Note that the same trick applies if in addition to your "we prove a new strong result" story, you want to include "in a special case our proof simplifies and gives a much easier proof of a classical result" story. Do this in exactly the same place—after the main results are stated, but before the main proof. Even if this creates a little redundancy, so what? The simplicity of the self-contained new proof far outweighs the extra few pages it occupies. And the reader who wishes to finish reading the paper will also be better prepared to tackle the proof of the main result having gone through the simple proof of an earlier result. Of course, if the special case is much too small, elementary, and not really helpful to understand the main proof, you are better off publishing this proof in a popular general audience journal.

Very occasionally, fields *abc* and *pqr* are too far apart, and both stories are equally valuable. This difficult decision can be resolved by writing two separate papers aimed at two different audiences. Let me emphasize: this is rare and ethically dubious unless both papers have some extra results the other does not, and you discuss the existence of the other paper straight in the introduction of each paper. Still, this is better than spoiling your story or making it more complicated. In summary, keep it simple!

Final practical advice. As always, the best way is to first try consulting with your friends and colleagues, then try again. Your writing will probably improve over time. Practice makes perfect stories. But if such help is unavailable, try to emulate others. If you are proving a special case of conjecture *X* in paper [M], search in GoogleScholar for papers which cite [M] and mention *X*. If there are too many, use MathSciNet® which tends to have fewer citations. If there are still too many, use some keywords, MSC numbers, or "From Reviews" link.

Now, go over all these papers to see what stories they are telling. Most likely you can frame your paper to tell a story along similar lines. As an important side benefit you will also learn the state of the art on X, and perhaps find some new examples and special cases your results can be applied to.



Igor Pak

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Collaborative Writing: What, How, and Why

Margaret Symington and Daniele Sepe

So, you're in a collaboration. You've had fun bouncing ideas back and forth, trying to jot down precise statements on the back of napkins in crowded bars or on your tablet in the comfort of your home, staring at collaborators through your computer screen. The thrill of working out the main arguments is gone, leaving behind a quiet satisfaction. It is time to face what is probably your fiercest critic: a blank page. In short, it is time to write up. While not exactly the most exciting aspect of your mathematical life, it may be seen as the necessary cost of turning ideas into publications, the heavyweights on your CV. What's more, this time you have to do it together with your collaborators, each of whom comes with personal views on notation, style, presentation, and so forth.⁵ Writing mathematics can feel like a hard, tedious task, but we're here to say that it can be interesting, and when done together can be both instructive and fun.

We're by no means experts on collaborative writing. We're merely sharing thoughts based on our experiences, especially a joint multiyear ongoing project with four coauthors, and what one of us has learned from Deneen Senasi, the Writing Director at Mercer University, while teaching (nonmathematical) writing to sophomores.

To discuss writing in the context of collaborations, we draw a distinction between *coauthoring* and *cowriting*. We use the former to refer to the mechanism for producing every paper that has more than one author, and the latter to refer to collaboration on the complex, multilayered process of writing. In what follows, we name and describe some aspects of coauthoring and cowriting, in the hopes of making the collaborative writing process seem less mysterious and more attractive

The "Co" of Coauthoring

Back to that collaboration of yours. You will all, automatically, be coauthors because you all will have contributed to the contents and production of the paper. But the flavor of your coauthorship may vary significantly. In particular, the overarching structure of your coauthorship will probably be a mix, sometimes shifting, of the following three:

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⁵You'd be surprised by how strongly opinionated some mathematicians are on the age-old debate of italics vs. boldface.

DOI: https://dx.doi.org/10.1090/noti2304

- *Follow the Leader*—one person writes; the others follow along, ask questions, and make suggestions;
- *Divide and Conquer*—everybody gets their own sections to write and the end result is obtained by collating;
- Round and Round—drafts get passed from one of you to the next for revision, amending, and polishing, hopefully converging in finite time.

While Round and Round may sound like the only truly collaborative structure, fear not the others. Even if you end up leading or following a leader, or are in charge of a specific section, there's plenty of room for everyone to have a strong influence on the final product. (Our highly collaborative four-way coauthorship largely utilized Follow the Leader, though it sometimes felt like Round and Round, with a touch of Divide and Conquer.) In our experience, the essentials for a constructive and engaging coauthorship are listening to each other, never refraining from pitching ideas, and always being willing to say "I don't understand," or ask "What do you mean?" even though doing so may feel daunting.

The "Co" of Cowriting

Onward to the write-up of your work. You certainly want to produce a manuscript that contains your interesting results and proofs, presented in a clear and concise fashion. But you can do more: as a writer of mathematics, you can shape your reader's understanding. Achieving this goal involves making choices upon choices—both big-picture and microlevel: how to present and structure the proofs of your results; what background material to include and from what point of view; notation, terminology, and wording, etc. That's where cowriting comes in: you are cowriting if at least one of your coauthors becomes a partner in making those choices.

We find that, to make such decisions in a coherent manner and with confidence, we need to wrestle with the following big-picture aspects of any piece of writing:

- Audience—Who are you writing for? What do they need to know?
- Purpose—What is the fundamental message of your manuscript? Are you trying to instruct, refine, or shift current understanding, fill in a gap, break new ground, etc.?
- Context—How do your results or your perspective relate to existing ones? What were your inspirations and what future directions do you envision?

Just like the math that you already sorted out with your collaborators, core questions concerning audience, purpose, and context are often best addressed in conversation, which gets your creative juices flowing. Furthermore, the push and pull of different perspectives can ultimately lead you to a richer understanding of how readers might experience your final product.

If you and your cowriter(s) come to a common understanding about context, audience, and purpose, then you are well-positioned to help each other out with all the microlevel choices, which will be advancing your objectives. In turn, thinking about your goals as you make a small-scale choice, such as what details to include in a particular proof, can change your perspective on big-picture aspects. This can cause you to write and rewrite several versions of proofs as your thinking about the results and the paper evolve.... But wait: this is beginning to sound like one step forward, two steps backwards. Why would you want to do that?

What's in It for You?

There's no question that a collaborative effort on crafting a manuscript takes time—and also runs the risk of feeding any tendencies toward perfectionism. But there's plenty to be gained. Shifts in thinking that occur while writing are part of understanding the mathematical problem at hand. We found that, through our discussions about how to frame our results, we learned a lot and deepened our understanding of our own work-more than we would have if we were writing it up separately. In fact, our cowriting also led to ideas for further math projects. The most important gain, however, is the likelihood that cowriting leads to a final manuscript that tells a mathematical story that better enables others to appreciate your ideas, thereby increasing your readership and the impact of your paper. Along the way, you will have gained skills as a writer (which will help you when you're on your own), and, if you are lucky, formed or solidified a friendship.

What kind of collaborative writing you choose on any project depends primarily on your circumstances, from deadlines to workload to individual personalities and each author's expertise. The key is to establish a working routine that plays to the strength of each coauthor and makes everyone feel comfortable. We hope this piece gave you some ideas of options. Final disclaimer: of course, we cowrote this piece and, by the end, we realized we'd written a piece about collaborative writing, when we had set out to write only about cowriting!





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Credits

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