How do you teach undergraduate abstract algebra? Your answer surely depends on to whom you are teaching it and your educational goals. Tom Sibley, in his excellent MAA text, explains that he wants to teach his students to "think algebraically." Joe Silverman, in his beautiful new AMS textbook, wants his students to understand the skeletal axiomatic structures (and maps preserving those structures) underlying the mathematical objects with which they are familiar when specific details are stripped away and similarities are highlighted. Which is precisely what Sibley means by his title. Both authors transcend the groups-first versus rings-first debate and move to a presentation we might call "structure-first." Technically both introduce groups first, but both also introduce rings and fields early on and go back and forth emphasizing common structural features.

Despite the harmony of pedagogical approaches, Silverman and Sibley have meaningful differences of emphasis. Silverman is primarily interested in exploring the properties of algebraic structures. His content coverage is deeper and goes further than Sibley’s. And he recognizes that a good course is built on an underlying narrative that knits everything together, propels development, and makes the course a coherent story. He knows that a good story contains episodes of surprise and wonder. Silverman calls these “punchlines,” by which he means results whose depth and beauty reward the explorer for the, sometimes sterile-seeming, work of abstraction and generalization. His chapters beautifully build towards denouements. Sibley also builds a compelling narrative but he is more invested in connecting his story to ideas his students already know. He expends considerable energy connecting abstract algebra to the algebra his students learned in high school. Sibley doesn’t get as far as Silverman, but he trades that coverage for giving his reader a clear idea of how and why we got to where we are.

I think of the Silverman and Sibley texts as centering the content of algebra in contrast to the books by John Osoinach and Ryota Matsuura, which emphasize the process of learning algebra. The books by Osoinach and Matsuura are informed by current pedagogical theory. Both use a form of inquiry-based pedagogy that is rooted in extensive experience and familiarity with examples before abstraction occurs. Osoinach is a committed IBLer, his goal is that students construct all the proofs themselves but he is extraordinarily adept at making it all feel natural and organic. For example, before presenting the definition of a group falls right out of the analysis. Matsuura teaches a course that does not assume that students have already been introduced to proof-writing. Matsuura’s mantra, formed from his decades of studying teaching and learning, is "experience before formality." He is very skilled at giving students structured space to play with examples, to formulate concepts and conjectures, and to uncover the insights and connections in examples that lead them forward while simultaneously introducing proof-writing. (His book will be released in summer 2022.)

All of these authors want their students to learn to think algebraically. All are extraordinary pedagogues. All have profound ideas about what it means to understand algebra and how their particular students, and yours, might get to that understanding.