When he entered the courtroom on October 16, 2017, mathematician Jonathan Mattingly expected to watch the proceedings all morning before being called to give testimony. He would be the fourth or fifth witness for Common Cause, an advocacy group working to end gerrymandering. But the case was on a tight four-day timeline—one day each for the plaintiffs, Common Cause and the League of Women Voters, and two days for the defendants, North Carolina redistricting officials. Right away, the three-judge panel announced that there was no disagreement about the facts, so they wanted to jump straight into expert testimony.

“[The lawyers] came over to me and said, ‘Okay, Mattingly, you’re on in 10 minutes.’ So I was the first witness of the case, having never ever done this before, and having never been in a courtroom in my life,” Mattingly recalls. “I used to watch Perry Mason a lot as a kid, but that was about it.”

Mattingly’s appearance in federal district court for Common Cause v. Rucho was the highest-stakes mathematics lecture of his life to that point. His goal: to convince the judges that North Carolina’s 2016 congressional map was a partisan gerrymander and that mathematics offered the tools to quantify such maps.

Scott Hershberger is the communications and outreach content specialist at the AMS. His email address is slh@ams.org.

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Figure 1. A version of this figure, showing the S-shape that Mattingly refers to as the “signature” of gerrymandering, appeared in the district court decision in Common Cause v. Rucho. Democratic votes are packed into Districts 12, 4, and 1, thus making Republicans more likely to be elected in the remaining districts. The blue line is the actual North Carolina congressional map, the green line is the map drawn during the “Beyond Gerrymandering” event at Duke, the box plots represent Mattingly’s ensemble of more than 24,000 maps, and the yellow line represents the medians of the ensemble.
The chair of Duke University’s mathematics department at the time, Mattingly was one of an emerging group of mathematicians using Markov chain Monte Carlo methods to sample from the enormous space of possible district boundaries available to map-drawers. By revealing the properties of the underlying distribution, he argued, this ensemble method clearly showed that the North Carolina map was a statistical outlier intentionally crafted to benefit the Republican party.

His testimony stretched through the entire morning session and continued after lunch. The judges did have some initial confusions: Was he trying to predict the geography of future votes? “No, I was using historical votes to analyze a map and compare it to […] other possible maps.” How many maps are there? Trillions upon trillions, but “that’s not really the right question,” since the underlying distribution matters more than the raw number of maps. Eventually, Mattingly’s argument seemed to land home. The court ruled that the map drawn by the North Carolina General Assembly was indeed an unconstitutional partisan gerrymander, citing Mattingly’s testimony extensively in its decision.

In the United States, the practice of gerrymandering—deliberately manipulating the boundaries of voting districts to achieve a political end—dates back to the early 19th century. But mathematicians have turned their attention to it only recently. The 2010s saw an explosion of interest and the development of sophisticated techniques to study gerrymandering. By the end of the decade, mathematicians had played prominent roles in all three of the highest-profile partisan gerrymandering court cases: those in Pennsylvania, Wisconsin, and North Carolina.

Now, in the wake of the 2020 census, all 50 states are in the process of drawing, implementing, and in some cases litigating the maps that will influence the balance of power in state legislatures and the US House of Representatives for the next decade. For the first time, mathematicians have been embedded in this redistricting cycle from the get-go.

In their shift from the classroom to the courtroom, from department committee meetings to redistricting commission meetings, mathematicians have contributed much to legal discussions around map-drawing. Staunchly nonpartisan in their work, these scholars are doing everything they can to end gerrymandering—regardless of which party benefits. “The most basic tenet of a democracy is however many seats you had before, if there’s a substantial change in public opinion one way or the other, your seats should change. And that doesn’t happen often under our current maps,” says Mattingly, now a James B. Duke Distinguished Professor. “This lack of responsiveness, especially at critical moments, is anti-democracy.”

Mathematicians Enter the Redistricting Scene

An expert in stochastic systems, Mattingly was among the first mathematicians to study gerrymandering using ensemble methods, starting in late 2013. Mattingly and his student Christy Vaughn (now Vaughn Graves) examined the 2012 congressional elections in North Carolina, in which 4 out of the 13 seats went to Democrats. The pair found that 95% of their randomly sampled redistrictings produced between 6 and 9 Democratic seats with the same vote counts, providing the first rigorous mathematical evidence that the map drawn by the Republican legislature was an intentional gerrymander.

The project was covered by North Carolina Public Radio and The Conversation, but it didn’t make waves in mathematical or public policy circles. Mattingly returned to his previous lines of research. He might never have studied gerrymandering again had it not been for a 2016 policy advocacy program at Duke called “Beyond Gerrymandering.” The event convened 10 retired North Carolina judges as a mock nonpartisan redistricting commission to test how such a commission might function in the state. Mattingly and colleagues analyzed the new, unofficial map using their mathematical framework, finding that it was much more typical in the universe of possibilities than the actual map. This time, the result attracted more attention. Mattingly found himself writing public policy articles, giving talks on gerrymandering, and before long, being asked to write a court declaration for Common Cause v. Rucho.

Figure 2. Jonathan Mattingly, a James B. Duke Distinguished Professor at Duke University.
Quantifying Gerrymandering research group at Duke⁴ has been active ever since. Meanwhile, Moon Duchin, a geometer at Tufts University, began researching gerrymandering through the mathematical lens of compactness in 2016. She entered the public policy realm more directly than Mattingly, finding it helpful that he had paved a path.

"I came to understand that there was a need for more people to do expert work specifically in voting rights litigation [...] on behalf of minority groups, people of color," says Duchin, an associate professor of mathematics. She resolved to build a network of subject-matter experts trained in advocacy. Starting from an informal Metric Geometry and Gerrymandering Group, she initiated a series of workshops and conferences across the country in 2017 and eventually formalized the MGGG Redistricting Lab.⁵ Regardless of the details of different mathematical methods, she says, all reasonable approaches consistently give results that agree qualitatively whether a map is gerrymandered.

Wes Pegden, an associate professor at Carnegie Mellon University, entered the scene with a background in combinatorics and discrete probability. The word “gerrymandering” didn’t even appear in the title of his first paper on the topic [6]. The 2017 paper proved a theorem about Markov chains, using the district boundaries in Pennsylvania as an example application at the end.

The timing of mathematicians’ arrival in the redistricting policy world was fortuitous. A flurry of gerrymandering litigation was underway across the country, and voting rights advocates sought new ways to argue their cases in court. At the heart of these cases was a decades-old question with mathematical undertones: Is there a “judicially manageable standard” that courts could use to determine when a map’s partisan bias renders it unconstitutional? Put another way, is gerrymandering quantifiable?

Mathematicians who study gerrymandering agree that the answer is yes. But convincing the courts of that has been just as tough as proving an elusive theorem.

**Pennsylvania: Expert Testimony and a Scramble for Data**

Mere months after Pegden published his first gerrymandering paper, the League of Women Voters of Pennsylvania (LWV) and a group of Democratic Pennsylvanians filed suit against the Commonwealth of Pennsylvania. The plaintiffs hoped to have the state’s 2011 congressional map invalidated as an unconstitutional partisan gerrymander. The lawsuit alleged that the map, drawn by a majority Republican legislature and signed into law by a Republican governor, was an extreme example of “packing and cracking”: Democratic voters were crammed into five districts and spread thinly among the other 13. Lawyers for the LWV saw Pegden’s paper and asked him to testify in the case as an expert witness. "Certainly, it’s not something I expected to happen," he says.

Despite the unfamiliar venue, years of giving mathematics presentations had prepared Pegden to explain his findings in court. The key difference, he says, was that all of his testimony had to be drawn out through questions asked by the LWV lawyers. Although “there wasn’t a lot of rehearsal, [...] a lot of the preparation was me explaining to the lawyers [the mathematics] I was doing so that they were prepared to ask me questions.” In advance of the December 2017 trial, he also submitted a written report to the court so that the defendants would know what to expect [7].

Trial judge Kevin Brobson, who heard *League of Women Voters v. Commonwealth* in the Pennsylvania Commonwealth Court, is a Republican with a conservative judicial philosophy. Trial judges are elected by a partisan process in Pennsylvania, so Pegden was “pleasantly surprised by how seriously [Brobson] took” his testimony. “There were several times when he interrupted me to ask me a direct question, because he was concerned about whether he was really understanding what I was saying. There was this fun part of the trial where we put up the theorem from our paper as an exhibit.” The judge, wanting to make sure he understood the theorem, asked detailed questions about the notation and terminology.

Although Brobson agreed that the map was gerrymandered in favor of Republicans, he held that it did not violate the state constitution. With the 2018 midterm elections looming, the case then went on a fast track to the Pennsylvania Supreme Court, which promptly ruled that the existing map “clearly, plainly and palpably” violated the state constitution. The Republican-majority General Assembly had mere weeks to come up with a new remedial map.

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⁴https://sites.duke.edu/quantifyinggerrymandering/
⁵https://mggg.org/
That’s when Moon Duchin suddenly got involved in the case. ‘Court cases move at this bizarre pace where it’s like, ‘nothing, nothing, nothing’ and then everything happens really fast,” she says. The Democratic governor, Tom Wolf, tapped her to analyze the General Assembly’s remedial map and write a report within two and a half weeks. To do so rigorously would require detailed data on the precise locations of all the precinct boundaries within voting districts. There was just one problem: “Nobody knows where the precinct boundaries are in most states,” Duchin says.

Duchin found herself in “a wild scramble to get usable data.” The state legislature didn’t know the locations of the precinct boundaries. The secretary of state didn’t know, either. Duchin ended up contacting individual counties to compile the information she needed. “Across Pennsylvania, it’s county officials who get to move these precincts around—and they don’t have to tell anybody when they do so,” she says.

After the frenzy of data-gathering, Duchin compared the Assembly’s remedial map to an ensemble of more than three billion maps with similar or higher levels of compactness and county preservation. There was a less than 0.1% chance that the map was drawn in a non-partisan way, she found [8]. The Pennsylvania Supreme Court rejected the Assembly’s plan, instead implementing a map drawn for the court by Nate Persily, a law professor at Stanford University.

According to Duchin, the court was likely to reject the Assembly’s remedial plan even without her mathematical analysis. Still, she says, it was important to demonstrate quantitatively that the proposed map was just as extreme as the one it was supposed to replace. And when she later analyzed Persily’s map, she found it to be a good one that would be quite responsive to changes in voting patterns.

Pegden’s and Duchin’s work had an immediate impact. In the 2018 elections under the new map, Pennsylvanians elected 9 Republicans and 9 Democrats to the US House of Representatives—a stark contrast to the 13 Republicans and 5 Democrats elected in 2016.

**Wisconsin: Mathematics Reaches the Supreme Court**

As the drama in Pennsylvania unfolded, another high-stakes case about partisan gerrymandering was wending through the courts. *Gill v. Whitford* centered on Wisconsin’s 2011 state house map, drawn by a Republican-majority legislature with the intent to cement Republican control. In November 2016, a district court declared the map unconstitutional—the first time in more than 30 years that a federal court had struck down a redistricting plan due to partisan bias.6

The decision relied heavily on the “efficiency gap,” a measure of the vote share wasted by each party in an election. First described by a political scientist in 2014 [9, 10], the efficiency gap was an early candidate for a quantitative test that courts could use to evaluate claims of gerrymandering.7

When *Gill v. Whitford* made its way to the Supreme Court in 2017, mathematicians got involved through amicus curiae (literally “friend of the court”) briefs. Amicus briefs provide a way for people not directly implicated in a case to share their expertise with the court. “You’re lending a helping hand,” Duchin says. “It’s like, ‘Hey, you guys are brilliant, but you’re not experts in everything. Here, let me tell you about the thing I know really well.”

Eric Lander, a mathematician and geneticist who was the founding director of the Broad Institute, wrote an amicus brief arguing for the adoption of an “extreme outlier standard” based on ensemble sampling. Mattingly and fellow Duke mathematician Gregory Herschlag provided the quantitative analysis for Lander’s brief, showing that Wisconsin’s map was more extreme than 99.4% of comparable alternatives. Duchin read drafts of the brief and offered edits. Meanwhile, Pegden wrote a separate amicus brief with Jowei Chen and Jonathan Rodden as coauthors. Like Lander, Pegden argued that ensemble methods could provide a feasible standard for addressing partisan gerrymandering claims.

Pegden found the process of writing an amicus brief quite different from writing an ordinary mathematical paper. The brief contained no original research, but rather had to explain the consequences of existing research to a nontechnical audience. And the timeline for writing the brief was much shorter than for writing a research article. Still, he says, he was able to adjust fairly easily.

6The only previous such case was *Davis v. Bandemer* in 1982, but that district court decision was reversed at the Supreme Court. Claims of racial gerrymandering have historically met with more success.

In the end, the Supreme Court didn’t even rule on whether the map was constitutional. Instead the June 2018 decision held that the plaintiffs, a dozen Democratic Wisconsinites, had not presented sufficient evidence that they had been personally harmed by the gerrymandering. The Supreme Court sent the case back to district court to give the plaintiffs another chance to make their case. In the meantime, no new map would be drawn for the 2018 elections.

Although the court kicked the can down the road on the fundamental question, the mathematicians saw “how legitimately useful” the amicus briefs were to the justices and clerks, Duchin says. “They referenced Lander’s brief, and they clearly took it as a quite helpful explanation of how [random sampling] could be relevant.” That experience served as a dress rehearsal for the next gerrymandering case before the Supreme Court, the one whose outcome would reverberate throughout the current redistricting cycle.

**North Carolina: The Mathematicians’ Brief and a Frustrating Outcome**

North Carolina’s district boundaries spent most of the 2010s under scrutiny in court. Both the legislative and congressional maps from 2011 were thrown out as racial gerrymanders, and the replacement maps were immediately challenged as partisan gerrymanders. The resulting case about the congressional map, *Common Cause v. Rucho*, was the one that first brought Jonathan Mattingly to the courtroom in 2017. But Mattingly’s successful testimony and the district court’s ruling that the map was unconstitutional did not end the drama in North Carolina.

While *Gill v. Whitford* was underway, the North Carolina case bounced around in limbo. It was eventually combined with a case from Maryland, a state gerrymandered in favor of Democrats, and heard by the Supreme Court in early 2019 as *Rucho v. Common Cause*. Again, mathematicians rallied in support of voting rights. Eric Lander wrote another amicus brief, as did Pegden and two coauthors. Duchin, in turn, spearheaded the “Mathematicians’ Brief,” an amicus brief coauthored by 11 mathematical scientists, two law professors, and three law students. Duchin and her colleagues spent about a month writing the brief, culminating in what she remembers as a “frantic week” to complete the data demonstrations based on the latest scientific advances. “In this world, deadlines are hard deadlines,” Duchin says.

As before, the mathematicians argued that the quantitative methods they had developed offered a judicially manageable standard for evaluating gerrymandering claims. But the court was not convinced. According to Duchin, all nine justices seemed to acknowledge the relevance of the mathematics. Yet even given a baseline and a bell curve, the majority said that determining a cutoff would be too hard. So in a landmark 5–4 decision on June 27, 2019, the Supreme Court ruled that partisan gerrymandering was beyond the reach of federal courts. From then on, only individual states could address gerrymandering issues.

The outcome frustrated mathematicians. “Trial courts around the country had done so much good work in carefully digesting […] the factual evidence brought by expert witnesses,” Pegden says, “and then the Supreme Court re-wrote that story.” On top of that, Chief Justice John Roberts’ majority opinion miscited Pegden’s amicus brief as if it argued for merely protecting traditional districting criteria. “And then [the opinion] says, ‘But the natural political geography can inherently lead to packing […]’, which is our whole point,” Pegden explains. Since cities tend to skew Democratic, while rural regions tend to skew Republican, a truly average map in the ensemble of possibilities generally slightly favors Republicans [12].

“Yes, political geography can do all sorts of things,” Pegden continues. “And so only by comparing a map to other alternative maps of the same state do you really understand when it’s been intentionally gerrymandered—but that’s exactly what these [mathematical] methods do.”

On the other hand, Justice Elena Kagan authored a scathing dissent that accurately reflected the mathematicians’ major points. That Kagan cited the Mathematicians’ Brief “was thrilling to see,” Duchin says.

As a result of the landmark ruling, the case in Wisconsin was dismissed and the state house map was allowed to stand for the 2020 election. The North Carolina congressional map was taken up in a state court, which ordered the map redrawn later in 2019.

That same summer, Mattingly and Pegden testified in a state court about North Carolina’s legislative maps.8 An opposing expert witness, economist Janet Thornton of Berkeley Research Group, tried to use binomial distributions to claim that the differences in seat numbers in the

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8Gregory Herschlag, an assistant research professor of mathematics at Duke, wrote a court declaration in collaboration with Mattingly during the remedial map-drawing phase of the racial gerrymandering case that preceded this partisan gerrymandering case.
mathematicians’ ensembles were not statistically significant. Pegden spent much of his rebuttal explaining why that argument was “bogus.”

Luckily, the three judges paid close attention to Mattingly and Pegden and ultimately threw out the legislative maps due to partisan gerrymandering, too. “Their [ruling] read like a math term paper,” Mattingly says. “They’ve internalized the arguments, and they’ve given them back out.” Despite the defeat at the Supreme Court, the mathematicians had partially succeeded in North Carolina: All of that state’s maps were redrawn more fairly (but still imperfectly, accordingly to Mattingly) by the legislature for the 2020 election.

A New Redistricting Landscape

As the 2020 census neared, the redistricting landscape looked quite different than it had a decade earlier. Gerrymandered maps had been successfully challenged in state courts and replaced with fairer ones—but only after the biased maps were used in multiple elections. The Supreme Court had ruled that federal courts were powerless to address partisan gerrymandering. And the mathematics of analyzing political maps had gone from a novel curiosity to a rich tapestry of research widely cited by reform advocates. With each court appearance, each conversation with an advocacy group, mathematicians refined how they communicated their analyses. “By the time we got to [2019], we were showing movies in court and visualizations that were significantly more compelling,” Mattingly says. For example, one animation his team has used shows how the percentage of Democratic votes statewide determines the number of Democrats elected under a given map in comparison to an ensemble of possible alternatives [13]. Although their efforts have had mixed results in court, mathematicians point out their tangible influence on redistricting policy discussions. For a long time, the prevailing public attitude echoed the pornography test: You know it when you see it. Strangely contorted district shapes? They must be gerrymandered. Nice round districts? They must be fair. Now, according to Mattingly, journalists and lawyers have begun to understand that “you need to have the microscope” of mathematics to know when a map has been rigged.

“The 2012 maps in North Carolina and the 2016 maps were equally gerrymandered—and they look completely different [in terms of] compactness,” Mattingly says. What’s more, “the 2016 maps to the eye look like the 2020 maps, but the 2020 maps are much better maps in terms of fairness.”

Similarly, before the advent of ensemble methods, people often assumed that a fair map must yield proportional representation (for example, if 55% of the votes statewide were Republican, Republicans should earn roughly 55% of the seats). But as mathematicians have emphasized when discussing political geography, this is not an appropriate baseline [12]. In addition to contributing these insights, researchers empower the public conversation by releasing their ensembles of maps publicly so that enterprising journalists and citizen data scientists can run their own analyses.

As public awareness increases, movements to reform redistricting processes state-by-state are gaining steam. Here, too, mathematicians have gotten involved, seeking to prevent gerrymandered maps from being created in the first place. For instance, Wes Pegden was one of two academics to serve on the Pennsylvania Redistricting Reform Commission, created by Governor Tom Wolf in the wake of League of Women Voters v. Commonwealth.

Pegden traveled across the state with the commission to hear directly from Pennsylvania citizens. At the public meetings, Pegden remembers, “one thing that stood out is how much it’s not a partisan issue for people.” For example, some voters lamented that their town had been cleaved into two districts with distant congressional offices, making it harder to talk with their elected representatives. Above all, “they want a process that is basically fair and transparent,” Pegden says.

After nine months of work, the commission released its report in August 2019 [14]. It recommended that in future redistricting cycles, a politically independent commission should submit three maps to the General Assembly (or a bipartisan subset) for final selection. But in 2020, the Assembly shot down the creation of such a commission. An alternative proposal, which would have placed stricter guardrails on the process by which the legislature draws maps, also failed [15]. Other states have successfully begun implementing reforms. In the 2010 cycle, only Arizona and California relied on independent redistricting commissions whose
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members were not political appointees. This time around, Colorado and Michigan also have such commissions [17]. An increasing number of states have adopted a compromise approach where an advisory body draws initial maps that the legislature then puts to a vote. Even in states like North Carolina and Wisconsin where the legislature maintains its grip on the pen, the current cycle features increased transparency overall. But the pandemic-induced delay in the 2020 census data, which was released in August 2021 instead of April, gives states a worryingly small window to draw and approve new maps in time for the 2022 primary elections—a situation that some legislatures are leveraging to ram through their plans with minimal public input.

Scrubbing Proposed Maps in Real Time

As of this writing, in early November 2021, the redistricting cycle is still underway. But already mathematicians are immersed in conversations with advocacy groups, lawmakers, independent commissions, and the public.

Duchin’s interdisciplinary MGGG Redistricting Lab is fielding upwards of 20 emails daily from groups asking for input on various stages of the redistricting process. Before any maps were proposed, she helped mapmakers in Ohio, Michigan, Alaska, and New Mexico collect the public’s suggested district boundaries through Districtr, an open-source map-drawing tool that her team launched in 2018. The independent commission in Michigan also used Districtr to gather over 1,200 public submissions about “communities of interest,” an amorphous criterion that Duchin’s interdisciplinary MGGG Redistricting Lab is showing how the commission’s proposed plans (colored dots) compare to an ensemble of alternative maps under proportionality, “ Duchin says. “With mathematicians in the room, you get the gray stuff, which is 100,000 alternatives that are all population-balanced, compact, contiguous, and […] preserve counties when possible.”

Figure 7. Moon Duchin presented this graphic to Michigan’s independent redistricting commission in October 2021, showing how the commission’s proposed plans (colored dots) compare to an ensemble of alternative maps under several historical voting patterns. “Without mathematicians in the room, they’re just comparing these dots to each other and to the blue line [proportionality],” Duchin says. “With mathematicians in the room, you get the gray stuff, which is 100,000 alternatives that are all population-balanced, compact, contiguous, and […] preserve counties when possible.”

Scrutinizing Proposed Maps in Real Time

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9 Washington, Idaho, Montana, and Alaska also use non-politician commissions, but most of their members are chosen by the legislature [16].

10 In most of these states, including Wisconsin but not North Carolina, the governor can veto the legislature’s proposed maps [18].

11 https://districtr.org/
A list of additional references (court documents and news articles) is available at https://www.ams.org/about-us/scotthershberger.

References


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