

development of a new course, implementation of teaching innovations, or a seed project that increases the likelihood of future external funding, for example.

Once you have a draft, you should first go back to the guidelines and check very carefully that all have been followed. Do not give the reviewer a trivial reason to reject the proposal. Make sure you have addressed all items requested in the call for proposals or program description and that you have the nitty gritty details (margins, font sizes) correct. For programs that evaluate proposals using a rubric, following the guidelines carefully and in order will make it easier for the evaluator to find each section, which is likely to lead to a better outcome. Now enlist an experienced colleague or your department chair for feedback. For a technical grant, someone in your area who has received a similar type of funding is ideal. For a more general (internal) grant, someone not in your field might be better able to advise you on whether or not you have explained your work well for a general audience. Seek feedback and then listen to it. Revise as needed to improve the proposal and to reach the correct audience. If you have reviewers' comments from an earlier version of your proposal, read those carefully and modify your proposal to address any concerns raised. If you are unsure how to interpret the comments, a trusted experienced colleague can help. You can also contact the program officer or review committee chair (for internal grants) to ask about feedback received, to ask for additional feedback on earlier proposals, or to clear up any confusion regarding guidelines.

By the time the proposal is ready to be submitted, you will have spent a great deal of time and energy on it. Ideally the experience of thinking about your proposed work and how to present it along with how it fits into the bigger picture will have been a benefit to you whether or not the grant is funded. Use the process to learn, connect with new mentors, and get feedback on your ideas. If the grant is not funded, you will still have benefited from the experience. This is particularly important since it typically requires several rounds of application, denial, revision and improvement, and reapplication to become successful in obtaining funding. If your proposal is funded, congratulations! The hard work of making the most of the opportunity is just beginning.

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Susan Morey

Credits

Photo of Susan Morey is courtesy of Susan Morey.

Communicating Mathematics in a Research Proposal

Emily Clader

Good proposal writing is, in many ways, just good mathematical writing, which is just good writing, period. Neither of these equivalences is entirely true, however, and examining their nuances is a helpful way to probe the question of what exactly makes a strong research proposal in mathematics.

To be clear, when I say “research proposal,” I have in mind the Project Description in an application for a National Science Foundation grant, but also the part of any grant application—whether through the NSF, the Simons Foundation, your home institution, or some other funding body—in which you, the applicant, explain the work for which you are requesting funding and why it deserves to be supported. How can you comprehensibly and convincingly explain the significance of your research within the constraints of a proposal?

Good Proposal Writing is Good Mathematical Writing

Let's start by considering, in the specific context of a research proposal, some of the aspects that good mathematical writing generally exhibits.

- **It invites interest from the entire intended audience.** “Know your audience,” they say. In the case of a grant proposal, that audience is your reviewers, who are likely to be members of your broad research community—algebraic geometry, for example, or number theory—but unaware of the technical details of your more focused field. On the other hand, they could very well be the world's leading expert in your microtopic. For the sake of the nonexperts, it's helpful to highlight some of your subfield's flashier or more well-publicized results, and to situate your own work within that story. The experts will catch you if you tell any outright lies, but they're likely to be impressed rather than bored if you can gloss over the details enough to give an engaging and accessible account of the mathematics they already know.
- **It leads with motivation and chooses details with intention.** No first sentence should ever be “Let X be a projective variety.” This is true of papers, and it's doubly true of research proposals. Before being presented with mathematical details, a reader's interest should

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be primed with clear yet nontechnical motivation. Of course, the time for details will come later in the proposal, but not every detail necessarily needs to make the cut. Is it important to the story you're telling that the reader sees the subtleties that arise in a certain special case, or the construction that underlies a certain definition? If not, acknowledge to the reader that you are leaving something out and give yourself permission to move on.

- **It anticipates questions, but not all of them.** Pointing out natural follow-up questions is a great way to lend motivation and structure to a research proposal: you present the current state-of-the-art, and you do so in such a way that your own research problems are framed as the next logical things to explore. Be careful of the temptation to over-explain, though. As an expert in your own subject, you've probably been confused about some subtle point that eventually led to an epiphany. But if that subtle point is not something a novice would pick up on, then clarifying that potential confusion may be more distracting than illuminating.

The above are things I find helpful to keep in mind whenever I write about mathematics, but a research proposal is not the same as other types of mathematical writing; it's not a textbook, it's not a paper, and it's not a research statement like the ones you wrote when applying for graduate school, postdocs, or faculty positions. Some of the peculiar qualities of proposal writing are the following.

- **It is forward-looking.** Highlight what you've already done, but remember that the real purpose of the proposal is to describe the work you intend to do in the future. Your existing work has paved the way for that future research, and it provides evidence of your capability, but it's a supporting player in this story.
- **Its scope is broad yet cohesive.** I find this one especially difficult, because I want my proposal to show off every good idea I've ever had. There's something to be said for flexing the breadth of your interests and expertise in a proposal—it's certainly not meant to be as focused as a paper—but be careful not to appear scattered. One good test is to make sure you can come up with a descriptive title for the entire proposal. If not, you may need to narrow your focus.

Good Mathematical Writing is Good Writing

I made another (perhaps controversial) claim at the beginning of this article, which is that good mathematical writing is, in at least some ways, just good writing. This is something that many mathematicians overlook, focusing their attention on whether information is accurately and understandably conveyed without considering how (un)pleasant the reader's experience of receiving that information might be. I believe that we ignore writing quality at our own peril, however, especially when our future funding depends on the vagaries of a reviewer's reaction to what

we have written. To bring good writing practice into your proposal writing, think about whether your proposal has the following qualities.

- **It's enjoyable to read.** This is perhaps the hardest piece of advice to implement, the one that requires the most genuine artistry, but it matters too much to ignore. Read your writing aloud to get a feel for its cadence. Have you repeated the same words in quick succession, or structured every sentence in the same way? If you have a favorite writer of nonmathematical nonfiction, look at their work with an eye toward what makes it flow.
- **It acknowledges the limits of a reader's attention.** Long paragraphs tend to make a reader's eyes glaze over, whereas careful formatting choices—boldface section titles, frequent signposts like “Theorem” or “Question”—keep them focused. Imagine that your reader's attention fades after the first couple of sentences of each paragraph and skims ahead for the next key idea. (Have you done that while reading this article? It's okay, I forgive you.) From this perspective, it's critical not to “bury the lede” by hiding an important point in the middle of a big block of text.

- **It's just a little bit repetitive.** Certain ideas are liable to recur in your proposal (your commitment to student-led research, for example, or your interest in extending technique A to setting B), and it creates a sense of cohesion to point out these threads as they arise. Try not to reiterate the very same idea more than once, however. While having recurring themes shows clarity of purpose, phrases like “as mentioned previously” tend more to demonstrate poor planning.

The relationship between good writing and good proposal writing certainly does have its limitations, however. Some qualities I would probably shun in a magazine article but I embrace in a grant proposal are the following.

- **It explicitly uses buzzwords from the grant solicitation.** Does the funding agency want to know that the work is “transformative” and that the applicant is “well-qualified” to carry it out? It may feel clunky, but sentences like “The proposed work has the potential to be transformative because...” or “The PI is particularly well-qualified to answer these questions in light of...” show that you understand and meet the funder's expectations.
- **It brags.** This one might go without saying, but it can be surprisingly difficult to implement. You may have to reassure yourself multiple times that the reader wants to be convinced of your capability and expertise. Writing in the third person (which is another of the idiosyncratic but common qualities of proposal writing) can help to make these assertions feel less like bragging and more like statements of fact.

In the end, applying for grants is going to be nerve wracking, time consuming, and often a source of no small amount of dread. But it will also be an opportunity to put

into words—perhaps more eloquent words than you’ve ever previously sought—what you find exciting about your research field and where you fit into that mathematical story. Try to have fun with it, to take pride in both your writing and your research. Rejections are inevitable, but the experience of crafting a beautifully written proposal will serve you well no matter what.



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What is Broader Impact?

Max Lieblich

Introduction

This is a brief discussion of the notion of “Broader Impact” in an NSF proposal. The NSF has a nice website [NSF21] about what Broader Impact means, with several examples. The NSF Proposal and Award Policies and Procedures Guide (PAPPG) [NSF20] currently states: “The Broader Impacts criterion encompasses the potential to benefit society and contribute to the achievement of specific, desired societal outcomes.” Peter March wrote a useful (but perhaps slightly dated) memo [Mar07] in 2007, laying out a vision for the types of societal goals that a proposal could seek to achieve.

This article is written more or less as a series of examples meant to complement the documents above, phrased in the form of questions that I hope will stimulate thought and (perhaps) conversations with mentors. My examples start with things that are not Broader Impacts, and proceed to those that are.

I have drawn on my own experience talking to people, reading proposals, and serving on panels to generate examples. One reason I encourage you to talk to other senior

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mathematicians is to have the opportunity to draw on their experiences, too.

Broader Impact Develops over a Career, not over a Year

Just as with research, Broader Impact activities evolve over time.

You can’t do it all

I certainly do not want anyone to think that trying to do *all* of these things at once is a remotely good idea. Doing something well takes effort and time, and that includes the Broader Impact work you do. Your career will change over time, and you should expect that you will start and stop activities as you go. Choose a Broader Impact (or a few) to work on for the next five years. Each year think about your goals and accomplishments, and tweak your plan. If, after five years, you feel like it might be time to move on, then switch to something new! Eliminating activities is just as important as starting new ones. On the other hand, if you are still passionate about the work, by all means keep doing it, making it better as you go.

The best any of us can do is to leave things better than we found them.

Start simple and grow

Many of the things we see around us took people years of hard work to achieve. As a young PI, you are at the start of your journey. If you’re interested in writing a textbook, start writing it and talk about your ideas, plan, etc. If you have a vision for a series of professional development conferences for young people, talk about your vision and think of your proposal as a request for seed money.

In my experience, NSF panels work very hard to understand what different career stages look like, and read proposals with this in mind.

What Broader Impacts Are Not

Doing your job

If you can substitute “I will do my job” for whatever it is you think might be a Broader Impact, it is probably not a Broader Impact.

- Are you planning to teach a graduate course on geometric measure theory from scratch?
- Are you planning to put your papers on the internet and give talks about them at conferences?
- Are your office hours a welcoming space for undergraduates?
- Are you collaborating with female and BIPOC faculty in your field?
- Are you serving on several departmental committees or organizing the department colloquium?

It is good that you are doing your job. Please keep doing your job.