Contact Geometry and the Mapping Class Group

Joan E. Licata

Three-manifolds are tantalizing objects. With physical senses adapted to a three-dimensional physical world, we might expect to have geometric intuition, yet few of us can easily visualize how to rotate the Poincaré homology sphere or navigate $\mathbb{R}P^3$. One trick for making three-manifolds more accessible is cutting them into pieces: a lens space may be hard to imagine, but anyone who's enjoyed a donut or two can picture a pair of solid tori. Decomposing large complicated objects into smaller, simpler pieces is ubiquitous in topology, but ultimately, the infamous Pottery Barn aphorism kicks in: if you break it, you fix it. Once we cut, we need to be able to glue these pieces back together.

A map from the boundary of one manifold to another "glues" the manifolds; more formally, we form the quotient that identifies each point with its image. Surfaces are only two-dimensional, but they are absolutely central to the study of three-manifolds because they are the natural domain and codomain for maps gluing three-manifolds

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along their boundaries. As we'll show shortly, when the pieces are simple enough, these gluing maps capture the full complexity of the manifold.

To ensure the result of gluing is again a topological manifold, the gluing map must be a homeomorphism, while in the smooth category, a diffeomorphism is required. Fixing an identification of each boundary component with some model surface S lets us view these gluing maps as topological automorphisms of S, so they form a group. It turns out to be most profitable to study a mutual quotient of $\mathrm{Homeo}^+(S)$ and $\mathrm{Diff}^+(S)$ called the *mapping class group*. There are several standard notations for the mapping class group of a surface S, but we'll stick with the easy-to-remember "MCG(S)."

For topologists, the mapping class group is a tool to study manifolds, but it appeals to many flavors of mathematicians for many different reasons. Group theorists study it as an algebraic object, while the geometric interpretation of mapping classes leads to connections with dynamics and Teichmüller space.

The mapping class group is well behaved in some ways, while not so nice in others. For example, it's finitely presented, but none of the presentations is particularly intuitive or easy to state. One can't do justice to such an interesting object in just a few pages, but this has a silver lining: since completeness is impossible, I'll declare myself free to focus on a narrow but interesting piece of the picture without guilt. I recommend [FM12] for a far more thorough treatment, while this article will attempt three goals: introduce the mapping class group of a surface; describe some specific techniques for building three-manifolds from mapping classes; and finally, explore a relationship between algebraic structures in the mapping class group and some geometric properties of the resulting manifolds.

1. What is the Mapping Class Group?

Mapping class groups are defined for manifolds of any dimension, but invoking the prerogative of low dimensions, we'll focus exclusively on mapping class groups of surfaces. Given a smooth surface S, two diffeomorphisms are *isotopic* if they are homotopic through a path of diffeomorphisms. This gives us a way to identify related functions, and the mapping class group MCG(S) is the group of isotopy classes of orientation-preserving diffeomorphisms. There are alternative definitions: isotopy classes of homeomorphisms form an isomorphic group, and MCG(S) can also be identified with the 0^{th} homotopy group of the space of diffeomorphisms of S. With an eye to a later application, it will be most useful for us to view each element of MCG(S) as an equivalence class of diffeomorphisms

We'll allow both closed surfaces and those with boundary, but we'll assume all surfaces are compact, orientable, and connected. In the case that S has non-empty boundary, each diffeomorphisms must fix the boundary pointwise.

An element of the mapping class group of S is called a *mapping class*, and the identity map on S represents the identity mapping class. With this first example in hand, we need to find a second—and more interesting—one.

1.1. Dehn twists.

Let S be an annulus parameterized as

$$\{(t,\phi) \mid t \in [0,1], \phi \in [0,1]/0 \sim 1\}.$$

The map $\tau(t,\phi) = (t, \phi - t)$ is called a *positive Dehn twist*. To see that τ is not isotopic to the identity, we consider how it acts on curves in the surface —this is a technique we rely on often. Let γ be the arc given by fixing $\phi = \phi_0$ and letting t vary. The image of γ under τ shares the same endpoints but wraps once around the annulus in the S^1 direction. See Figure 1. As an oriented loop, $\gamma \cup \tau(-\gamma)$ generates $\pi_1(S;(0,\phi_0)) \cong \mathbb{Z}$, so the two arcs are not homotopic. There's no isotopy fixing the boundary that takes $\tau(\gamma)$ to γ, so there's certainly no isotopy taking τ to the identity on all of S. This shows that a positive Dehn twist is a non-identity element in the mapping class group of the annulus, and in fact, it generates the entire group.

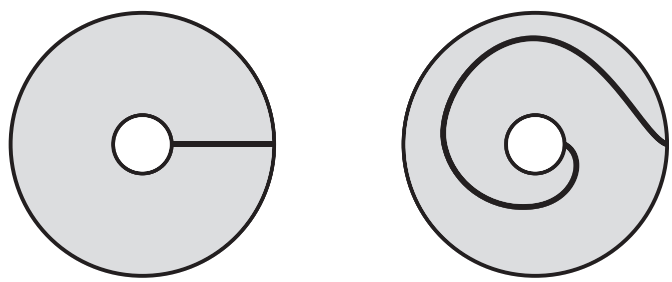

Figure 1. Left: γ connects the two boundary components of the annulus. Right: the image of γ under a positive Dehn twist.

To generalize this example to a more complicated setting, pick a simple closed curve C in an arbitrary surface S. The curve has an annular neighborhood in S which may be parameterized as above. Define a *Dehn twist along C* to be a diffeomorphism which restricts to the annulus as the Dehn twist defined above and extends smoothly to the identity elsewhere. The mapping class of this diffeomorphism depends on the curve C: if C bounds either an embedded disc or a once-punctured disc in S, then rotating the disc defines an isotopy to the identity. We call such curves *inessential*, and any Dehn twist along an inessential curve lies in the trivial mapping class, as seen in Figure 2.

However, Dehn twists along essential curves are the building blocks for the entire mapping class group:

Theorem ([Lic64], [Deh87]). *Dehn twists generate MCG(S).*

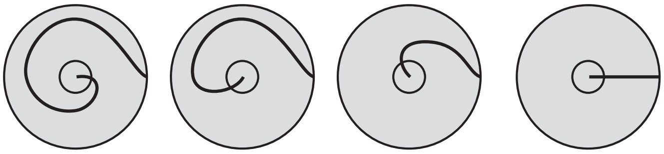

Figure 2. If C bounds a disc, then a Dehn twist along C is isotopic to the identity.

This is a wonderful result, because it establishes that any mapping class factors as a composition of simple maps, but there's an even better statement:

Theorem ([Hum79]). *If S has genus $g > 1$, $MCG(S)$ is generated by Dehn twists around the set of $2g+1$ curves shown in Figure 3.*

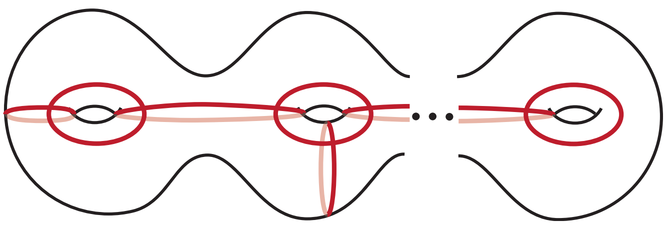

Figure 3. The Humphries generators of MCG(S) are the positive Dehn twists around the indicated curves.

Just as it might seem that mapping class groups are too accessible to be interesting, we note that the relations among distinct Dehn twists are many and varied. Some of these take familiar forms: if C_i and C_j intersect exactly once, then twists around these curves obey the braid relation: $\tau_i\tau_j\tau_i = \tau_j\tau_i\tau_i$. Other relations are more surprising. A Dehn twist around any boundary-parallel curve lies in the center of the mapping class group, but the "lantern relations" equate certain products of boundary parallel twists with products of non-commuting Dehn twists. The simplest lantern relation is shown in Figure 4.

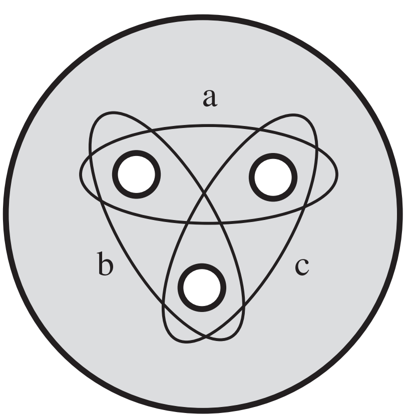

Figure 4. In the surface formed by removing three open discs from a closed disc, the product of positive Dehn twists $\tau_a\tau_b\tau_c$ around the labeled curves equals the product of Dehn twists around curves parallel to the four boundary components.

Although we've barely scratched the surface, this provides enough tools to move on to the promised topic of this article: the relationship between mapping class groups and three-manifolds. In the next section we'll describe a first technique for constructing a three-manifold from a surface mapping class.

2. Building by Gluing

The two constructions coming next—mapping tori and open book decompositions—each produce a closed, compact, connected three-manifold. The input for each construction is a specific map, but manifolds constructed from diffeomorphisms in the same mapping class are diffeomorphic. Since manifolds in three-manifold topology are often studied only up to diffeomorphism, it's reasonable to think of the manifold as the output of a mapping class rather than its particular representative.

2.1. Construction I: The mapping torus.

Using a surface diffeomorphism to build a mapping torus is a simple way to promote two-dimensional data to a three-dimensional object.

Let h be a diffeomorphism of a closed, compact surface S. The product $S \times I$ has two copies of S as its boundary, and the mapping torus T_h of h is the closed manifold obtained from $S \times I$ by identifying $(x, 1) \sim (h(x), 0)$ for all x in S. A manifold that can be constructed as a mapping torus is said to be *fibered*, and each surface $S \times \{t\}$ in a fibered manifold is called a *fiber*. Interestingly, a fibered manifold may fiber in multiple ways, and there is a rich theory associated to understanding all possible fiberings of a fixed manifold. However, we'll bypass this rabbit hole in order to begin probing how properties of a mapping class determine properties of three-manifolds. We'll start with some special types of classes known as *periodic* and *reducible*, respectively.

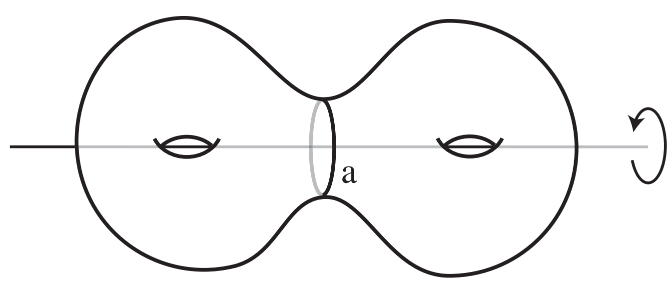

Figure 5. Rotating the genus two surface by π around the indicated axis is periodic because it has order two, and it's reducible because it fixes the curve a.

The first definition is straightforward and makes sense in purely group-theoretic terms: a mapping class is *periodic* if it has finite order. For an easy example, consider rotating a closed surface by π as shown in Figure 5. A mapping class is *reducible* if it fixes the isotopy class of some collection of essential simple closed curves. In fact, the previous example is also reducible, as it fixes the curve on the surface labeled "a." Within this mapping class one may select a representative that fixes a pointwise, and if we cut along the curve to create two new boundary components, this diffeomorphism descends to a well defined mapping class on each component of the new surface.

Mapping classes that are neither reducible nor periodic are called *pseudo-Anosov*, and they can be identified independently — as opposed to by a process of elimination — using dynamics. The sorting of mapping classes into periodic, reducible, and pseudo-Anosov is know as the Nielsen-Thurston classification, and one of its most striking applications is the following result of Thurston:

Theorem ([Thu98]). *The mapping torus T_h is a hyperbolic three-manifold if and only if $h \in MCG(S)$ is pseudo-Anosov.*

Hyperbolic manifolds are those admitting negative curvature metrics, and the classification of three-manifolds in terms of their supported metrics is one of the huge breakthroughs of modern topology. The program of proving Thurston's Geometrization Conjecture was completed by Perelman in 2003 and included a proof of the infamous Poincaré Conjecture, the only one of the million-dollar Millennium Problems solved to date.

As noted above, Thurston's theorem only tells us something about the manifolds which can be realized as mapping tori, and some three-manifolds simply don't admit any fibered structures. In order to relate mapping classes to all three-manifolds, we need to consider a different construction.

2.2. Construction II: Open books.

Not every three-manifold is fibered, but we'll now see that every three-manifold is "almost fibered." That is, inside every closed, oriented three-manifold one may find a collection of disjointly embedded loops whose complement is fibered. As a starting point, we'll show how to build manifolds that clearly have this structure.

As before, start by considering a mapping torus, but this time, let h be a diffeomorphism of a surface S with nonempty boundary. Since h fixes the boundary pointwise, each component of ∂S becomes a toroidal boundary component in the mapping torus T_h. The boundary of T_h inherits a natural product structure $\partial S \times I / \sim$. To complete the process of building a closed manifold, collapse the second factor of each boundary component. Although the surfaces $S \times t$ have distinct interiors, they now share the same copy of ∂S. These shared boundary curves form a link in the manifold, called the *binding*, and the complement of the binding is fibered by copies of $S \setminus \partial S$, which are called the *pages*.

Although this seems more involved than constructing the mapping torus of a closed surface, the binding always

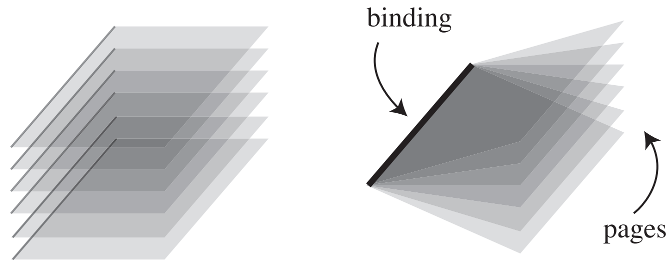

Figure 6. Collapsing the binding of an open book.

collapses in the same way; thus, the complexity again lies only in the choice of mapping class. The data (S, h) is called an *abstract open book*, and we say that the manifold built from this data has an *open book decomposition*. Not only can we construct manifolds from abstract open books, but up to diffeomorphism, every three-manifold can be realised this way [Ale23].

It's said that "classical" mathematics is anything published before your own PhD, but open book decompositions are classical mathematics even by a more restrictive standard. The existence result above dates back to the 1920's, but the prominence of open book decompositions in twenty-first century topology stems from their applications to contact geometry. We'll begin the next section wandering around an apparently new branch of mathematics before we can make the connection to open book decompositions and the mapping class group. Be patient, and enjoy the ride.

3. Contact Geometry

A *contact structure* on a smooth, orientable three-manifold is a nowhere integrable oriented two-plane field. That is, a contact structure is a two-plane field which can't be integrated,—even locally—to give a surface. The existence of such two-dimensional objects stands in sharp contrast to the one-dimensional case, as integral curves of vector fields can always be found.

It's not terribly difficult to imagine an integrable plane field, as we can construct these directly. For example, on any fibered manifold T_h we may take the tangent planes to each of the fibers, and by construction these form an integrable plane field. However, this is exactly the behavior we need to exclude: there is no open subset of any surface where the tangent planes and contact structure coincide.

To illustrate contact structures, we'll start with an example: the *standard contact structure* on $\mathbb{R}^3$ is the kernel of the one-form $dz - ydx$. At each point (x, y, z), this form vanishes on the linearly independent vectors $\{\partial_y, y\partial_z + \partial_x\}$ in the tangent space $T_{(x,y,z)}\mathbb{R}^3$. Notice that these vectors are independent of the x- and z-coordinates, so it suffices to understand how their span varies along the y-axis. At the origin, the contact plane is horizontal, while it makes a quarter twist as y approaches $\pm\infty$. Translating this line of twisting planes horizontally and vertically exhausts $\mathbb{R}^3$ and completes the contact structure.

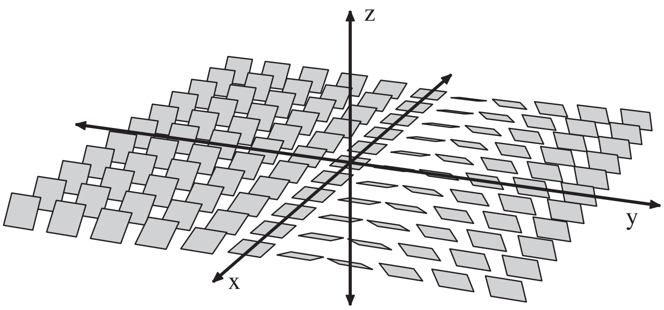

Figure 7. The standard contact structure restricted to the xy-plane.

The technical tool proving the planes of the standard contact structure are not tangent planes of any surface is the Frobenius Integrability criterion. Qualitatively, the twisting seen in this example is the obstruction to integration and characterizes all contact manifolds. In fact, the standard contact structure provides a universal local model, a statement made precise by the contact Darboux Theorem. Like its symplectic analogue, the Darboux Theorem states that one may associate to every point in a contact manifold a diffeomorphism taking the contact planes in a neighborhood of the point to the contact planes in a neighborhood of the origin in the standard $\mathbb{R}^3$. This gives us a special family of charts to describe a contact manifold and implies that contact planes always twist along parallel curves.

3.1. Tight and overtwisted contact structures.

Recall that the planes in the kernel of $dz-ydx$ make a half twist as y varies from negative to positive infinity. In contrast, consider the kernel of the one-form $\cos rdz + r\sin rd\theta$. This contact structure is similarly invariant under translation in the z-direction, but it sees the planes spinning countably many times around each horizontal ray from the z-axis. Contact structures always exhibit enough twisting to preclude integrability, but as it turns out, more twisting makes contact structures less interesting. A central theorem in the field partitions all contact structures into two classes, called *tight* and *overtwisted*.

Although we won't state the precise definition of an overtwisted contact structure here, we note that they're completely classified by algebraic topology: there's a unique overtwisted contact structure in every homotopy class of two-plane fields [Eli89]. The tight contact structures — those that twist enough but not too much — are far more mysterious. Intriguingly, some manifolds have no tight contact structures, some have a unique tight contact structure, and some have infinitely many [EH01], [Eli92], [Kan97]. Tight contact structures are at the heart of the field, and they tend to be more useful in applications and have more intriguing ties to other mathematical objects. When meeting a new contact structure for the first time, "Tight or overtwisted?" is the first question to ask.

3.2. Contact structure and open books.

There are many approaches to studying contact structures on three-manifolds, but we'll focus on the astonishing fact that the data of an open book not only determines a manifold, but also an equivalence class of contact structures on that manifold. Before explaining this correspondence, observe that if we can associate a contact three-manifold to any surface mapping class we get a new version of the motivating question: do structures in the mapping class group determine phenomena in the world of contact manifolds? In many cases the answer is yes, and this alchemy turning algebra into geometry will be our final topic.

As claimed above, contact plane fields are characterized by their twisting, and an open book decomposition localises this rotation in a neighborhood of the binding. More precisely, we say that an open book decomposition *supports* a contact structure on the ambient manifold whenever two properties are satisfied. First, in a neighborhood of the binding we require a standard model: after parameterizing each component of B by ϑ, there exists a neighborhood $D^2 \times S^1 = \{r, \theta, \vartheta \mid r \in \mathbb{R}^+, \theta, \vartheta \in S^1\}$ where the contact structure is the kernel of $d\vartheta + r^2 d\theta$ and ∂_ϑ induces the boundary orientation on the page. This neighborhood is invariant under rotation by ϑ and θ, but the planes twist radially with r. The second compatibility condition states that this is "most" of the twisting: away from the binding, there exists a vector field simultaneously transverse to the pages and contact planes whose flow preserves the contact planes. Informally, this prevents the contact planes from rotating too much with respect to any page.

Theorem ([Gir02], [TW75]). *Each open book supports a unique isotopy class of contact structures.*

In addition to making contact structures easier to visualize, we'll see next that open book decompositions are a powerful technical tool.

4. Contact Properties and Mapping Classes

With the relationship between mapping classes and contact three-manifolds in hand, it's time to examine some examples of structures on both sides of the equivalence.

Positive Dehn twists were defined above, and the inverse of a positive Dehn twist is unsurprisingly called a negative Dehn twist. In terms of coordinates, a negative Dehn twist maps $(t, \phi) \in I \times S^1$ to $(t, \phi + t)$. Mapping classes which can be expressed as a product of Dehn twists of a fixed sign form a monoid in the mapping class group. One can always associate a monoid to a subset of generators, but these positive and negative monoids in the mapping class group determine fundamental properties of the associated contact manifolds: if the mapping class h lies in the positive monoid in MCG(S), the contact manifold associated to the open book (S, h) always supports a tight contact structure, and if $h \neq 1$ lies in the negative monoid, the contact manifold associated to (S, h) always supports an overtwisted contact structure [Gir02] [AO01] [LP01] [Pla04].

Here, the contact phenomenon is a consequence of a structure we could define purely algebraically, but in other examples the algebraic substructure in the mapping class group comes from the contact geometry. Just as a topological three-manifold can arise as the boundary of a four-manifold, some contact structures are induced by realizing the ambient manifold as the boundary of a symplectic four-manifold. There are several different notions of such *fillability*, and for each flavor of fillability, consider the mapping classes whose associated open book supports a fillable contact structure. For each surface S and each notion of fillability, these mapping classes form a monoid in MCG(S) [BEVHM12], [EVHM15].

4.1. The Giroux correspondence. In general, the flow of information from contact geometry back to the mapping class group is rather subtle, as there are many distinct open books which support the same contact manifold. Luckily, the relationship among these open books is understood. An operation called *positive stabilization* replaces the pair (S, h) with a new open book (S', h') supporting the same contact structure. The surface S' is built from S by attaching a band along two intervals of the boundary. Since S embeds into S', h naturally extends to a diffeomorphism on the new, larger surface which we again call h. Then h' is the composition $\tau \circ h$ for τ a positive Dehn twist along some curve that runs once along the new band and closes up in the original surface. This sounds complicated, but the celebrated Giroux Correspondence says this single operation captures the full complexity of the relationship:

Giroux Correspondence [Gir02]. Any two open book decompositions supporting a fixed contact structure can be made to agree after a finite sequence of positive stabilizations.

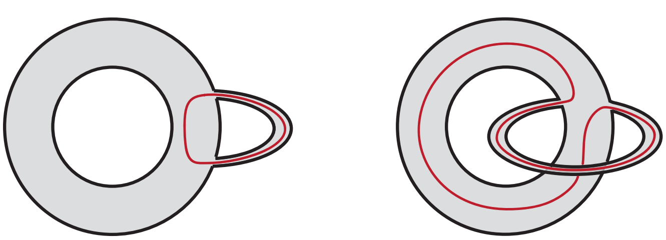

Figure 8. This figure shows distinct stabilizations of an open book with annular pages. The new monodromy is the composition of the original monodromy with a postive Dehn twist around the thin red curve.

Since S embeds naturally into S', stabilization provides a framework for relating maps on different surfaces, and indeed, structures in different mapping class groups. It's clear that any product of positive Dehn twists in MCG(S) must stabilize to a product of positive Dehn twists in MCG(S′), since stabilization simply adds an extra positive factor. More interestingly, consider the fillability monoids introduced in Section 4. Since these are defined by a property of the supported contact structure, stabilization must respect this structure, as well.

The Giroux Correspondence shows us that the relationships between different mapping class groups are central to understanding how surface diffeomorphisms dictate contact behavior. However, these relationships are not always so easy to understand. Another monoid of interest in the mapping class group, Veer^+, consists of *right-veering maps* and strictly contains the monoid generated by positive Dehn twists. Mapping classes in the complement of Veer^+ always produce overtwisted contact structures, but any mapping class can be promoted to something right-veering with sufficiently many carefully chosen stabilizations [HKM07].

Theorem ([HKM07]). *A contact structure is tight if and only if the monodromy of every supporting open book lies in* Veer^+.

The notion of a right-veering monodromy is rather intuitive, but replacing this condition with a more technical one known as 'consistency' leads to a stronger result:

Theorem ([Wan15]). *A contact structure is tight if and only if the monodromy of any supporting open book is consistent.*

Results like these help clarify two general types of questions we might ask.

1. Given a class of contact manifolds, can one characterize all the mapping classes which produce supporting open books?
2. Can one start with some class of diffeomorphisms and characterize all the contact structures supported by the associated open books?

These are difficult questions, and there are interesting results even if the set of mapping classes is dictated simply by the surface. For example, every overtwisted contact structure can be supported by an open book built from a planar page, but for an arbitrary tight contact structure it's an open question when this is possible [Etn04].

There are still more unknowns than knowns when it comes to understanding the relationship between mapping class groups and contact geometry, which makes this a wonderful sandbox in which to play.

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