

An arXiv for Lecture Notes

Instructors and authors share how they have benefited from AMS Open Math Notes

Scott Hershberger

After earning his bachelor's degree in mathematics and economics in 1993, Todd Fasanella built a career in finance. He used mathematics to predict loan prepayments and defaults, as well as to invest in hotels. But over the years, he dreamed of returning to school to earn a PhD in mathematics.

To prepare, Fasanella began self-studying graduate-level topics using Open Math Notes,¹ the American Mathematical Society's free repository for lecture notes spanning all levels of post-secondary mathematics. "It's the equivalent of bumming notes off of someone when you miss class," he says.

Launched in 2016, Open Math Notes (OMN) is a centralized source for notes on standard undergraduate courses, topics courses, and graduate courses. Screened by an advisory board, these draft works range from textbooks in progress to informal lecture notes from unique courses. OMN also contains classroom-tested, thoroughly vetted notes published in the *Journal of Inquiry-Based Learning in Mathematics*. OMN contributes to the growing Open Educational Resources movement by helping lower financial barriers to university-level mathematics around the world.

OMN was the brainchild of Karen Vogtmann, a professor emeritus of mathematics at Cornell University, who served for a decade on the board of arXiv. Just as arXiv provides a forum to post and download research papers in progress, Vogtmann envisioned "a place for people to put their notes—whether or not they were planning on publishing them—that would be available to everybody." Readers can

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¹<https://www.ams.org/open-math-notes>

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Karen Vogtmann, the founder of Open Math Notes and the chair of its advisory board.

send feedback to authors, and authors can upload revisions of their notes.

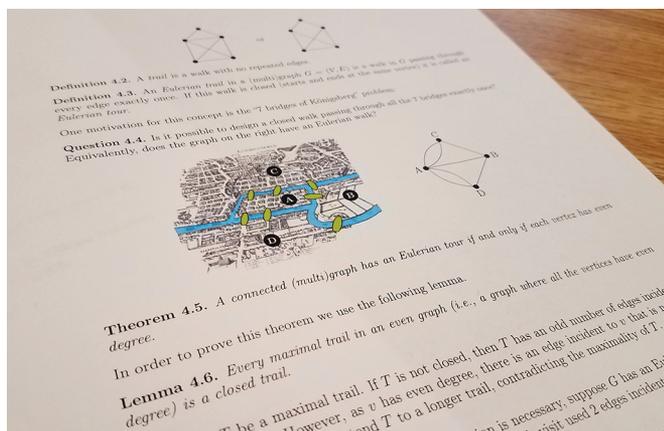
Fasanella estimates that he has referenced some two dozen sets of notes on OMN covering probability, analysis, ergodic theory, dynamical systems, Lie groups, and other topics. By the time Fasanella enters a PhD program, he expects to have learned the content of the entire first year. Several of the notes he has studied were written by Michael Taylor, the William R. Kenan Professor of Mathematics at the University of North Carolina and a prolific contributor to OMN.

Taylor has written course notes and textbooks since 1972. "Whenever I teach a course, the available books do not suit me, and so I take matters into my own hands," he says. Taylor has posted eight sets of notes on OMN, four of which went on to become textbooks published by the AMS. He has not decided if he wants to publish the others

as textbooks—but even if Taylor doesn't, his notes will still be useful to Fasanella and others.

"Having libraries like this available is of immense value," Taylor says. In the past, lecture notes rarely circulated beyond the walls of an instructor's classroom. Most were simply not available to the public, while others could only be found on a mathematician's personal website. Now, Open Math Notes provides a way for instructors and students alike to discover notes that they might not have otherwise been able to access.

At the Indian Statistical Institute, Yogeshwaran Dhandapani taught a master's course from Dmitry Panchenko's Lecture Notes on Probability Theory in fall 2020. "The material was quite perfect for my class," says Dhandapani, an associate professor. This semester, he is teaching a course on graph theory using notes by Benny Sudakov.



A page from Benny Sudakov's notes on graph theory, posted on OMN in 2019.

In Spain, Jerónimo Alaminos Prats, a professor at the University of Granada, found new exercises to assign his students by browsing the calculus and real analysis notes on OMN. Meanwhile in Romania, Ion Mihai teaches differential geometry courses at the University of Bucharest using notes by Theodore Shifrin and Sigmundur Gudmundsson. Mihai uses Shifrin's notes to teach computer science students, while Gudmundsson's match the needs of his pure mathematics students.

Dhandapani especially appreciates OMN's feature that allows readers to "follow" certain notes and receive emails when authors post revisions. Always in search of new resources for teaching, he takes a look at the site every time more notes on probability theory are added.

For some authors, OMN serves as a stepping stone in the lengthy process of writing and publishing a textbook. Roger Plymen was long fascinated by the Skewes number, the smallest number for which the prime number theorem undercounts the number of primes. An emeritus professor at the University of Manchester, Plymen wrote a rough manuscript on the topic but didn't know what to do next.



Jerónimo Alaminos Prats, a professor in Spain who has used Open Math Notes.

Then, at the 2017 Joint Mathematics Meetings in Atlanta, he met Eriko Hironaka, a consultant for AMS Book Acquisitions and the main point of contact for OMN authors. Hironaka suggested that he post the next draft on OMN.

That goal kept Plymen occupied for a year. Preparing the notes for OMN clarified how to structure the final book and motivated him to include additional historical sources. Two years after appearing on OMN, *The Great Prime Number Race* was published in the AMS Student Mathematical Library. "If I had never written the Open Math Notes in the first place, I would never have written the book," Plymen says.

OMN's informal format often allows the personal style of authors to shine. In the 1990s, Jane Gilman, John Conway, William Thurston, and Peter Doyle team-taught "Geometry and the Imagination," an innovative Princeton course with an emphasis on hands-on activities and discussions. Years later, two publishers approached Gilman about reworking the notes for formal publication. She wasn't interested: "Part of their charm is their unfinished nature." Yet thanks to OMN, the next generation of mathematicians will still be able to learn from them, she says.

When the COVID-19 pandemic prompted mathematics instructors to seek out more online resources, many turned to Open Math Notes. The repository saw a 29% increase in quarterly downloads from the first to second quarter of 2020. Toward the end of the year, more than four dozen sets of notes for inquiry-based learning appeared through a partnership with the *Journal of Inquiry-Based Learning in Mathematics*² (JIBLM). Founded by Ted Mahavier and David Clark in 2007, JIBLM contains notes on calculus, real analysis, topology, and more, all geared toward instructors trying out inquiry-based learning for the first time.

"The thought that you could learn how to be an effective educator in the classroom using inquiry-based learning strategies at the same time that you are going to write your own course materials is the equivalent of saying, 'I will

²<https://jiblm.org/index.php>

OPEN MATH NOTES *by the numbers*

137,000
TOTAL DOWNLOADS

Most-downloaded

10,000 *Linear algebra course notes*
DOWNLOADS by Terence Tao

200 SETS of
NOTES

52 NOTES FROM THE
*Journal of Inquiry-Based
Learning in Mathematics*

build my own boat and get in and sail across the Atlantic," Mahavier says. "You would do better to buy a well-tested boat [...] and try to sail that one across." By making past and future JIBLM notes available on OMN, Mahavier hopes to encourage more instructors to adopt inquiry-based learning strategies.

In just its first five years, Open Math Notes has already demonstrated the value of an arXiv-like site for lecture notes on topics as ubiquitous as calculus and as specialized as the Skewes number. Founder Vogtmann especially encourages authors to post notes that introduce current research topics with a ground-up approach. As more people publish notes on the site, its reach will continue to grow among mathematicians and students—both current and future.

A version of this article that includes direct links to all the sets of notes mentioned is available at https://www.ams.org/news?news_id=6871.



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Credits

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