returned as a project mentor where his team worked on sequence-to-sequence modeling.

Alumni of the Boot Camp have taken positions throughout all sectors of BIG (Business, Industry, and Government). In government, we have alumni in the Federal Reserve, the FDA, Pacific Northwest National Lab, and Sandia National Lab. In retail, our students have found jobs at Amazon, Sam's Club, and Target. Several went to "big tech" such as Google, eBay, and Microsoft. Other companies employing our alumni include JP Morgan, Lockheed Martin, Astra Zeneca, among many, many others.

Pivoting to Online

Rather than cancelling the Boot Camp during the COVID-19 crisis, the IMA decided to run the summer 2020 Boot Camp virtually, albeit with several modifications. As all instruction transitioned online, so too did the Boot Camp minicourses. However, the most critical part of the camp experience, working on an industry-sponsored team project, was a cause of concern. We especially wanted students to experience the pitfalls and challenges as well as the exhilaration of quick discovery and prototyping. In order to encourage team building and collaborative communication, the IMA experimented with software platforms and arrived at a suitable framework:

- Instruction, skills development, and cohort meetings worked exceptionally well on Zoom, which also allowed for archiving of lectures and course material.
- Frequent check-ins with the full cohort proved to be an excellent way to build rapport among students.
- Project teams messaged and provided running updates on Slack. Git repositories were used to collaborate on data manipulation and software development.
- Online team presentations, provided at several stages during the last three weeks, worked very well. Zoom allowed for seamless handoffs between team members with easy integration of slides.

The results proved so promising that six months later, the IMA held an inaugural two-week winter Boot Camp in January 2021. Instead of offering a three-week instruction period during the camp followed by team projects, students were asked to prep for the courses with recorded lectures and tutorials. Though the January 2021 cohort skewed more towards applied mathematics, many of the students started with little to no experience in programming and data science.

Conclusion

Programs like the IMA Math-to-Industry Boot Camp provide a framework for training a broad array of math PhD students for employment outside of academia. Many of the lessons learned could be applied in departments offering PhD degrees in the mathematical sciences. A program similar to the one offered at the IMA can be created either by a department, or better still, by a consortium of several geographically nearby departments. The consortium model allows for sharing of resources and having a critical mass of students. We hope such programs will provide PhD graduate students across the country the opportunity to prepare quickly for mathematics jobs outside of academia.

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Pset Partners

Andrew V. Sutherland

Introduction

The brutal murder of George Floyd last year and the protests that followed sparked some difficult discussions within our department, and some serious introspection. Some of the ways in which structural racism and inequity impact the mathematical community are obvious, most notably in the lack of diversity in our ranks. But some can be less obvious, especially to those not directly impacted by them.

One issue that has been particularly challenging for us at MIT is inclusivity. Reading over some of the discussions from last year I am still struck by the opening lines of "A Message to the MIT Math Community," an anonymous post to our discussion forum for mathematics majors that was

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the starting point for a long conversation about inclusion and diversity among students, faculty, and staff.

> *I find a community in the math department in the sense that we all like math, but I have never felt completely included.*

The sense of community I feel with my fellow mathematicians is what gives me the greatest sense of fulfillment in my work, but this post and the discussion that followed made it clear to me that many students never really find or feel this sense of community. Indeed, it took a while for me to find it myself.

One of the first things I learned as an undergraduate at MIT was a new term for what I had previously called "doing my homework." At MIT this activity is affectionately known as "psetting" (the term "pset"[1] is shorthand for "problem set," a set of problems to be solved). But it took me a while to learn that psetting (and mathematics research in general) is more rewarding when done in the company of others.

Most classes at MIT have a policy of permitting and even encouraging collaboration on problem sets. Students are generally required to name their collaborators and to write up their own solutions, but they are welcome to discuss the problems and their ideas for solving them with others. Styles of collaboration vary widely. Some students work side-by-side and solve the problems as a team, while others prefer to tackle the problems on their own but may consult their collaborators when they find themselves stuck.

Student collaboration has many pedagogical and psychological benefits. It allows students to develop intuition they might not easily glean from a lecture or textbook, it teaches them the value and pleasure of working with others, and it can help to build a sense of community and common purpose. But for those without collaborators, it can be very isolating.

Study groups often form naturally among students who are in the same living group or who have worked together before, and they may arise when students meet during office hours or recitation. But this does not always work. Many students who would welcome the opportunity to work with others can find it difficult to get a collaboration started. The process of forming partnerships tends to favor those who are already connected in some way, and these advantages accumulate over time. This can leave those who are not connected feeling ever more alone, and it puts them at an academic disadvantage. Finding the right partner can mean the difference between success and failure in a challenging class (just as finding the right collaborator can mean the difference between your latest research idea panning out or not).

The pandemic has exacerbated all of these issues. Students working remotely have few opportunities to form partnerships, and in the fall of 2020 most of our incoming class had to begin their academic career at MIT online, having never set foot on campus or meeting any of their classmates in person.

[1] `https://mitadmissions.org/help/faq/psets/`

The Pset Partners Project

Several students involved in the discussion on diversity and inclusion suggested creating a website to help students find partners to work with on problem sets. There are many web services that provide platforms for online collaboration, but these tools do not solve the problem of finding collaborators. Some of the courseware platforms we use do provide features intended to help students find teammates, but their functionality is quite limited. We held several brainstorming sessions in June and early July 2020 to flesh out the students' ideas. These sessions were primarily student led, but also included a few faculty members and researchers who were formerly MIT students.

A summary of their ideas were presented to all math majors on our department forum for feedback, including the following list of explicit goals:

- Make it easier for students to find collaborators.
- Promote community within the department.
- Encourage inclusiveness and reduce isolation.
- Preserve students' privacy and promote respectful interactions.
- Build a system that is lightweight and minimize the friction of using it.

With less than two months to go before the fall term, we set out to build a "Pset Partners" website to help students find collaborators. This time frame might seem wildly optimistic, but we were fortunately able to build on infrastructure that had been created for `researchseminars.org`, the online listing of lectures and conferences that I expect many readers have used. Even better, the creators of `researchseminars.org`, Edgar Costa and David Roe, were available for consultation, and their assistance proved to be invaluable. I should note that Costa and Roe created the initial version of `researchseminars.org` in an even shorter time frame by building on database infrastructure they had previously created for the *L-functions and modular forms database* (LMFDB)[2] as part of their work with the Simons Collaboration in Arithmetic Geometry, Number Theory, and Computation.[3]

An Initial Design

To start using Pset Partners, students provide some information about themselves, such as the classes they are taking, the times they are available, and their preferences for collaboration, including

- **size:** 2, 3–4, 5–8, or more than 8 people;
- **when:** *early* (shortly after a pset is assigned), *late* (a day or two before it is due), or *middle* (somewhere in between);

[2] `https://www.lmfdb.org/`

[3] `https://simonscollab.icerm.brown.edu/`

- **forum:** *text* (including team chat tools like Discord/Slack/Zulip), *video* (typically Zoom), or *inperson* (disabled for fall 2020);
- **style:** *solo* (independent but check answers), *collegial* (discuss strategy, collaborate if stuck), or *together* (team problem-solving).

Students can also express affinity/diversity preferences related to their department, year, and gender identity (if specified), as well as class-specific information, such as how committed they are to a class and how familiar they are with the material, all of which can be customized by class and assigned a weighting to indicate the strength of the preference. Most students fill in just the preferences listed above.

From the perspective of the matching algorithm, the most crucial information provided by students is the times they are available for collaboration, which they set by filling in a 7×24 hour grid for the week. The system tries to ensure a minimum overlap of four hours of availability in each of the groups it forms, which can be challenging with students spread across dozens of time zones as they were last fall. Indeed, most cases where the system was unable to match students arose in small classes with students who had mutually incompatible schedules.

To encourage students to be as flexible as possible when filling in the hour grid, students are shown the average number of hours of overlap they have with other students in each of their classes. When setting preferences, students can also see the distribution of preferences set by other students in their class. This can help nudge them toward making compatible choices, or at least warn them when when they are about to make a highly incompatible choice.

The system provides four ways for students to find partners. They can

- create or join a public group,
- create a private group and invite others to join,
- join a group open to new members that has been recommended to them by the system based on their schedule and preferences, or
- place themselves in a pool of students who will be automatically matched by the system.

A key design goal was to make it easy for students to find or change partners throughout the term. At MIT, class composition tends to be fairly fluid in the first several weeks, and students who might initially be happy working on their own may change their minds as the assignments get more difficult. Students are free to drop classes without penalty up until the last five weeks of the term, so it is not uncommon to lose a partner, and there will always be groups impacted by changes in circumstances or that simply never quite work. There are also social and pedagogical advantages to having students work with multiple groups over the course of the term, so we wanted to make that as easy as possible.

In addition to matching students that have placed themselves in the match pool, the system may offer students the opportunity to join an existing group that is compatible with their schedule and preferences, if that group has been configured to accept new members whenever it is below its size limit. This is the default setting for groups that are created by the system during the matching process (students can change this if they wish), and in large classes there will typically be several groups that can accommodate new members. This means that if a student is looking to join a group, perhaps because they just left a group that was not working for them, the system is often able to immediately suggest another group.

One of the more whimsical aspects of Pset Partners is the randomly generated group names. We initially planned to let students choose group names, but groups created by the system need to be given an initial name. The decision to name the website "Pset Partners" was made by the students in part because of its pleasant alliteration. We decided to lean into this by using random alliterative adjective-animal combinations like "Acrobatic Armadillos," "Boisterous Beavers," or "Charismatic Chipmunks" for group names. We then realized that it could be confusing to students and instructors if the group names were allowed to change over time, and decided to make the randomly generated names immutable.

I think this small bit of whimsy helped generate interest, not only from students, but also from faculty. One of the first emails I received from a faculty member regarding Pset Partners was a bemused query about the "Yodeling Yellowhammers" group that had formed in his class. I think we all welcomed something to smile about as we began what we knew would be a very challenging term of remote teaching.

The Matching Algorithm

You are probably familiar with the Gayle-Shapely algorithm [2], which efficiently solves the *stable matching problem*: given sets A and B of size n and bijections $\rho_a : B \to \{1,\dots,n\}$ and $\rho_b : A \to \{1,\dots,n\}$ for each $a \in A$ and $b \in B$, find a bijection $\pi : A \to B$ that is *stable* in the sense that there is no pair $(a,b) \in A \times B$ with $\pi(a) \neq b$ such that $\rho_a(b) < \rho_a(\pi(a))$ and $\rho_b(a) < \rho_b(\pi^{-1}(b))$. The elements $a \in A$ and $b \in B$ can be interpreted as individuals with preference rankings ρ_a and ρ_b, respectively; a stable matching is one in which there is no pair (a,b) who would both prefer to be matched with each other rather than the person they have been matched with.

The Gayle-Shapely algorithm has many important applications to resource allocation, but it is not useful in our setting where we do not want to partition students into sets A and B. The problem of constructing a bipartite matching within a single set C of $2n$ people given preference rankings is known as the *stable roommate problem*. For this problem a stable matching need not exist. There are algorithms to efficiently find one when it does [3], but when it does not, even the problem of closely approximating the best matching possible is NP-hard [1]. This also applies to the more general problem in which each $c \in C$ ranks all the subsets that contain them in order of preference and the system is asked to find an optimal partition of C relative to these preferences.

We took a different approach. Rather than trying to optimize each individual's satisfaction, we seek to optimize the satisfaction of the pool as a whole by constructing a function $\rho : 2^C \to \mathbb{Z}_{>0}$ that assigns a positive integer to each subset $S \subseteq C$ that reflects the degree to which the set S satisfies the preferences and scheduling constraints of the students in S, with lower values indicating greater satisfaction. Given a partition π of C into subsets $S_1, \dots, S_n$, the weighted average $\sum \rho(S_i)/|S_i|$ measures the average satisfaction of each student in the pool. One can then ask whether there is an incremental change to π, such as swapping two students or moving a student from one subset to another, that improves average satisfaction. If one starts with an arbitrary partition π and repeatedly applies incremental changes that improve average satisfaction until none exist, the resulting partition will be "stable" in the sense that there is no single exchange of students or migration of a student from one group to another that would improve the average satisfaction of the pool as a whole.

The fact that our satisfaction metric is discrete and bounded below guarantees that this process will terminate, and in practice it terminates quickly. Like all local search algorithms, it finds a local minimum that is not necessarily a global minimum, and it is sensitive to the starting conditions, especially the shape of the initial partition. For this reason we give priority to size preferences and attempt to construct an initial partition that satisfies all expressed size preferences before applying any incremental changes.

The Launch of Pset Partners

The Pset Partners website went live on August 31, 2020, with about sixty mathematics classes, ranging from 18.01 (Calculus) to 18.965 (Geometry of Manifolds); this included large service classes taken by hundreds of students as well as small upper level courses, including graduate seminars. We initially considered restricting the site to the larger classes, but we were encouraged by faculty to include all classes in the department. We made a conscious decision not to include classes outside the mathematics department. Given the short development time frame and the minimal amount of testing we were able to do, it seemed prudent to limit the scope of this experiment.

Several hundred students signed up in the first week, and the total was close to a thousand by the third week. Aside from a few minor hiccups that were corrected by manual intervention, the system worked more or less as designed. The participation rate varied widely by class: in some classes more than 3/4 of the students used Pset Partners, while a few classes had no participants. Overall, the average participation rate was about 28 percent. Over the course of the fall term more than 300 pset groups were created, involving more than 900 students (some students were members of groups in multiple classes).

We sent an end-of-term survey to all students and instructors who used the Pset Partners website in the fall of 2020, and received about 200 responses. More than half the respondents said they found the site very or extremely useful, and over 75 percent found it at least somewhat useful. Over 95 percent said they hoped the system would be used again in the spring, and several respondents asked that it be made available to classes in other departments.

While most of the comments were positive, several students noted difficulties connecting with the partners the system had found for them. The system sends an initial email to all members of a group when it is formed, but it is then up to the students to coordinate their activities from that point. In several cases this did not happen.

Based on student feedback and interest from other departments we decided to make the site available to all MIT classes in the 2021 spring term, subject to an activation step that instructors need to take before their class becomes visible to students. This was done to ensure that we had the instructors' approval, and in the hope of motivating them to encourage their students to use the site. It was clear from our experience in the fall that the level of an instructor's enthusiasm for Pset Partners had a significant impact on the rate of student participation in their class. About 160 instructors chose to use Pset Partners in the spring, including most of the larger classes in mathematics, biology, chemistry, physics, economics, and computer science.

We made three changes to the system between the fall and spring terms in an effort to help students matched by the system to connect with each other. First, we added a Code of Conduct[4] emphasizing that in addition to the standards that every MIT student is expected to uphold, students using Pset Partners are expected to actively reach out and include partners they have been matched with. Second, the initial email that is sent to students who have been matched now includes a suggested time for a first meeting, based on the schedules of the students. Third, the system sends a follow-up email three days after the initial matching to ask students how things are going, and gives them an option to immediately request a new match if they have not been able to connect with their partners.

Final Thoughts

We conducted another end-of-term survey after the spring term. The results were encouraging in that a majority of respondents said they found the system very or extremely useful, and over 90 percent of those not graduating said they planned to use the system again in the fall in classes where it is available. But comments included in the survey responses made it clear that we still have not solved the problem of ensuring that students who are matched actually connect with each other. We received multiple comments from students who expressed frustration with classmates who signed up for Pset Partners but did not

[4] `https://psetpartners.mit.edu/conduct`

actually work with (or in some cases even contact) the group with whom they were matched. I am not sure that this is a problem technology can solve.

It is surely too soon to declare the project a success or failure. The system is still very new, and last year was exceptional in so many ways that I hesitate to draw any long-term conclusions. But I was heartened by the following survey comment:

> *I would not have been able to get through ... if it wasn't for the support and community that my pset group brought.*

At least in this one case, the project achieved its goal.

If you would like to learn more about Pset Partners, I invite you to visit `https://psetpartners.mit.edu/about`, which gives a brief overview of the site and includes a link to our sandbox,[5] where you can try out the system for yourself, both as a student and as an instructor. The software that runs the website can be found at our GitHub repository[6] and is available to everyone under an open source AGPL license.

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Credits

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[5] `https://psetpartners-test.mit.edu/`

[6] `https://github.com/AndrewVSutherland/psetpartners`

How to Run a Math Group

Tom Gannon

Often, we meet students (and others!) who believe that mathematics is a series of rote memorization of formulae and algorithms. Even students more interested in mathematics can still earn a high school diploma while believing that the only valid proofs are two-column proofs used in a geometry class. In 1993, the University of Texas at Austin created a program now known as the *Sunday Morning Math Group* to fight these stereotypes, and to help spread the fun that critical thinking in a mathematical context can really have. I had the privilege of running this group for two years, and in this article, I'll give you tips on how to start your own!

What Is a Math Group?

A *math group* (also known as a *math circle*) is a tool invented by mathematicians to try to teach students the creativity and problem-solving skills that are often used in pure mathematics. All math groups are different—what works well for one group may not work as well for others. Most emphasize math that isn't typically covered in school curricula, such as the basics of proofs (such as the pigeonhole principle, the basics of logic, or induction) and other topics such as combinatorics. Like good mathematicians, we will give an example.

An example

At the University of Texas at Austin, our Sunday Morning Math Group (SMMG) ran as follows. Every other Sunday during the fall and spring semesters, students would arrive at noon and receive a slip of paper containing their first "riddle." The "riddle" is often a relatively easy question, to get any new students used to the flow of the riddle system. After receiving a riddle, students will think about the riddle, and then raise a hand to chat with a math department volunteer, either to explain the answer to the riddle or ask a question about it. If the student gets the answer to a question right, they receive the next "riddle" which explores the concept a bit deeper.

For example, one of our sessions had riddles which were designed to teach the students binary. The first question they received was "You have a 1 pound weight, a 2 pound weight, and a 4 pound weight. Can you combine the weights to make something that weighs 5 pounds? Can you combine the weights to make something that weighs 6 pounds?" Slowly, the riddles "build up" to introducing notation such as 1011_2, and later go on to explore other concepts (for example, other number bases and the algorithms

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