Dynamics in One Non-Archimedean Variable
by Robert L. Benedetto

Benedetto’s book is a graduate-level introduction to dynamical systems over a non-archimedean field. Many people may be more familiar with the subject of complex dynamical systems, if not from groundbreaking work of Sullivan, Douady, and Hubbard in the 1970s and 80s, then from the eye-grabbing computer generated graphics of fractals produced during the same period by Mandelbrot. In his book, Benedetto’s main focus is the analogous study where the complex numbers are replaced by non-archimedean fields.

One of the attractions of non-archimedean dynamics is the beautiful interplay between number theory and dynamical systems, and the analogies and contrasts between the $p$-adic numbers and the complex numbers. For example, the Mandelbrot set, whose complicated fractal structure has inspired a great deal of research and open problems in complex dynamical systems, is quite simple over non-archimedean fields. On the other hand, while the dynamical behavior of points on Fatou components is relatively controlled for actions of rational maps on the complex Riemann sphere, more complicated phenomena occurs for analogous actions on the non-archimedean projective line, as seen in the existence of so-called “wandering domains.”

With this book, Benedetto develops the foundations of the subject in a way that is accessible to the beginning graduate student and useful to interested researchers from neighboring fields. He begins with a review of the dynamical systems of rational maps acting on both archimedean and non-archimedean spaces. He then builds the necessary background on $p$-adic geometry and the Berkovich projective line before moving on to core topics such as repelling fixed points, density and equilibrium measures for actions on Berkovich space. The chapters are written and arranged so that one can either pick and choose topics and themes to supplement one’s knowledge, or work through the book sequentially, absorbing the full overarching narrative.

An Introduction to $q$-analysis
by Warren P. Johnson

Johnson’s book is a textbook for a one- or two-semester capstone course, and as such it is a survey of important results in mathematics that are accessible to seniors with a standard undergraduate mathematics background. What makes this book special and particularly engaging is that it centers itself not around a list of major results within a theory, but rather around the remarkable uses and properties of $q$-series.

At over 500 pages, the book covers a lot of ground, including seminal work of Gauss, Euler, and Ramanujan, yet it is anything but encyclopedic in feel. Instead Johnson takes us on a journey through classical topics in combinatorics, analysis, and number theory, using the ever-present $q$-series, not only as a way to count such things as inversions in permutations and numbers of partitions, but also as an organizational and conceptual centerpiece.

The mental gymnastics involved in translating a counting problem into a manipulation of formal sums can be daunting to beginners, and can take time and perseverance to master. To counter this, Johnson avoids using a dry definitions-statements-proofs style of pedagogy and instead keeps his readers engaged by skillfully employing a consistent narrative flow that simultaneously holds aloft historical perspectives, running examples, and clearly identifiable goals.

Johnson’s book balances unquestionable good taste and mathematical relevance with just a tinge of quirkiness of topic to make it an attractive basis for a bookend course aimed at senior math majors who may or may not be contemplating math graduate school. It is also the perfect bookshelf item for graduate students and researchers as a useful reference on how to use $q$-series as an elegant and powerful problem-solving tool or as a book to have on hand for an enjoyable read on a rainy day—it is one of those books that one can simply pull down off the shelf, open at random, and learn something cool.