Xingxing Yu, Georgia Institute of Technology, Title To Be Announced.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [https://www.ams.org/cgi-bin/abstracts/abstract.pl](https://www.ams.org/cgi-bin/abstracts/abstract.pl).

Active Learning Methods and Pedagogical Approaches in Teaching College Level Mathematics (Code: SS 6A), Hashim Saber, University of North Georgia.

Applied Knot Theory (Code: SS 1A), Jason Cantarella, University of Georgia, Eleni Panagiotou, University of Tennessee at Chattanooga, and Eric Rawdon, University of St Thomas.

Boundary Value Problems for Differential, Difference, and Fractional Equations (Code: SS 9A), John R Graef and Lingju Kong, University of Tennessee at Chattanooga, and Min Wang, Kennesaw State University.

Geometric and Topological Generalization of Groups (Code: SS 4A), Bikash C Das, University of North Georgia.

Nonstandard Elliptic and Parabolic Regularity Theory with Applications (Code: SS 2A), Hongjie Dong, Brown University, and Tuoc Phan, University of Tennessee, Knoxville.

Probability and Statistical Models with Applications (Code: SS 5A), Sher Chhetri, University of South Carolina, Sumter, and Cory Ball, Florida Atlantic University.

Quantitative Approaches to Social Justice (Code: SS 7A), Chad Topaz, Williams College.

Special Session on Combinatorial Commutative Algebra (Code: SS 8A), Michael Cowen, Hugh Geller, Todd Morra, and Sean Sather-Wagstaff, Clemson University.

Structural and Extremal Graph Theory (Code: SS 3A), Hao Huang, Emory University, and Xingxing Yu, Georgia Institute of Technology.
MEETINGS & CONFERENCES

Salt Lake City, Utah
University of Utah

October 22–23, 2022
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see https://www.ams.org/sectional.html.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at https://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic combinatorics and applications in harmonic analysis (Code: SS 3A), Joseph Iverson and Sung Y. Song, Iowa State University, and Bangteng Xu, Eastern Kentucky University.

Approximation Theory and Numerical Analysis (Code: SS 2A), Vira Babenko, Drake University, and Akil Narayan, University of Utah.

Building bridges between commutative algebra and nearby areas (Code: SS 5A), Benjamin Briggs and Josh Pollitz, University of Utah.

Commutative Algebra (Code: SS 4A), Adam Boocher, University of San Diego, Eloísa Grifo, University of California, Riverside, and Jennifer Kenkel, University of Michigan.

Extremal Graph Theory (Code: SS 1A), Bernard Lidický, Iowa State University.

Fractal Geometry, Dimension Theory, and Recent Advances in Diophantine Approximation (Code: SS 9A), Alexander M. Henderson, University of California, Machiel van Frankenhuijsen, Utah Valley University, and Edward K. Voskanian, The College of New Jersey.

Free boundary problems arising in applications (Code: SS 14A), Mark Allen, Brigham Young University, Mariana Smit Vega Garcia, Western Washington University, and Braxton Osting, University of Utah.

Geometry and Representation Theory of Quantum Algebras and Related Topics (Code: SS 6A), Mee Seong Im, United States Military Academy, West Point, Bach Nguyen, Xavier University of Louisiana, and Arik Wilbert, University of Georgia.

Graphs and Matrices (Code: SS 11A), Mark Kempton, Emily Evans, and Ben Webb, Brigham Young University.

Higher Topological and Algebraic K-Theories (Code: SS 18A), Agnès Beaudry, University of Colorado Boulder, Jonathan Campbell, Duke University, and John Lind, California State University, Chico.

Inverse Problems (Code: SS 12A), Hanna Makaruk, Los Alamos National Laboratory, and Robert Owczarek, University of New Mexico.

Knotted surfaces and concordances (Code: SS 15A), Mark Hughes, Brigham Young University, Jeffrey Meier, Western Washington University, and Maggie Miller, Princeton University.

Mathematics of Collective Behavior (Code: SS 10A), Roman Shvydkoy and Daniel Lear, University of Illinois at Chicago.

PDEs, data, and inverse problems (Code: SS 7A), Jared Whitehead, Brigham Young University.

Recent advances in algebraic geometry and commutative algebra in or near characteristic p (Code: SS 8A), Bhargav Bhatt, University of Michigan, and Karl Schwede, University of Utah.

Recent advances in the theory of fluid dynamics (Code: SS 17A), Elaine Cozzi, Oregon State University, and Magdalena Czubak, University of Colorado Boulder.

Recent Advances of Numerical Methods for Partial Differential Equations with Applications (Code: SS 16A), Joe Koebbe, Utah State University, Yunrong Zhu, Idaho State University, and Jia Zhao, Utah State University.

Several Complex Variables: Emerging Applications, Connections, And Synergies (Code: SS 13A), Jennifer Brooks, Brigham Young University, and Dusty Grundmeier, Harvard University.

Topics in graphs, hypergraphs and set systems (Code: SS 19A), David Galvin, University of Notre Dame, John Engbers, Marquette University, and Cliff Smyth, The University of North Carolina at Greensboro.
Boston, Massachusetts
John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

**January 4–7, 2023**
Wednesday – Saturday
Associate secretary: Steven H. Weintraub
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Atlanta, Georgia
Georgia Institute of Technology

**March 18–19, 2023**
Saturday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Fresno, California
California State University, Fresno

**May 6–7, 2023**
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: October 4, 2022
For abstracts: March 7, 2023

The scientific information listed below may be dated. For the latest information, see [https://www.ams.org/sectional.html](https://www.ams.org/sectional.html).

**Special Sessions**
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [https://www.ams.org/cgi-bin/abstracts/abstract.pl](https://www.ams.org/cgi-bin/abstracts/abstract.pl).

- **Advances by Scholars in the Pacific Math Alliance** (Code: SS 22A), Andrea Arauza Rivera, California State University, East Bay, Mario Banuelos, California State University, Fresno, and Jessica De Silva, California State University, Stanislaus.
- **Advances in Functional Analysis and Operator Theory** (Code: SS 6A), Michel L. Lapidus, University of California, Riverside, Marat V. Markin, California State University, Fresno, and Igor Nikolaev, St. John’s University.
- **Algebraic Structures in Knot Theory** (Code: SS 4A), Carmen Caprau, California State University, Fresno, and Sam Nelson, Claremont McKenna College.
- **Algorithms in the study of hyperbolic 3-manifolds** (Code: SS 26A), Robert Haraway, III and Maria Trnkova, University of California, Davis.
- **Analysis of Fractional Differential and Difference Equations with its Application** (Code: SS 20A), Bhuvaneswari Sambandham, Dixie State University, and Aghalaya S. Vatsala, University of Louisiana at Lafayette.
- **Artin-Schelter regular algebras and related topics** (Code: SS 27A), Ellen Kirkman, Wake Forest University, and James Zhang, University of Washington.
- **Combinatorics Arising from Representations** (associated with the Invited Address by Sami Assaf) (Code: SS 16A), Sami Assaf, University of Southern California, Nicolle Gonzalez, University of California, Los Angeles, and Brendan Pawloski, University of Southern California.
- **Complexity in Low-Dimensional Topology** (Code: SS 14A), Jennifer Schultens, University of California, Davis, and Eric Sedgwick, DePaul University.
Data Analysis and Predictive Modeling (Code: SS 8A), Earvin Balderama, California State University, Fresno, and Adriano Zambom, California State University, Northridge.

Inverse Problems (Code: SS 5A), Hanna Makaruk, Los Alamos National Laboratory, and Robert Owczarek, University of New Mexico, Albuquerque and University of New Mexico, Los Alamos.

Math Circle Games and Puzzles that Teach Deep Mathematics (Code: SS 13A), Maria Nogin and Agnes Tuska, California State University, Fresno.

Mathematical Biology: Confronting Models with Data (Code: SS 21A), Erica Rutter, University of California, Merced.

Mathematical Methods in Evolution and Medicine (associated with the Invited Address by Natalia Komarova) (Code: SS 1A), Natalia Komarova and Jesse Kreger, University of California, Irvine.

Methods in Non-Semisimple Representation Categories (Code: SS 11A), Eric Friedlander, University of Southern California, Los Angeles, Julia Pevtsova, University of Washington, Seattle, and Paul Sobaje, Georgia Southern University, Statesboro.

Recent Advances in Mathematical Biology, Ecology, Epidemiology, and Evolution (Code: SS 10A), Lale Asik, Texas Tech University, Khanh Phuong Nguyen, University of Houston, and Angela Peace, Texas Tech University.

Research in Mathematics by Early Career Graduate Students (Code: SS 7A), Doreen De Leon, Marat Markin, and Khang Tran, California State University, Fresno.

Scientific Computing (Code: SS 19A), Changho Kim, University of California, Merced, and Roummel Marcia.

The use of computational tools and new augmented methods in networked collective problem solving (Code: SS 18A), Mario Banuelos, California State University, Fresno, Andrew G. Benedek, Research Centre for the Humanities, Hungary, and Agnes Tuska, California State University, Fresno.

Women in Mathematics (Code: SS 12A), Doreen De Leon, Katherine Kelm, and Oscar Vega, California State University, Fresno.

Zero Distribution of Entire Functions (Code: SS 9A), Khang Tran and Tamás Forgács, California State University, Fresno.
MATHEMATICAL MOMENTS
PODCASTS

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Thinking Algebraically
An Introduction to Abstract Algebra
Thomas Q. Sibley, St. John’s University, Collegeville, MN

Thinking Algebraically presents the insights of abstract algebra in a welcoming and accessible way. It succeeds in combining the advantages of rings-first and groups-first approaches while avoiding the disadvantages. After an historical overview, the first chapter studies familiar examples and elementary properties of groups and rings simultaneously to motivate the modern understanding of algebra. The text builds intuition for abstract algebra starting from high school algebra. In addition to the standard number systems, polynomials, vectors, and matrices, the first chapter introduces modular arithmetic and dihedral groups.

The second chapter builds on these basic examples and properties, enabling students to learn structural ideas common to rings and groups: isomorphism, homomorphism, and direct product. The third chapter investigates introductory group theory. Later chapters delve more deeply into groups, rings, and fields, including Galois theory, and they also introduce other topics, such as lattices. The exposition is clear and conversational throughout.

The book has numerous exercises in each section as well as supplemental exercises and projects for each chapter. Many examples and well over 100 figures provide support for learning. Short biographies introduce the mathematicians who proved many of the results. The book presents a pathway to algebraic thinking in a semester- or year-long algebra course.

Ten years from now, what do you want or expect your students to remember from your course? We realized that in ten years what matters will be how students approach a problem using the tools they carry with them—common sense and common knowledge—not the particular mathematics we chose for the curriculum. Using our text, students work regularly with real data in moderately complex everyday contexts, using mathematics as a tool and common sense as a guide. The focus is on problems suggested by the news of the day and topics that matter to students, like inflation, credit card debt, and loans. We use search engines, calculators, and spreadsheet programs as tools to reduce drudgery, explore patterns, and get information. Technology is an integral part of today’s world—this text helps students use it thoughtfully and wisely.

This second edition contains revised chapters and additional sections, updated examples and exercises, and complete rewrites of critical material based on feedback from students and teachers who have used this text. Our focus remains the same: to help students to think carefully—and critically—about numerical information in everyday contexts.

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