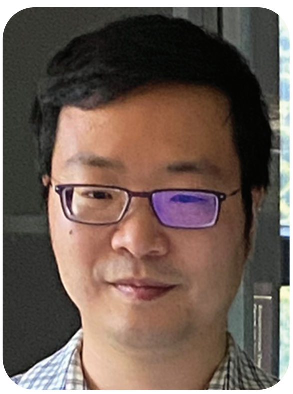
Chevalley Prize in Lie Theory

The Chevalley Prize was established in 2014 by George Lusztig to honor Claude Chevalley (1909–1984). It is awarded for notable work in Lie Theory published during the preceding six years; a recipient should be at most twenty-five years past the PhD. The prize is awarded in even-numbered years, without restriction on society membership, citizenship, or venue of publication.



Xuhua He

Citation

Xuhua He

The 2022 Chevalley Prize is awarded to Xuhua He for substantial contributions to the representation theory of p-adic groups and the structure theory of affine Deligne-Lusztig varieties.

This award is based on three papers of He: "Kottwitz-Rapoport conjecture on unions of affine Deligne-Lusztig varieties," published in the *Annales Scientifiques de l' École Normale Supérieure*, "Cocenters of p-adic groups, I: Newton decomposition," published in *Forum Math. Pi*, and "Cocenters of p-adic groups, III: Elliptic cocenter and rigid cocenter" (joint with Dan Ciubotaru), published in *Peking Math Journal*. In these papers, Xuhua He has given several striking applications of a breakthrough result (established with Sian Nie in 2014) on the cocentre of an affine Hecke algebra to classical problems in representation theory and arithmetic geometry.

Fundamental to representation theory is the notion of a character, given by the trace of a group element acting in a representation. We may also view the character as a function on the group algebra, in which case it factors over the cocentre of the group algebra. (Recall that the cocentre of an algebra is the quotient of A by the subspace of commutators ab–ba for all elements a, b in A. A trace vanishes on commutators, and hence factors over the cocentre.) In the case of the group algebra, conjugate elements of the group become equal in the cocentre, and we recover the character table.

Iwahori-Hecke algebras are extremely important algebras in Lie theory. They first arose in the work of Iwahori, when studying the algebra of endomorphisms of certain induced modules (principal series modules) for finite reductive groups. They are very similar to group algebras of Weyl groups, but one of the relations (the fact that a simple reection squares to 1) is deformed in an interesting way. Motivated by the representation theory of finite reductive groups, it is natural to ask whether one can develop a character theory for Hecke algebras. This was pursued by several authors in the 1990s, most notably Geck and Pfeiffer. A crucial observation is that the cocentre has a basis given by the images of minimal length representatives in their conjugacy class (with respect to the natural length function coming from the Coxeter group structure). The proof of this fundamental fact was case by case, and used computer for exceptional types.

In a breakthrough result from 2014, He (together with Sian Nie, and outside of the time relevant to this prize) established a conceptual proof of Geck and Pfeiffer's result in the finite case, and extended it to affine Weyl groups. Thus, for the first time we have a good notion of the "character table" of an affine Hecke algebra. Just as Iwahori-Hecke algebras for Weyl groups are fundamental to the representation theory of finite groups of Lie type, affine Hecke algebras are fundamental to the representation theory of p-adic groups, and capture its tamely-ramified representations, meaning irreducible and with a vector that is fixed under the Iwahori subgroup.

In "Cocenters of p-adic groups, I: Newton decomposition," He makes a systematic study of the "big Hecke algebra" of a reductive group (which sees all smooth, admissible representations), and establishes a remarkable description of its cocentre, indexed by "Newton points." As an application he is able to give a very short new proof of Howe's conjecture on invariant distributions, which was first established by Clozel (1989), and by Barbasch and Moy (2000). In "Cocenters of p-adic groups, III: Elliptic cocenter and rigid cocenter" (with Ciubotaru), the authors establish the trace Paley-Wiener theorem for mod-l representations of p-adic groups, extending the work of Bernstein-Deligne-Kazhdan (1986) which applied for complex representations.

In a separate line of work, Xuhua He has also applied his knowledge of the cocentre to deduce important results concerning the structure of affine Deligne-Lusztig varieties. Affine Deligne-Lusztig varieties were introduced by Rapoport and are fundamental to the representation theory of p-adic groups, and the theory of Shimura varieties. For example, in "Kottwitz-Rapoport conjecture on unions of affine Deligne-Lusztig varieties," He solved a conjecture of Kottwitz and Rapoport on the nonemptiness of these varieties. In joint work with Zhou ("On the connected components of affine Deligne-Lusztig varieties" which appeared in *Duke Mathematical Journal*), He describes the connected components of affine Deligne-Lusztig varieties. Combined with results of Zhou, this settles a 1987 conjecture of Langlands and Rapoport on the rational points of Shimura varieties for residually split groups.

In summary, He's recent work has had a major impact on the applications of Lie theory to number theory and beyond.

Biographical Note

Xuhua He is the Choh-Ming Professor of Mathematics at the Chinese University of Hong Kong. His research interests include arithmetic geometry, algebraic groups, and representation theory. He received his bachelor's degree in mathematics from Peking University in 2001 and a PhD degree from MIT in 2005 under the supervision of George Lusztig. He worked as a member at the Institute for Advanced Study during 2005–2006 and as Simons Instructor at Stony Brook University during 2006–2008. He worked at the Hong Kong University of Science and Technology during 2008–2014 as an assistant professor and associate professor, and then moved to the University of Maryland during 2014–2019 as a full professor of mathematics before joining CUHK in 2019. He received the Morningside Gold Medal of Mathematics in 2013, the Xplorer Prize in 2020, and was an invited sectional speaker of the International Congress of Mathematicians in 2018.

Response from Xuhua He

It is a great honor to be awarded the 2022 Chevalley Prize in Lie Theory. First of all, I would like to thank my family, my advisors, my colleagues, and all of my friends for their long-time support and encouragement. I would also like to take this opportunity to show my appreciation to all the people who have helped me in my career and an opportunity to say a bit about my career in Lie Theory.

I started my mathematical career as a graduate student under the supervision of George Lusztig. It was very fortunate for me to have George as an advisor, who shaped my way of thinking about Lie Theory. What attracts me most in Lie Theory is the intriguing connections between the reductive groups and their Weyl groups.

A fundamental work in Lie Theory is the Bruhat-Chevalley decomposition: a connected reductive algebraic group is a union of the Bruhat cells, indexed by the Weyl group. Many questions in Lie Theory are related to understanding how the Bruhat-Chevalley decomposition is compatible with the conjugation action. For example, the Deligne-Lusztig varieties describe the interaction of the Bruhat cells with the Frobenius-twisted conjugation action in a reductive group; and the intersection of a Bruhat cell with a conjugacy class gives an essential ingredient in Lusztig's theory of character sheaves.

A major part of my research in the last ten years has been to understand the interaction of the Bruhat-Chevalley decomposition with the conjugation action for loop groups and p-adic groups. I am very glad to make progress on this problem, and in turn, make new developments and discoveries in the study of cocenters of Hecke algebras of p-adic groups and in the study of affine Deligne-Lusztig varieties.

Lie Theory is a fascinating field, and I am incredibly proud and honored to contribute to it. I am very grateful to my colleagues and collaborators; four of them, in particular, I would like to mention by name: Dan Ciubotaru for the long-term collaboration on the representations of p-adic groups, Sian Nie for his insight on the combinatorics of Weyl groups, Michael Rapoport for teaching me the Shimura varieties, and Geordie Williamson for explaining to me his beautiful work with Elias on the Soergel bimodules. Of course, the list can go on and on for a long time. It is great to work with so many brilliant mathematicians. And I would like to thank them all for what they have taught me and for so many hours working together—struggling, failing, and a little better understanding of Lie Theory.

Credits

Photo of Xuhua He is courtesy of Xuhua He.