AMS Mathematical Moments
& Mathematics of Planet Earth

Harnessing Wind Power

MATHEMATICAL MOMENTS

Mathematics contributes in many ways to the process of converting wind power into usable energy. Large-scale weather models are used to find suitable locations for wind farms, while more narrowly focused models—incorporating interactions arising from factors such as wake effects and turbulence—specify how to situate individual turbines within a farm. In addition, computational fluid dynamics describes air flow and drag around turbines. This helps determine the optimal shapes for the blades, both structurally and aerodynamically, to extract as much energy as possible, and keep noise levels and costs down.

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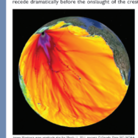
Sounding the Alarm

Nothing can prevent a tsunami from happening—they are enormously powerful events of nature. But in many cases networks of seismic detectors, sea-level monitors and deep ocean buoys can allow authorities to provide adequate warning to those at risk. Mathematical models constructed from partial differential equations use the generated data to determine estimates of the speed and magnitude of a tsunami and its arrival time on coastlines. These models may predict whether a trough or a crest will be the first to arrive on shore. In only about half the cases (not all) does the trough arrive first, making the water level recede dramatically before the onslaught of the crest.



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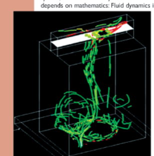
Building Efficiently

Buildings use so much energy that cutting consumption by one-half would be equivalent to taking every passenger vehicle and light truck off the roads for a year. Such a reduction can be done by integrating components, such as air conditioning and lighting, into one continually monitored system, as is done with hybrid cars. This integrated approach depends on mathematics: Fluid dynamics is used to locate vents and heat sensors optimally, while graph theory and linear algebra help identify the most important parameters in the huge amount of data the sensors collect. This makes it possible to make real-time adjustments essential to the system's efficient operation, which is cool for the occupants and the planet, too.



Coupled thermal and air flow computational fluid dynamics study in a building.

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Knowing Rogues

It doesn't take a perfect storm to generate a rogue wave—an open-ocean wave much steeper and more massive than its neighbors that appears with little or no warning. Sometimes winds and currents collide causing waves to combine non-linearly and produce these towering walls of water. Mathematicians and other researchers are collecting data from rogue waves and modeling them with partial differential equations to understand how and why they form. A deeper understanding of both their origins and their frequency will result in safer shipping and offshore platform operations.

Photo courtesy of Karsten Petersen.

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Resisting the Spread of Disease

One of the most useful tools in analyzing the spread of disease is a system of evolutionary equations that reflects the dynamics among three distinct categories of a population: those susceptible (S) to a disease, those infected (I) with it, and those recovered (R) from it. This SIR model is applicable to a range of diseases, from smallpox to the flu. To predict the impact of a particular disease it is crucial to determine certain parameters associated with it, such as the average number of people that a typical infected person will infect. Researchers estimate these parameters by applying statistical methods to gathered data, which aren't complete because, for example, some cases aren't reported. Armed with reliable models, mathematicians help public health officials battle the complex, rapidly changing world of modern disease.

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Predicting Climate

What's in store for our climate and us? It's an extraordinarily complex question whose answer requires physics, chemistry, earth science, and mathematics (among other subjects) along with massive computing power. Mathematicians use partial differential equations to model the movement of the atmosphere; dynamical systems to describe the feedback between land, ocean, air, and ice; and statistics to quantify the uncertainty of current projections. Although there is some discrepancy among different climate forecasts, researchers all agree on the tremendous need for people to join this effort and create new approaches to help understand our climate.

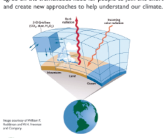

Image courtesy of William F. Ruddiman and W.H. Freeman and Company.

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Going with the Floes

Sea ice is one of the least understood components of our climate. Naturally its abundance or scarcity is a telling sign of climate change, but sea ice is also an important actor in change as well, insulating the ocean and reflecting sunlight. A branch of mathematics called percolation theory helps explain how salt water travels through sea ice, a process that is crucial both to the amount of sea ice present and to the microscopic communities that sustain polar ecosystems. By taking samples, doing on-site experiments, and then incorporating the data into models of porous materials, mathematicians are working to understand sea ice and help refine climate predictions.

For More Information: "Thermal evolution of permeability and microstructure in sea ice," K. M. Golden, et al., *Geophysical Research Letters*, August 28, 2007.

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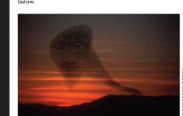
Getting It Together

The collective motion of many groups of animals can be stunning. Flocks of birds and schools of fish are able to remain cohesive, find food, and avoid predators without leaders and without awareness of all but a few other members in their groups. Research using vector analysis and statistics has led to the discovery of simple principles, such as members maintaining a minimum distance between neighbors while still aligning with them, which help explain shapes such as the one below.



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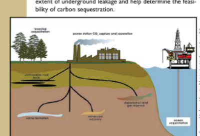
Burying Carbon Dioxide

One possible way (along with improving energy efficiency and finding alternate fuels) to deal with the huge amount of carbon dioxide going into the atmosphere is *carbon sequestration*: burying CO_2 thousands of feet underground in old or unusable reservoirs, before it is emitted. Naturally geology is involved, but so too is mathematics. Linear algebra, numerical analysis, and partial differential equations underlie the models that combine with small-scale experiments to predict the extent of underground leakage and help determine the feasibility of carbon sequestration.



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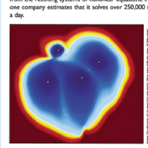
Finding Oil

As high as gas prices are, they would be much higher without modern oil exploration techniques, which make operations more efficient (and cleaner). Drilling a well can cost 20 million dollars, so drillers now rely on mathematical models of reservoirs, rather than hunches, to choose sites. The models approximate a reservoir's characteristics from data collected using sound waves beamed underground, and from the resulting systems of nonlinear equations. In fact, one company estimates that it solves over 250,000 systems a day.



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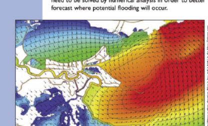
Predicting Storm Surge

Storm surge is often the most devastating part of a hurricane. Mathematical models used to predict surge must incorporate the effects of winds, atmospheric pressure, tides, waves and river flows, as well as the geometry and topography of the coastal ocean and the adjacent floodplain. Equations from fluid dynamics describe the movement of water, but most often such huge systems of equations need to be solved by numerical analysis in order to better forecast where potential flooding will occur.



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Locating, locating, locating

Originally designed for military use, the Global Positioning System (GPS) now lets boaters, drivers, and hikers pinpoint their location to within a few meters. Most of GPS's functionality is derived from arithmetic, algebra, and geometry. The time it takes for a signal to travel from a transmitting satellite to a GPS receiver establishes the distance between the two, which places the GPS user on an imaginary sphere centered at the satellite. Similar calculations are done concurrently using other satellites. Once corrections for the difference between satellite and receiver clocks are made, the GPS user's location must be one of the points of intersection of three spheres.

Image courtesy of The Aerospace Corporation.

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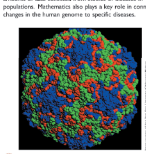
Defeating Disease

From modeling microscopic genes and proteins to tracing the progression of an epidemic through a country, mathematics plays an important role in combating disease. For example, the basic model used to analyze the dynamics of infectious disease is a system of differential equations. A new field called "data mining", involving statistics and pattern recognition, helps locate significant information in the vast amounts of data collected from studies of diseases in populations. Mathematics also plays a key role in connecting changes in the human genome to specific diseases.



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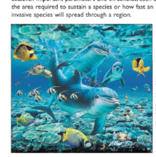
Revealing Nature's Secrets

Mathematical ecology is a growing and active area of interdisciplinary research between mathematics and ecology, using almost every part of mathematics (linear algebra, analysis, differential equations, stochastic processes, numerical simulations, statistics) to understand and model complex biosystems. This modeling helps establish important parameters and thresholds, such as the area required to sustain a species or how fast an invasive species will spread through a region.



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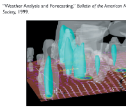
Forecasting Weather

Forecasting the weather requires enormous amounts of data and computation. In order to have an accurate model of the weather, one must know the temperature, humidity, air pressure and wind speed (among other things) at different points and elevations. Although incorrect forecasts may be more memorable, current three- to seven-day forecasts are better than 36-hour forecasts were just 20 years ago. Increases in computing power have helped improve weather forecasts, but it's the mathematics behind the models that has led to the great increase in accuracy.

Collected information is the basis for numerical calculations that output approximate solutions to the relevant nonlinear partial differential equations. Weather models take into account the rotation of the Earth and the perpetual interaction among land, sea, and air. While more data and better computers are obvious sources of improved forecasting, the not-so-obvious sources of better sampling techniques and better use of data have helped as well.

For More Information:

"Weather Analysis and Forecasting," *Bulletin of the American Meteorological Society*, 1999.



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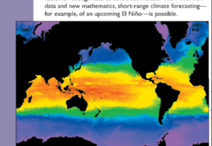
Describing the Oceans

Imagine trying to describe the circulation and temperatures across the vast expanse of our oceans. Good models of our oceans not only benefit fishermen on our coasts but farmers inland as well. Until recently, there were neither adequate tools nor enough data to construct models. Now with new data and new mathematics, short-range climate forecasting—for example, of an upcoming El Niño—is possible.



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See how math is connected to understanding our planet and its many dynamic processes at www.ams.org/mathmoments and www.ams.org/samplings/mpe-2013.

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