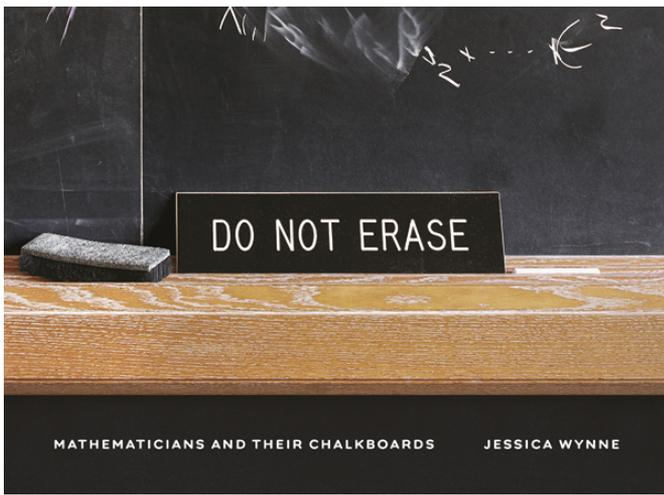




Erasable You

Reviewed by Daniel S. Silver



Do Not Erase

by Jessica Wynne

Princeton University Press, 2021

Photographs can revive our appreciation of common, everyday things. A bicycle leaning against a fence or a wheelbarrow left out in the rain might acquire new, unexpected beauty. So might a blackboard that has been filled with mathematical symbols, reflecting sunlight from a window on a late afternoon.

Do Not Erase is an homage to the humble blackboard, a loving photo essay about those black and green planar surfaces on which we have all focused our attentions for so many years, whether in classrooms, lecture halls, common rooms, or offices. The book's creator, Jessica Wynne, is a photographer but not a mathematician. She is simply

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enchanted by the symbols and drawings that she has seen on blackboards. Wynne's work has been featured in *The New York Times*, *Fortune Magazine*, *The New Yorker*, and *National Geographic*. She is a professor at the Fashion Institute of Technology in New York City.

With guidance and encouragement from friends Amie Wilkinson and Benson Farb at the University of Chicago, Wynne recorded images of mathematicians' blackboards at institutions around the world. More than a hundred are presented in the book. Each is preceded by a personal statement by the mathematician. The result is a unique book, one that will enhance anyone's appreciation of this lowest of all low-tech mathematical tools. When you are finished, you might even think that you have chalk dust on your hands and clothes.

Printing and binding of *Do Not Erase* were done by Graphicom, S.p.A, in Verona, Italy. The company, noted for its museum-quality color reproduction, is used regularly by publishers such as Taschen and Rizzoli. They use 100% green energy and engage in sustainable production. The results we find in *Do Not Erase* do not disappoint. Colors are rich and seductive. The pages are large (8 by 11 inches). Impressive also is the fact that the book's price is only \$35.

As low-tech as blackboards are, they were not the first erasable medium. In the Middle Ages, students wrote on wax tablets. And long before that, one simply drew on sand. There is a popular (but doubtful) legend about Archimedes of Syracuse ordering an approaching Roman soldier not to disturb his circles in the sand. According to the story, his warning was met by the sharp end of a Roman sword. It's a pity that Archimedes didn't run away, first placing a sign that read: DO NOT ERASE.

We do not know with certainty when writing on slate began. Geoffrey Chaucer mentioned it in his late fourteenth century *Treatise on the Astrolabe*. The practice became widespread four centuries later as land and sea transportation improved and quarried slate from Wales could be obtained [1].

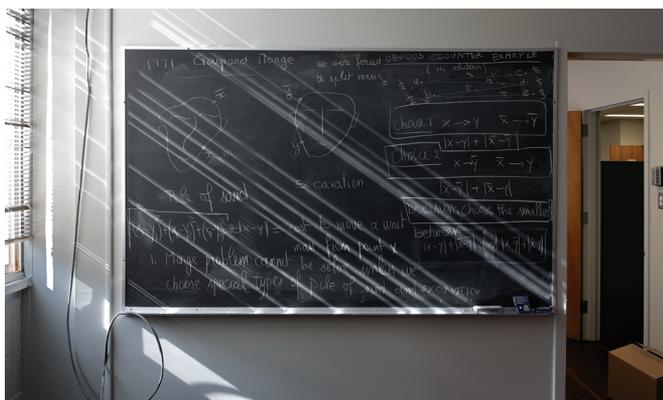


Figure 1. Sunlight streaks across the blackboard of Wilfred Gangbo at the University of California, Los Angeles.

By the early 1800s, blackboards were being introduced into classrooms. Not everyone was thrilled by the new technology. In 1830, Yale students refused to go up to the board, preferring to recite their assignments from their desks with their geometry textbooks in hand. The dissenting students numbered forty-two, almost half of Yale's class. The faculty dismissed them all. The episode became known as the "Conic Sections Rebellion" [2].

Most of the blackboards in *Do Not Erase* are black. A few are green. There are no whiteboards. It is difficult to be sentimental about whiteboards and those dry, somewhat erasable markers, which have been in classrooms only since the mid 1990s. Perhaps a book like this one will appear one day, waxing nostalgic about the smell of SD-Alcohol-40 and isopropanol, but don't hold your breath. Or maybe, do.

How fond of blackboards can one be? If there were a prize given, it would go to Étienne Ghys at École Normal Supérieure in Lyon. Ghys confesses that when he was beginning his career, he convinced his wife to let him hang a blackboard in their bedroom. He removed it some months later after they both had gotten tired of chalk dust on the sheets.

While blackboards are usually associated with classrooms, most of the boards in *Do Not Erase* are in offices or common areas, where they are tools for research. Mathematics is a social activity, as we are reminded by Jan Vonk at the Institute for Advanced Study in Princeton: "We keep gathering religiously around our blackboards, where we share our aspirations and doubts, our successes and failures, our insights and confusions, and most importantly, our excitement and passion for mathematics." Vonk's board contains ideas from number theory. On one corner of the chalk rail is a sign that reads "ERASE," perhaps a Zen statement to contrast with the book's title.

Photography can be voyeuristic. We are uninvited guests. But a photograph can also involve us in the scene. Looking at the blackboards in *Do Not Erase* one might get the impulse to jump up and write something.

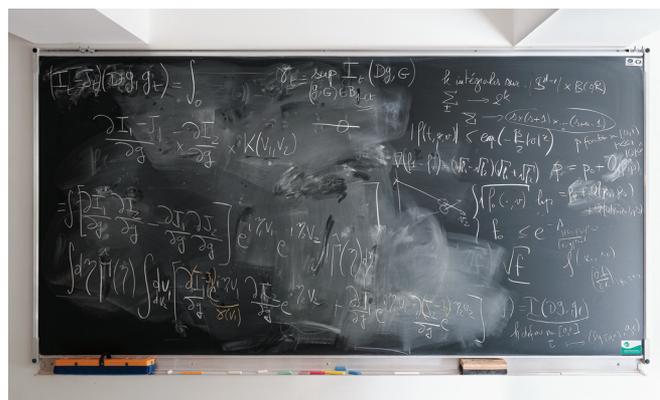


Figure 2. Isabelle Gallagher records a "good week's work" on her blackboard at Université de Paris. Like many mathematicians, she uses her board for teaching students and working with collaborators.

Collaboration in front of a blackboard is an athletic event, with participants taking turns getting up to add details or erase mistakes. If nothing substantial is proved, at least we get some exercise.

Benson Farb has a different perspective. He describes working at a blackboard with another person as a dance: "She can jump up and start writing on the board during my explanation, amending my computations, noting possible problems, unwrapping some equation into a flurry of computations of her own. Doing this dance at the blackboard with someone is an intense, frustrating, energizing, and sometimes moving experience." He concludes, "Chalkboards are a major part of my life. I couldn't live without them." The drawing on Farb's board describes an example of the late William Thurston, a pioneer of low-dimensional topology. Pink and yellow curves are the end results of circles dragged by points in the plane. Hazy white bands of chalk left by his eraser give the drawing a ghostly touch. The hands of a plain clock above are frozen at 11:20. Like all of the photographs in *Do Not Erase*, this one functions in two ways, telling a deep mathematical story but also offering us a still life that can be enjoyed by anyone.

Speaking of chalk, without it a blackboard is useless. Every mathematician understands good chalk to be a blessing. With it, writing flows easily and can be read from the back of the room. There appears to be a consensus among mathematicians that the world's best chalk is Hagoromoro Full Touch Chalk, from Japan (now made in Korea). It is mentioned three times in *Do Not Erase*.

On the other hand, bad chalk can ruin a blackboard lecture. If the chalk is too long, it can squeak horribly. Years ago, faculty and students at Yale would be amused by the angry reaction of the French-American Yale mathematician Serge Lang whenever the sound of chalk offended his ear. They could depend on Lang calling out to the speaker, in a loud voice: "Break zee chalk! Break zee chalk!"

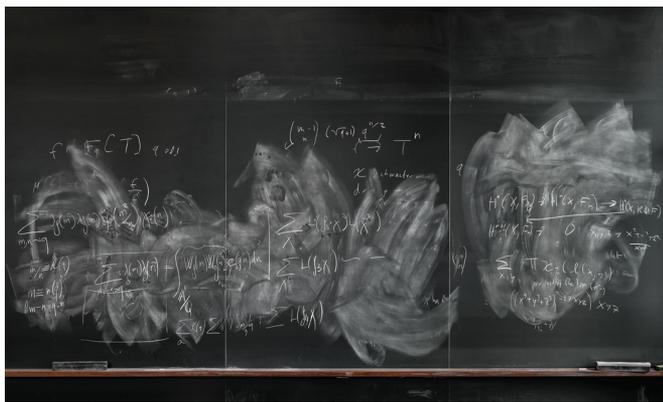


Figure 3. Philippe Michel displays a chalk-hazy blackboard at the Swiss Institute of Technology, Lausanne. He describes a European method for cleaning boards, using water and wipers. “With a clean board and a clear mind, you’ll be ready to start anew.”

Many of the contributors to *Do Not Erase* described the artistic experience of working on a blackboard. For Boya Song, a PhD student at MIT, “it is like painting a portrait of the mathematical problems in my mind.” There is an awareness of beauty that can appear on the board unexpectedly. Others reflect on the sensual nature of writing on a smooth, hard surface with soft chalk.

Such thoughts might strike some as ludicrous. What is beautiful about a blackboard? It is an artist’s task to make us see overlooked beauty around us. “While most people taking photographs are only seconding received notions of the beautiful, ambitious professionals usually think they are challenging them,” wrote Susan Sontag in her famous essay *On Photography* [3]. Wynne has expanded our notions of what is beautiful with her photographs of blackboards.

In addition to reflections on art, perceptive psychological observations about blackboards are found in *Do Not Erase*. Nathan Dowlin at Columbia University explains how the impermanence of blackboard writing encourages risk-taking. “There is no pressure to get it perfect the first time, or even to get it right, since it’s going to be erased in an hour or two anyway.”

Michael Harris, his colleague at Columbia, goes even further. The mathematics we begin to formulate is tentative, but “making it visible on the board is one way we give each other permission to continue.” Harris’s board is a wide expanse of black slate with writing in white chalk. A note in the upper right-hand corner reminds us of the impermanence of blackboard writing: DO NOT ERASE BEFORE NOON.

Often mathematics graduate students who are learning to teach are told that when you present a well-known theorem in front of a class, you are recreating it. For that moment, you take possession of it. H el ene Esnault at the Free University of Berlin understands this. For her, the

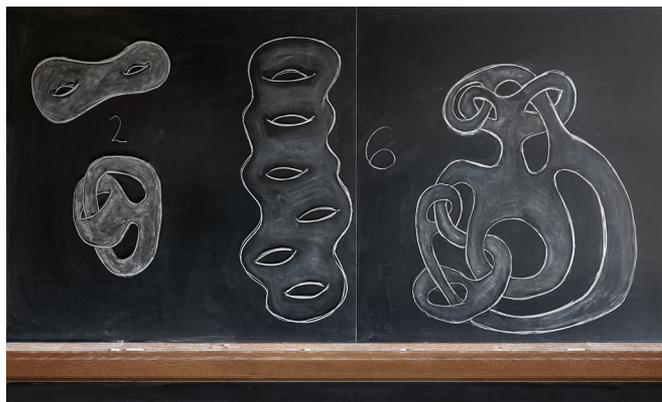


Figure 4. The fluid drawings of Nancy Hingston of the College of New Jersey will surrender to the eraser when her blackboard is needed again.

ephemeral nature of blackboard writing evokes a note of poignancy. “A proof, when we hold it, is a moment of absolute joy. It is also a farewell. Once it has been produced, it no longer belongs to us: anyone can reproduce it, and use it further. The blackboard witnesses the privileged moment when we meet the proof. In today’s isolation, it feels almost like a souvenir of a past time.” Esnault’s blackboard has the statement and proof of a theorem about projective varieties, which was joint work with Moritz Kerz. It is outdoors on grass, in front of trees. It is summer. Naturally, the blackboard is green, not black.

The large size of blackboards is part of their suggestive power, something that paper does not share. Perhaps that is the main reason why many mathematicians like to work with chalk and eraser in hand. Sun-Yung Alice Chang of Princeton University calls her big blackboard the best part of her office. She admits, “I often go in to work just to use it.” Hers is big indeed. In the photograph, it displays the statement of the theorem of which she is most proud, the “4-dimensional conformal sphere theorem.” It was joint work with Matthew Gursky and Paul Yang. The small table in front is piled high with pads of paper and reprints. It is difficult not to believe that we are interrupting them at work.

There is inspiration and optimism to be gained from blackboards. When stuck on a difficult point, you can always stand back and gaze at a large blackboard, letting the symbols and drawings wash over you, mixing together in your mind. A drawing that you left up on the board in your office will be there to greet you every day, reminding you of what you need to think about (or accusing you of what you haven’t done). Some drawings might remain unerased for months or even years.

Virginia Urban of the Fashion Institute of Technology gets inspiration in another way. She reflects: “...there is the knowledge that I am writing on a surface that has held the ideas of decades of people that came before me, and that I am now a part of that legacy.” Musicians speak in a

similar way about old, historic instruments. Ironically, Urban's board is covered with a workaday lesson about IRS reimbursements for professional travel. A NO SMOKING sign, glued to the top right-hand corner of the board, at-tests to its age.

Blackboards make most people think about teaching rather than research. Despite all of the electronic teaching devices in modern classrooms, blackboards continue to be preferred by many. Paul Apisa at the University of Michigan offers two axioms that help us understand why. First, mathematics can be complicated. Second, humans think slowly. Using a blackboard rather than Beamer slides takes time and effort, and we have time to digest the thoughts presented. Ronen Mukamel, at Harvard Medical School and Brigham and Women's Hospital in Boston, expresses the idea this way: chalkboard communication occurs at the speed of thought rather than the speed of light. And "chalk-covered fingers are the physical reminder of a presentation well made."

Computer presentations with their fleeting images are not only frustrating for many audience members, their replacement of blackboard lectures can discourage the healthy skepticism that students need to develop. Writing that appears to be typeset and professionally rendered diagrams look too good to be wrong. In contrast, lectures presented at a blackboard usually contain many small—one hopes, trivial—mistakes that encourage questions. Simion Philip at the University of Chicago sees the blackboard as an "active space, ready to change and to carry any thought." We modify, we erase, we fix our mistakes and improve our thoughts when we work at the board. Philip's board, describing relationships among surface symmetries, has only the essential features needed for comprehension. It is easy on the eye, but it also has the white haze of an erasure, the scars of intellectual battle.

Do Not Erase is being published as the world struggles to emerge from a grim pandemic. Staying at home for more than a year has made all of us more mindful of the people and things that encourage our work. Colleagues. Students. Campuses. And for many, blackboards. (Alas, not for Lorenzo Diaz of Pontifical Catholic University, Brazil, who confesses that he has an allergy to chalk.) Change is the nature of the world, and nothing is permanent. But those who read *Do Not Erase* and admire its artful photographs will share a hope that blackboards remain with us at least a little bit longer.

SEE DO NOT ERASE PRINTS AT AMS OFFICES. If you visit the AMS headquarters in Providence, RI, or the Office of Government Relations in Washington, DC, you can see some of the full-size prints featured in *Do Not Erase* along with the mathematicians' explanations of their work. Purchase of the prints was made possible by contributions from an anonymous donor, AMS Executive Director Catherine Roberts, and Edwynn Houk Gallery.

The photographs in Providence feature blackboards of the following mathematicians, all of whom have been actively involved in the AMS community:

- Kasso Okoudjou (Tufts University)
- Sun-Yung Alice Chang (Princeton University)
- Hee Oh (Yale University)
- Bryna R. Kra (Northwestern University)
- Terence Tao (University of California, Los Angeles)

The print on display in Washington, DC, shows the blackboard of Amie Wilkinson (University of Chicago). The AMS is proud to showcase this mathematical art and the work of these mathematicians. Read more about the AMS installation at https://www.ams.org/news?news_id=6826.

References

- [1] Peter Davies, *Writing slates and schooling*, *Australasian Historical Archeology* 23 (2005), 63–69.
- [2] Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts, *Tools of American mathematics teaching, 1800–2000*, Johns Hopkins Studies in the History of Mathematics, Smithsonian Institution, Washington, DC; Johns Hopkins University Press, Baltimore, MD, 2008. MR2426819
- [3] Susan Sontag, *On photography*, Farrar, Strauss and Giroux, New York, 1973.



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