The Man Who Loved Problems: Richard K. Guy

Andrew Granville and Carl Pomerance

"Richard Kenneth Guy on his hundredth birthday
Working at his figures round the clock non-stop. . . "
—Andrew Bremner

Introduction

This article celebrates the life of one of the most colorful figures in contemporary mathematics. Richard Guy, popular author, columnist, educator, researcher, and avid mountain climber, passed away in 2020 at the age of 103.

Richard's research interests were primarily in number theory, combinatorics, game theory, and geometry, and his books on these subjects fascinated, and continue to fascinate, generations attracted by the lure of solving an accessible research problem. His extraordinary books not only appealed to a wide audience but brought readers in, encouraging them to play with the mathematics and learn by doing. For many years he ran the Research Problems column in the *American Mathematical Monthly*, which each month invited people to write an essay about an accessible problem. Besides his research and his inspiring books, Richard was a mentor to many students, enjoyed outreach, and was an extraordinary expositor.

Early life. Guy grew up in Warwickshire, England, and, reading Dickson's *History of the Theory of Numbers* at the age of 17, discovered his lifelong passion for number theory problems. As an undergraduate at Cambridge he became an avid composer of chess problems, which he blamed for not getting a "first" when he graduated in 1938. He then trained to become a teacher at the University of Birmingham. He married his wife Louise in 1940, bonding over their mutual passion for mountain climbing and for dancing. They had three children, Elizabeth Anne, Peter, and computer scientist and mathematician, Michael J. T. Guy.

Moving around. In 1941, Guy trained as a meteorologist, forecasting for bombers and other aircraft during the Second World War. Posted to Reykjavik, he was commissioned as flight lieutenant in the Royal Air Force in 1942; in Iceland, he did some glacier travel, skiing, and

Figure 1. Flight Lieutenant Richard Guy, RAF, in 1942.

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To Richard Guy (9/30/1916–3/9/20), a friend, mentor and colleague.

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Figure 2. On the fire escape at University College London in 1948. (The same one is there today.)

mountain climbing, marking the beginning of another long love affair, this one with snow and ice. Subsequently he was posted to Bermuda. After the war, he first taught mathematics at Stockport Grammar School and then at Goldsmiths' College in London. From 1951 to 1962, Guy taught at the University of Malaya in Singapore, (now NUS), and then spent a few years at IIT Delhi, before finally moving to the University of Calgary in Alberta, Canada in 1965.

An amateur mathematician. Like others of the WW2 generation, Guy never went back to do a PhD, priding himself on being an "enthusiastic amateur" who enjoyed people who did quality research. In 1960, Paul Erdős visited Singapore and suggested some problems to Guy: "I made some progress in each of them. This gave me encouragement, and I began to think of myself as possibly being something of a research mathematician, which I hadn't done before."[1] He ended up writing four papers with Erdős, even solving an Erdős problem and developing his voracious appetite for unsolved problems. From then on, Guy would collect problems, both for his column in the *Monthly* and also for his wonderful book *Unsolved Problems in Number Theory*, commonly referred to as *UPINT*. Several notable mathematicians got their start trying to solve problems from *UPINT*.

[1]*Quote from Albers and Alexanderson's 2011 interview with Richard Guy in* "Fascinating mathematical people."

Guy published more than 100 research papers in number theory, geometry, graph theory, combinatorics, and combinatorial game theory, a subject he helped create, as well as many articles on "recreational mathematics." He worked with luminaries in each of these fields, such as Elwyn Berlekamp, Béla Bollobás, John H. Conway, Paul Erdős, Martin Gardner, Ron Graham, Frank Harary, Donald Knuth, Dick Lehmer, Yuri Matiyasevich, John L. Selfridge, Dan Shanks, and Neil Sloane.

A doctorate. Guy retired in 1982 and nine years later finally obtained an (honorary) doctorate from the University of Calgary. In its award they wrote, "his extensive research efforts and prolific writings in the field of number theory and combinatorics have added much to the underpinnings of game theory and its extensive application to many forms of human activity." Guy worked on research for almost another 40 years, in the office five days a week, even at 100, prolific to the end.

Figure 3. Richard Guy, a Calgary professor in the 1960s.

A teacher. Richard enjoyed involving people in mathematics, particularly getting them hooked on unsolved problems. He taught at the first Canada/USA Mathcamp for high school students. Even as a centenarian, Richard would pose suitable problems, some recreational, some serious, always aiming to gently tempt the student into going further. Guy's last PhD student, Jia Shen, graduated when Richard was 92. He never really stopped, indeed he supervised undergraduates until he was 101. Participating in Calgary Math Nites, "an interactive event for students in Grades 7 through 12 who want to try more challenging mathematics,"[2] he "discovered" Alex Fink, (now at Queen Mary University, London) and Julian Salazar (now in machine learning with Amazon) with whom he published a paper [16] when Julian was 20 and Richard 98.

Graduate students were always impressed by his openness, and desire to include them, for example inviting students to meet with, and be inspired by, famous visiting mathematicians [19].

The Richard and Louise Guy Lecture Series was endowed as a gift from his wife Louise to Richard for his 90th birthday. The speakers have included Richard's collaborators like Conway, Graham, and Berlekamp; students like Richard Nowakowski; other exciting talents like Manjul Bhargava, Erik Demaine, Mike Bennett, and Noam Elkies; as well as mathematics from a different perspective from Edward Doolittle (from the First Nations University of Canada) and Eugenia Cheng (Scientist in residence at the Art Institute of Chicago). The most recent Guy lectures were given by Terry Tao (2020) and Ben Green (2021).

Figure 4. Richard and his wife Louise, cross-country skiing in 1971 just after arriving in Canada.

A climber. Richard and his wife Louise were well-known as hikers and climbers. They had learned to love the Himalayas in their time in India, had climbed in the Swiss Alps, made their way to the summit of Mount Kinabalu in Northern Borneo and visited the Rhotang Pass and the Kulu Valley in India, so they were familiar with mountains. But then, at almost 50, they got serious and moved to Calgary, joining the Alpine Club of Canada and the Calgary Mountain Club. They became two of their most important members and remained committed to mountain hiking and environmentalism even in their later years. Indeed in 2016, the Alpine Club opened the Louise and Richard Guy Hut on the western side of the Wapta Icefield in Yoho National Park to house overnight backcountry skiers. There is even a book celebrating their mountaineering exploits [20] *Young at Heart? The Inspirational Lives of Richard and Louise Guy*. For his 100th birthday, Richard helicoptered up to Mount Assiniboine Lodge and hiked four miles to Wonder Pass and then, the next day, hiked to the top of the Niblet. When Richard died, the headline of *Gripped Magazine* read "*Canadian Climbing Legend Richard Guy Dies at 103. He leaves behind a life of inspirational achievements and amazing accomplishments.*"[3]

A political man. If you had met Richard at a conference at any time in the last forty years, he would have been wearing a tweed jacket with a button pinned on that read "Peace is a disarming concept." He and his wife Louise were outspoken pacifists, even during the Cold War. One of us (CP) recalls an amusing incident in Edmonton, Alberta, in 1983. Walking to the conference at the university, I was stopped by a camera crew interviewing people on the street for the local television station about their opinions of the US testing cruise missiles in northern Alberta. I said that I was American, and they said I could still give my opinion. I said, somewhat inarticulately, that I was opposed to nuclear weapons. Arriving at the conference coffee room, I related the incident to those there. Immediately, Richard said: "You should have said, I'm American and better to test them here than in the US!"

Richard was a community builder who believed in helping others: He donated to the Calgary mathematics department for a lecture series and helped the University of Calgary acquire the Eugene Strens Recreational Mathematics collection. He donated time and money to the needs of hiking communities. Richard and Louise were early conservationists. They participated in the annual Calgary Tower climb to raise funds for the Alberta Wilderness Association. Richard continued the tradition after Louise died in 2010, even climbing at 102 while carrying a photo of Louise.

We both remember him from conferences in the 1980s and before, playing the piano while others, led by John Selfridge's loud baritone, sang songs from a bygone era. He was a warm and compassionate man, and will be missed.

[2] *https://science.ucalgary.ca/mathematics-statistics/engagement/educational-outreach/math-nite*.

[3] *https://gripped.com/news/canadian-climbing-legend-richard-guy-dies-at-103/*

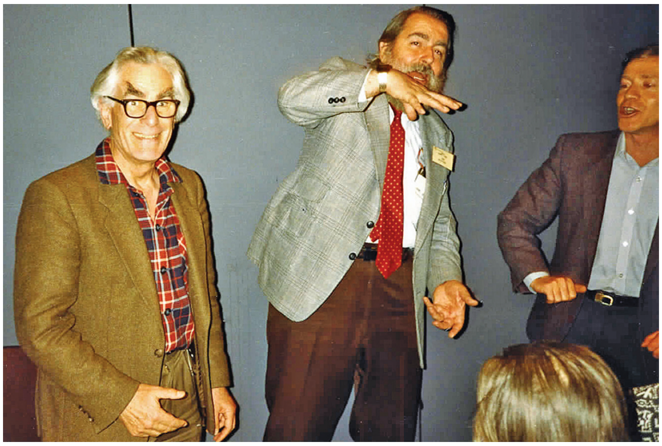

Figure 5. Richard shaking a leg with John Selfridge and Alf van der Poorten at a NATO Advanced Study Institute in Banff in 1988.

A mathematical artist. Richard loved the geometric side of both combinatorics and number theory, and would hand-draw beautifully inscribed diagrams. Here is one from a lecture back in 1993:

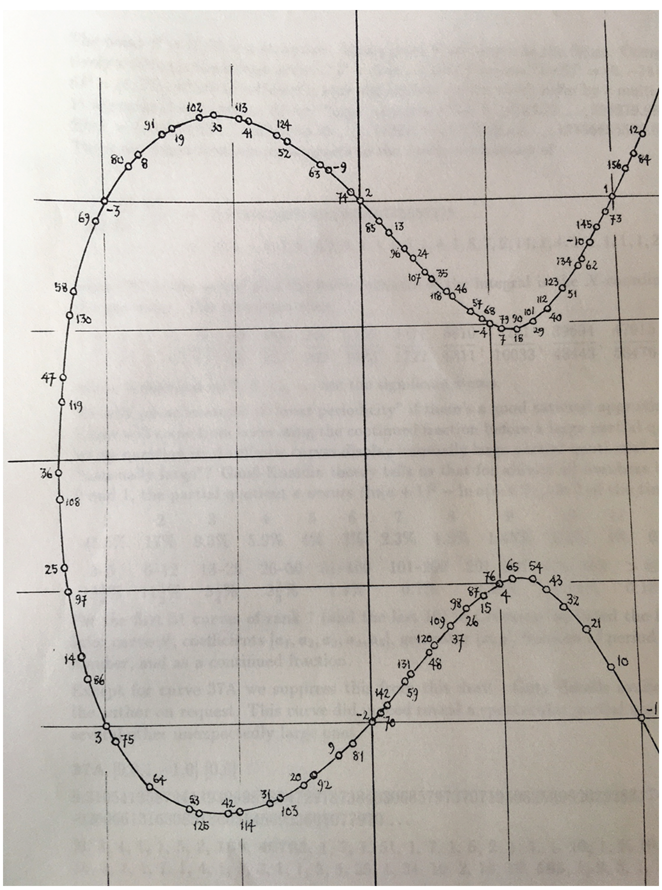

Figure 6.

In some old notes we found Richard's hand drawn picture of the multiples of the point $P = (2, 2)$ on the elliptic curve

$$E : y^2 = x^3 - 4x + 4,$$

an illustration in his talk *Are elliptic curves almost periodic?*

The idea is that $11P = (310, -5458)$ is not only, rather miraculously, an integral point, but also has surprisingly large coordinates. The zero of the group of rational points on E lies at "∞," so that $11P$ is "sufficiently near infinity that points which differ by a multiple of 11 are quite close together," as exhibited by the diagram. Thus the multiples of P are "almost periodic" with period 11.

A computational style. Besides the meticulous artistry of his rendition of the points, this exhibits Guy's style of doing extensive calculations and finding examples that attract the reader to the salient features. For example he goes on in the same talk to look for multiples of P with even larger coordinates (and therefore even better "pseudo-periods"). To do this he first calculates the continued fraction [0, 1, 4, 1, 1, 6, 3, 1, 4, ...] of $\frac{1}{2\Omega}\int_2^\infty \frac{dx}{y}$ where 2Ω is the period, He then identifies the large partial quotients which give good approximations to the value of the integral, and therefore Guy found

$$72P = (4543.72\ldots, 306279.98\ldots),$$

$$299P = (1544460, 66\ldots, 60705331.35\ldots), \text{ etc.}$$

In keeping with his self-image as a keen amateur, Guy did not explain the underlying theory (though he surely knew it), but rather drew readers (like us) in who wanted to understand where these extraordinary examples came from.

A man with problems. Richard loved promoting interesting unsolved problems. He wrote books and he ran a Research Problems column in the *American Mathematical Monthly* for many years. He led problem sessions and created sheets of problems at conferences, including at the West Coast Number Theory Conference, often held just before Christmas in Asilomar, California. At one of these problem sessions, Erdős was in the front row and appeared to be sleeping. Richard, who was running the session, pointedly said in a loud voice "Paul, do you have any problems?" At which point Erdős roused himself and said "I'll be all right in a minute." This passion for problems plays out in his research, indeed in Richard's papers he seems to be as interested in explaining what intrigues him and what he does not know, as he is in solving the actual research question.

Combinatorial Game Theory

When Richard started as a student at Cambridge in 1935, he "played 24 hours a day bridge, 24 hours chess and 24 hours snooker," (see [20, p. 6]), which might have affected his classical mathematical studies but assured, with his analytic mind, that he would be one of the principal founders of *Combinatorial Game Theory*.

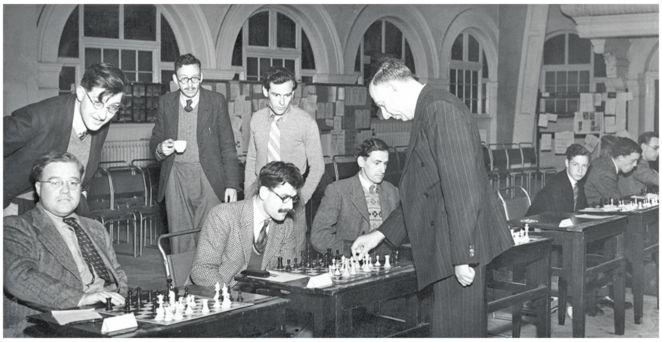

Figure 7. Playing a chess match against 1950s British champion, Harry Golombek.

Richard Guy was the "endings editor" for *British Chess Magazine* from 1947 to 1951, giving about 200 endgame studies. He coinvented the Guy–Blandford–Roycroft (GBR) code, used for representing chess positions, particularly for classifying endgames. Inspired by a chess game with pawns, he developed the Sprague–Grundy theory on impartial games so that it applied more generally. He discovered octal games, generating many intriguing conjectures; for example, his conjecture that the sequence of values for every finite octal game is periodic remains open.

To get the field off the ground, Richard helped organize an AMS Short Course on Combinatorial Games, as well as the first MSRI and BIRS[4] conferences on the subject. He collected problems and wrote the first four "Unsolved Problems in Combinatorial Game Theory" articles, as well as a book with this same title with Nowakowski. A little-known and hard-to-get gem is Richard's book *Fair Game: How to Play Impartial Combinatorial Games*. Richard was still pushing the boundaries of game theory at 90 with [9].

In late 1969, John Conway was investigating his *Game of Life* with a keen group of students and colleagues, including Michael Guy. Richard, visiting his son, joined in and carefully studied some of the "life-forms" emerging from the central body:

> "Guy noticed [something] that no one had ever seen before. It seemed to be wiggling, skittering, gliding its way diagonally across the board. He hollered to the others: 'Come over here, there's a piece that's walking!' This was the first step toward proving Life universal. Conway christened this walking piece the *Glider*...because after two moves its position differs from the starting position by a 'glide reflection,' a symmetry operation, and at generation 4 it looks exactly the same as it did at generation 0, but it has glided diagonally downward by a single place."
>
> From *Genius at Play: The Curious Mind of John Horton Conway* by Siobhan Roberts, 2015.

The Glider is a key object in our understanding of the Game of Life, exhibiting how information can be transmitted over long distances and contributing to the proof that the Game of Life is the most elegant universal computing machine of all (it is Turing complete and can be used to simulate any other Turing machine).

This work was the beginning of a beautiful friendship with Conway, Richard appreciating Conway's work and developing it in more games-like directions. This culminated in the well-known *Winning Ways for your Mathematical Plays* a four-volume discussion with Berlekamp and Conway of all sorts of Games and Plays (including word games) that has by now inspired everyone who works in Combinatorial Games. Martin Gardner described it in 1998 as "the greatest contribution to recreational mathematics in this century."

The last few years have seen the passing of a generation who brought color and rigor to combinatorial and mathematical games, and presented mathematics in a challenging and compelling manner. In the 15-month span from April 2019 to July 2020 we lost Elwyn Berlekamp, Richard Guy, John Horton Conway, and Ron Graham. Richard was there from the start and encouraged these other giants to participate.

Number Theory

Richard Guy had a gift for recognizing and appreciating patterns, separating structure from coincidences. He proposed the "Strong Law of Small Numbers" [11], giving 35 remarkable patterns that seem obvious when we check small values, but don't necessarily work for large values,[5] concluding that "There aren't enough small numbers to meet the many demands made of them."[6] His follow-up [12] claims "When two numbers look equal it ain't necessarily so." This can be inspiring stuff for an aspiring mathematician (and indeed did inspire the first-named author as a grad student).

Guy was mainly interested in properties of integer sequences and their appearance in geometry and Diophantine problems. His contributions are numerous

[4] *MSRI is the Mathematical Sciences Research Institute and BIRS is the Banff International Research Station.*

[5] *One mathematical confusion in the early Renaissance period was appreciating the difference between a proof by induction, and a verification for small values!*

[6] *A paper that won the 1989 Lester R. Ford award from the MAA for expository excellence.*

so we only discuss a few questions he came back to again and again:

Aliquot sequences. Let $\sigma(n)$ denote the sum of the divisors of n and $s(n) = \sigma(n) - n$, so $s(n)$ is the sum of the "proper" divisors of n. Pythagoras noted that on iterating the function $s(n)$, occasionally there were fixed points (perfect numbers) and he discovered the 2-cycle $s(220) = 284$ and $s(284) = 220$. Examples such as these stand out from the apparently more ordinary behavior where iterating $s(n)$ one arrives at a prime, after which the next value is 1 and the sequence stops. The iterates n, $s(n)$, $s(s(n))$, …form an "aliquot sequence" and they have fascinated mathematicians for millennia. The Catalan–Dickson conjecture claims that they all terminate or become periodic (so are bounded). The conjecture remains open even for $n = 276$. Guy spent many years accumulating a vast amount of data from different perspectives, leading Guy and Selfridge [17] to speculate that some of these sequences are unbounded!

The current reasoning is that viewed from afar, an aliquot sequence tends to act like a geometric progression for long stretches, with the contribution from small primes staying constant. Although Bosma and Kane showed that the geometric mean of $s(2n)/2n$ is less than 1, Pomerance recently showed that it is greater than 1, for $n \equiv 0$ mod 4. This was more or less predicted by Guy's calculations and heuristic models using Markov chains. The theory continues for $s(s(2n))/s(2n)$, but beyond the second iterate we only have these heuristics, [7]. Richard intrigued bright young minds into working on this problem for over 40 years, and at one point described it as his favorite problem.

How to factor a number? In 1976, Richard wrote a survey of then state-of-the-art integer factorization methods [10] that became very influential with the advent of the RSA cryptosystem in 1978. The title though was a play on words: in the article he exhibited many factoring ideas by factoring the same number (namely, 1037), again and again! In the survey he especially developed a discussion of the Pollard rho method, an enigmatic and eminently programmable factorization algorithm that is still not completely understood.

Linear divisibility sequences. These are linear recurrence sequences that are also divisibility sequences (that is, x_m divides x_n whenever m divides n). For example the Fibonacci numbers have both properties, and so they form a linear divisibility sequence. Indeed, all sequences derived from second order linear recurrences with zeroth term 0 are linear divisibility sequences. Moreover, products of linear divisibility sequences are also linear divisibility sequences. In other words, if n_i and m_i are two linear divisibility sequences then the sequence $n_i \cdot m_i$ is also a linear divisibility sequence. Richard was interested in finding other examples. With Hugh Williams [18] he determined that a_n, the number of distinct tilings of a 4-by-$(n-1)$ rectangle by dominos (1-by-2 tiles), is a linear divisibility sequence given by the fourth order linear recurrence

$$a_k = a_{k-1} + 5a_{k-2} + a_{k-3} - a_{k-4}$$

with $a_0 = 0, a_1 = 1, a_2 = 1, a_3 = 5, a_4 = 11, a_5 = 36, a_6 = 95$, etc. This sequence is not a product of second order sequences, so how can it be explained? They found a general pattern, where the recurrence sequence can be given in the form $a_n = \alpha^n + \beta^n - \gamma^n - \delta^n$ where $\alpha\beta = \gamma\delta$. Here we note the alternative representation $a_n = \alpha^{-n}(\alpha^n - \gamma^n)(\alpha^n - \delta^n)$ which related their construction to what was previously known. This work led to further understanding, including a series of papers with his former doctoral student Eric Roettger that culminated with a solution to Lucas' unsolved problem of generalizing the Lucas sequences to the setting of higher order recurrences, as well as an idea for public key cryptography.

Fun combinatorial number theory. In 2003, Elwin Berlekamp and Richard determined permutations of $\{1, \dots, n\}$ such that the sum of any two neighbors is a Fibonacci number. In 1986, Yuri Matiyasevich and Richard give the remarkable formula

$$\pi = \lim_{n\to\infty} \sqrt{\frac{6\log(F_1F_2\cdots F_n)}{\log \operatorname{lcm}[F_1, F_2, \dots, F_n]}}$$

where F_j is the jth Fibonacci number.

Diophantine problems. Richard was also interested in Diophantine problems; for example, in 1993 Bremner, Richard, and Nowakowski [5] settled which integers n can be represented as

$$n = (x + y + z)(1/x + 1/y + 1/z),$$

with integers x, y, z. They showed that solutions correspond to non-trivial rational points on the elliptic curve

$$Y^2 = X(X^2 + (n^2 - 6n - 3)X + 16n)$$

except for when $n = 0$, 1, or 9 (and so if there is one such representation of n there are infinitely many). They then determined the Mordell–Weil rank of this curve for all n with $|n| \leq 1000$.

Inspirational books. Richard's *Unsolved Problems in Number Theory* [13] inspired so many mathematicians of different generations. It is a celebration of problems, from the small to the large, from the famous to the unheralded. All authors are held in equal esteem (in fact reverence) be they an amateur who has calculated a few examples or a Fields' medalist revolutionizing the area. There is a conscious egalitarianism focused on a love of problems (with pertinent commentary). It has gone through three editions and continues to be an inspiration.

Richard's *Book of Numbers*, joint with John H. Conway, is the perfect gift for the aspiring mathematician. It features many beautiful examples, proofs, applications, and all things number theory. It has been translated into at least eight languages besides English.

Richard Guy never stopped doing research: indeed, the memorial article by Jacobson, Scheidler, and Williams in the Sept 2020 CMS Notes focusses on Richard Guy's many interesting contributions after the age of 95!

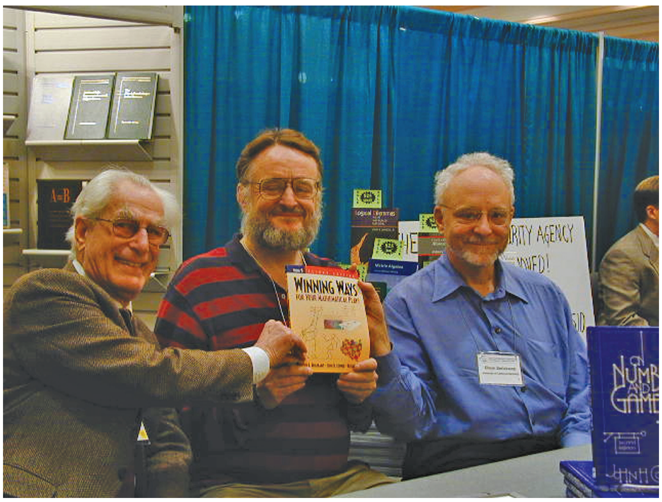

Figure 8. From left to right: Guy, Conway, and Berlekamp and their *Winning Ways* in 2004.

Geometry or Number Theory?

In 2007, Guy remarked on his favorite problems in geometry,

> "The combination of geometry and number theory is dear to my heart."[7]

Here are a few of his favorites:

Consider the N-by-N integer lattice $\Lambda_N \coloneqq \{(x,y) \in \mathbb{Z}^2 : 1 \le x, y \le N\}$.

- What is the size of the maximal subset of Λ_N with no three on a line? Is it $< 2N$ once N is large enough?
- What is the size of the maximal subset of Λ_N in which all distances between different points are distinct? Erdős and Guy [8] showed there can be more than $N^{2/3-\epsilon}$ such points.
- The *crossing number* of a graph G is the minimal number of edges that cross (other than at vertices) when G is drawn on the plane. Guy proved several conjectures on crossing numbers for different families of graphs. He also looked at the *slimming number*, the minimal number of edges one needs to remove to make G planar.
- There are four different candidate configurations of tilings of the unit square by four triangles with rational sides. One is conjectured to be impossible, and the others were carefully considered by Bremner and Guy [2, 3].

One special configuration asks for rational sided triangles with vertices at $(0,0)$, $(x,1)$, $(1,y)$ where $x, y \in \mathbb{Q}$. They showed [2] that this is equivalent to finding integer solutions to

$$2mnrs(u^2 - 2uv - v^2) = uv(m^2 - n^2)(r^2 - 2rs - s^2)$$

which they studied using the theory of rational points on the resulting elliptic curves. In [3], they consider when all vertices of the triangles lie on two opposite sides of the square, which correspond to rational points on the quartic surface

$$xyz(x + y + z) = xy + xz + yz.$$

Tilings that are symmetric under a 180 degree rotation correspond to an elliptic curve that has rank 1 lying on this surface, which allows them to get some results on such configurations.

- Morley proved that the trisectors of the angles of a triangle meet at the vertices of an equilateral triangle; there are actually 18 Morley triangles associated to a triangle, all equilateral. With Bremner, Goggins, and his son Michael, Richard [1] described which triangles with all edges of rational length have a Morley triangle with an edge of rational length. Indeed if the triangle is equilateral then 6 of the 18 have a rational edge, otherwise the triangle belongs to a one- or two-parameter family of triangles.
- A triangle has eight vertices but only one center—read [15].
- In [4], Bremner and Guy show that integer right-angle triangle-rectangle pairs with a common area and a common perimeter can be parametrized by an elliptic curve defined over $\mathbb{Q}(t)$.

We see how Richard Guy's research in geometry was primarily motivated by connections with number theory (especially elliptic curves), while focusing on easy-to-state geometry problems. He made many contributions to problems columns, and his book *Unsolved Problems in Geometry* with H. Croft and K. Falconer, engages the reader with many problems in convex, discrete and combinatorial geometry.

Personal Reminiscences Collected

Andrew Bremner. Our first meeting was in 1973. At that time, I was a graduate student in Cambridge, and Richard was in town visiting John Conway. There was a knock on my office door, and Richard walked in and introduced himself. This was a trait I recognized over the years: he was

[7] *From an essay of Ted Bisztriczky in a collection of Guy remembrances in the* CMS Notes, *September, 2020.*

Figure 9. Richard at his favorite hiking destination, Mt. Assiniboine in 2006 for his 90th birthday.

always interested in what other people were doing, particularly those whose interests might overlap with his own. He was at that time teaching himself about elliptic curves, and over the next couple of years with his repeated visits to Cambridge, a bond of friendship formed between us which resulted many years later in a couple of coauthored articles. Others will write of Richard's legacy to mathematics. I prefer here just to tell some tales. Richard's profound humanity, his generosity, his kindness, his wit, were a source of inspiration to many over the years.

He was fond of telling the following Erdős story. At a conference, he encountered Erdős going the other way down the street. Recognizing Guy, Erdős said, and I quote,"Reechard, you are eenfinitely reech, you buy me cup of coffee." What could I do otherwise but comply, said Richard. Reminding him of this one day after a long afternoon in the Department at Calgary illustrates his occasionally impish sense of humor. I ventured a "Reechard, you are eenfinitely reech, you buy me dinner," to which the reply was "How can I possibly refuse?" He said there was a nice restaurant that would break our walk home (Louise was otherwise engaged that evening); and indeed we had a very pleasant meal. The bill came. Richard looking a little sheepish announced that he did not have his wallet: "I never need cash on campus, so never bring it."

Richard's love of the outdoors was known to all. One delight of visits to Calgary was the Sunday outing to the mountains led by Richard and Louise: I became familiar with many of the peaks around Banff and Canmore. One story comes to mind. An accessible peak in May is Yamnuska, which we had climbed. While we were eating lunch at the summit, a twenty-something couple arrived to share our ridge perch, the girl curiously eyeing our group. "I hope I'm not being rude," she said to us, "but just how *old* are you?" Richard proffered the reply: "Well, our three ages total 200 years..." After a scrunch of brow, "Wow?, she said, "you must all be at least 60, that's amazing!? (For the record, I was 44). "And how old are you?" said Richard. "Well *we* add up to 45." "Aha," said Richard, "you must both be at least 10!"

Richard loved words and wordplay, as can be seen from his writings, in particular the alliteration and assonance to be found throughout *Winning Ways*. He would polish off the *Globe and Mail* cryptic crossword after dinner, with praise or condemnation for particular clues. He loved anagrams. A cryptic crossword clue making the rounds in the UK earlier in 2020 was "Carnivorous bats, cause of pandemic"—how Richard would have delighted in that particular anagram.

On the occasion of Richard's 100th birthday in 2016, I presented him with an ode inspired by John Masefield's "Cargoes," a poem that every English schoolboy of a certain vintage (Richard and myself both included) was made to learn by heart:

> *"Richard Kenneth Guy on his hundredth birthday*
> *Working at his figures round the clock non-stop..."*

He immediately recognized the rhyme and metre, but topped everything off with the following fortuitous fact. Masefield had attended the same public school as Richard, and was guest speaker at the school Speech Day one year, actually presenting Richard with a book prize for his work in mathematics. Richard also told me that on one occasion at lunch in an Oxford College he had been seated next to J. R. R. Tolkien; but he couldn't recall any of the conversation.

In 2016, the year of his hundredth birthday, the Canadian Number Theory Association meeting in Calgary had a special session in honor of Richard. One day of the meeting, I noticed that he had slipped out of the auditorium before the very last talk of the afternoon, so when the proceedings finished I walked round to his office to see if he would be found there (and to make sure he was feeling fine). Indeed he was, at his desk, and marking up a set of galley proofs! It was then that he insisted I should come home with him and he would make me dinner...

Our last mathematical communications were in October of 2019. He had unearthed a rough draft of something we had talked about many years ago, involving Diophantine properties of the geometric configuration of the Morley triangles of a given triangle and a related Apollonian packing. This brought together his lifelong interest in classical Euclidean geometry, and his love of elementary number theory. Sadly, a final manuscript was never produced. His parting email finished: "Try the degenerate specimen, in which the point $(0, -60/13)$ on the circumcircle of the (right-angled) triangle A $(0, -60/13)$, B $(-25/13, 0)$, C $(144/13, 0)$..." and rounded off with "I can't do arithmetic any more!" Then "See you ere long. 'air long' 'air long gone. R."

Richard, requiesce in pace.

Divakar Bhargava. In July 1962, I had the great privilege of joining the prestigious Indian Institute of Technology (IIT), Delhi, as a first year student in the Bachelor's degree engineering program. Professor Richard K. Guy was the head of the mathematics department and taught us curves, solid geometry, and calculus.

Professor Guy had an exceptional sense of humor. Our class started at 8:00 AM sharp each weekday, but once in a while someone would come 5-10 minutes late and would try to sneak in through the door hoping no one would notice. But Professor Guy always noticed. He would greet the student by saying "Good afternoon!" This had the desired effect and the students tried hard not to be late.

Another instance I remember is when he started teaching us integral calculus. He went over the basics of simple integration in one class. In the very next class, he started by thoroughly cleaning the blackboard (this was his practice). Then he wrote on the blackboard simply $\int x dx = x^2/2$, and turned around facing the class and stood there for 30 seconds or so. There was complete silence—neither he nor any student spoke. Then he turned around and cleaned the blackboard some more, including all corners (it was already clean), but not erasing the equation. He again turned towards the class and stood there. Again, complete silence. He had hoped that some student would tell him that the constant of integration was missing on the blackboard, which he had taught us in the previous class. Cleaning the blackboard again was his humorous way of indicating to us that nothing on the blackboard could be construed as the constant of integration. Alas, all of us had forgotten that part of the previous lesson.

Unfortunately, after teaching at IIT for about two years, Professor Guy had a disagreement with IIT's Director and resigned. It was a huge loss for IIT and its students.

Manjul Bhargava. When I was a boy, my uncle would tell me stories about his favorite mathematics teacher at IIT Delhi, a certain "Professor Guy."

As I was growing up, my love for number theory also grew. Like many aspiring number theorists, I came upon the inspirational book *Unsolved Problems in Number Theory*, written by Richard K. Guy, after which I also looked up and read many of his wonderful articles in MAA journals. Eventually, I became a professional mathematician, and regularly attended the Joint Meetings. At the Joint Meetings, I had the good fortune of meeting this Professor Richard K. Guy. I was starstruck, but we also quickly became friends and spent a lot of time together at the Joint meetings. We would have meals, and share stories and talk about our latest and favorite open problems. We attended each other's talks, and sat together for others' talks, often passing notes to each other like school children (sometimes about the talk, sometimes not!).

However, it was not until some years later that I somehow came to the realization that these two Guys/guys mentioned above were in fact the same Guy/guy!! That was one of the most amazing surprises I've ever had. By chance, I had been reading a biography of Professor Guy, which mentioned his time at IIT Delhi. I put two and two together.

The next time I saw Richard, I told him about this fantastic discovery, and he laughed and loved the story. My uncle was also so surprised and excited to learn that I knew his former calculus teacher so well—the same calculus teacher he used to tell me about when I was a boy!

From then on, it was a running joke with Richard. If anyone, say, came up to me after my talk to tell me, "Nice talk," and he was standing there, he would say, "Well, I taught his uncle."

Richard was a true inspiration. He attended so many mathematics meetings, encouraged generations of scientists and mathematicians (including my uncle and myself), gave illuminating, humorous, and entertaining lectures, and was a prolific writer of numerous outstanding books and articles. How did he stay so consistently productive for a century?

At the Joint Meetings, we all learned very quickly one of the reasons why he was so active and doing great mathematics even into his 100s. At the huge convention center where the Joint Meetings took place, the balcony seats in the large auditoriums were often accessible only after going up a several-flight escalator. One time a number of us were standing and going up the endless escalator to reach the balcony entrance for the next plenary lecture. Suddenly, we noticed someone rapidly passing us by on the otherwise completely empty staircase next to the escalator. It was 96-year old Richard putting us all to shame. We concluded that Richard's regular climbing (of stairs, towers, mountains, etc.) had to be one of the secrets behind his long, active, and productive life. A number of us switched to using the staircase rather than the escalator from that day forward!

I visited Richard for a few days in Calgary in celebration of his 99th birthday, which was a true delight. He came daily to get me at my hotel, and we then walked over to his office building. On the first day, when we entered the lobby, he said "My office is on the 5th floor. Are you okay with taking the stairs, or would you prefer to take the elevator?" Knowing what I had already learned from him, I of course chose the stairs. We went up and down the stairs multiple times each day to and from his office, doing mathematics all day with the exception of going out for lunch and taking walks around campus, when we would talk about our numerous common favorite subjects including mountain climbing, music, games, tennis, IITs in the 1960s, and more.

Richard was such a wonderful friend, mentor, and supporter for so many years. The family connection over two generations made our bond deeper. I will miss him profoundly.

Ben Green. I was just getting properly interested in maths when I was around 14 or 15. The internet didn't exist and so it was pretty difficult to find accessible and interesting material. The bookshop at the University of Bristol had a surprisingly good selection of books, so I used to spend quite a bit of time there browsing. Eventually I decided to shell out what seemed like a small fortune (it was at least 60 pounds) for their copy of *UPINT* (2nd Ed), which had been very well thumbed by me by that point. It's very clearly the case that the book strongly influenced my mathematical tastes. In fact my first paper was about bounds for $B_h[g]$ sets (a generalisation of Sidon sets) which are certainly mentioned in the book, and at the start of my PhD I spent quite a while thinking about a number of other questions about Sidon sets and related objects mentioned in the book, though mostly without success. And after that I started thinking about sum-free sets and sets without arithmetic progressions, both of which are topics covered quite extensively in *UPINT*. A couple of years ago I treated myself to the 3rd Edition and I still browse it quite regularly!

It is often remarked, with good reason, that Paul Erdős was a master of selecting problems which are easy to state, but hard to solve and moreover whose study (and, sometimes, eventual solution) reveals deep structure. *UPINT* has a lot of problems like this and indeed it features a number of Erdős problems. However the density of interesting problems, comments and remarks is perhaps even greater than in a typical Erdős problem paper.

The first correspondence I had with Richard was in April 2004, the day after Tao and I posted our preprint on progression of primes. He sent me a very nice email of congratulations. I am glad to have had the opportunity, somewhat later, to meet him in person and to tell him what a positive influence on my development his book had had.[8]

Mike Bennett. When I started University, I majored in History and dabbled in Mathematics. In my second year, I had the good fortune to take a first course in Analysis from Peter Borwein and, the next year, a course in Number Theory from a young postdoc by the name of Karl Dilcher. One of them suggested that I should take a look at a book entitled *Unsolved Problems in Number Theory* (*UPINT*), then in its first edition, by Richard Guy. In short order, I was hooked, switched my major to Mathematics and, eventually, became a Number Theorist. Some years later, when the second edition came out (1994), I rushed out to buy a copy and, to my amazement, found that Richard had even cited a result of mine (rather kindly, since my result could hardly have been said to solve a problem of much interest to anyone else). More recently, I got to know Richard better, mathematically and personally, and served with him, for many years, on the board of the Number Theory Foundation. I was deeply honored to be invited to give the Louise and Richard Guy Lecture at the University of Calgary in 2017.

Richard and I also had in common, with Louise Guy, a keen interest in hiking and mountaineering, an interest they'd shared together since the 1930s, on three continents. In the 1950s, living in Singapore, they climbed the highest point (Lowes Peak) on Mount Kinabalu in Borneo—Louise was likely the second woman ever to reach its summit. After moving to Calgary, Louise and Richard were fixtures in the Alberta mountaineering scene. In 2012, I ran, together with Nils Bruin, Yann Bugeaud, Bjorn Poonen and Samir Siksek, a summer school at the Banff International Research Station (BIRS), in the heart of the Canadian Rocky Mountains. The school was on Contemporary Methods for solving Diophantine equations and aimed at advanced PhD students and Postdoctoral Fellows. Rather surprisingly, Richard asked if he could participate as a student (I suspect that the proximity to the mountains was alluring!). We were happy to have him and he dedicated himself to the task of learning some new mathematics, attending all the lectures and working with mathematicians 70 years his junior on our problem sets. When the summer school ended, Richard and I drove down from Banff to attend the CNTA conference in Lethbridge, Alberta. On the way, as we passed through the town of Canmore and admired the stunning Ha Ling peak, he commented that it had been one of Louise's favorite mountains and that he'd likely never stand on its summit again (being then almost 96). I later learned that the very next month, Richard hiked the notoriously steep trail up Ha Ling and tossed some of Louise's ashes from the summit! Richard and Louise were remarkable people who led remarkable lives. I miss them very much.

[8] *I might add that Richard invited me to give the Guy Lecture in 2016 and suggested a fairly remarkable itinerary of backcountry hiking that he could help me arrange. I wasn't quite clear on whether he was planning to come along as well—he must have been almost 100! Sadly (in one respect, at least) I ended up having to decline the invitation due to the birth of my son. [Green in fact gave the 2021 Guy Lecture.]*

Figure 10. Richard Guy enjoying a talk at the joint annual meeting in 2016.

Other Tributes

We mention some other remembrances of Richard Guy:

Very soon after Guy's passing, the MAA's April 2020 Focus collected together personal thoughts from nine people.

A number of essays about Richard, his teaching and his research, authored by his Calgary colleagues appeared in the September, 2020 issue of the CMS *Notes*, see `https://notes.math.ca/wp-content/uploads/2020/06/September-Notes-1.pdf`.

One of Richard's last projects was a wonderful joint book with Ezra (Bud) Brown, *The Unity of Combinatorics*, published in the MAA's Carus Mathematical Monographs series; see [6] for an affectionate review.

Also notable is the column by Joseph Malkevitch.[9]

Chic Scott has written a fascinating book [20] about Richard and Louise Guy from the perspective of the Alberta mountaineering community, and has shared some of his photos with us. He adds:

> "Although my understanding of mathematics is limited, we connected on an intellectual level (history, poetry, music) and also through our love of mountains. I played chess with Richard at Mount Assiniboine Lodge when he was 100 years old and managed to give him a good game, but he always won."

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Andrew Granville

Carl Pomerance

Credits

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