The Linear Algebra Curriculum Study Group (LACSG 2.0) Recommendations

Sepideh Stewart, Sheldon Axler, Robert Beezer, Eugene Boman, Minerva Catral, Guershon Harel, Judith McDonald, David Strong, and Megan Wawro

Introduction

Linear algebra is a vital topic in Science, Technology, Engineering, and Mathematics (STEM) fields. It provides the computational and theoretical foundation for a wide variety of classical areas such as engineering, physics, biology, and economics. In addition, a foundational understanding of linear algebra has led to the explosion of career opportunities in data science, signal processing, cryptography, computer science, and quantum computing. Research into the teaching of linear algebra continues to provide new insight into better ways of sharing linear algebraic knowledge and ways of thinking with students. Technology has become accessible and affordable for most students, and as such, should be used as an asset in teaching and learning. Modern applications have grown and can motivate learning and insights into the workings of linear algebraic ideas. Hence, the teaching of linear algebra has become increasingly important in this quantitatively driven world.

In 1993, Carlson, Johnson, Lay, and Porter [1] initiated the first Linear Algebra Curriculum Study Group (LACSG) to examine the role of the first course in linear algebra in the mathematics curriculum and offered numerous recommendations intended to better meet the needs of students and client disciplines at that time. The recommendations in [1] were a major step in raising awareness and significantly impacted linear algebra education both nationally and internationally. For example, special sessions on the learning and teaching of linear algebra were added to the programs of national and international meetings (e.g., the Joint Mathematics Meetings, International Linear Algebra Society meetings), and several important introductory linear algebra textbooks adopted the LACSG recommendations (e.g., [2]). Many of these recommendations are still relevant today; however, the field of linear algebra has continued to grow since those recommendations were published in 1993. New enabling technologies have been

Sepideh Stewart is an associate professor in the Mathematics Department at the University of Oklahoma. Her email address is sepidehstewart@ou.edu.

Sheldon Axler is a professor emeritus in the Mathematics Department at San Francisco State University. His email address is axler@sfsu.edu.

Robert Beezer is a professor of mathematics and computer science at University of Puget Sound. His email address is beezer@ups.edu.

Eugene Boman is an associate professor of mathematics at Penn State University. His email address is ecb5@psu.edu.

Minerva Catral is an associate professor of mathematics at Xavier University. Her email address is catralm@xavier.edu.

Guershon Harel is a professor of mathematics at University of California, San Diego. His email address is gharel@ucsd.edu.

Judith McDonald is a professor of mathematics at Washington State University. Her email address is jmcdonald1@wsu.edu.

David Strong is a professor of mathematics at Pepperdine University. His email address is David.Strong@pepperdine.edu.

Megan Wawro is an associate professor in the Mathematics Department at Virginia Tech. Her email address is mwawro@vt.edu.

Communicated by Notices *Associate Editor William McCallum.*

For permission to reprint this article, please contact: reprint-permission@ams.org.

DOI: https://dx.doi.org/10.1090/noti2479

invented and are widely available, older technologies have improved or become obsolete, research in linear algebra has blossomed, and the application areas of linear algebra have continued to expand.

In 2018, funded by the National Science Foundation (DUE-1822247), experts from around the nation were invited to a 2-day workshop to re-evaluate the important work begun in [1], nearly thirty years ago. As a result, the LACSG 2.0 (the authors of this paper) was formed to revisit and update the recommendations in [1]. The new recommendations suggest teaching linear algebra sooner in the curriculum, removing calculus as a prerequisite, considering the needs of industry, being aware of the latest research in linear algebra education, taking advantage of technology in teaching, motivating concepts with applications, and developing second courses in linear algebra. The purpose of this paper is to communicate the LACSG 2.0 recommendations to mathematics program leaders and instructors of linear algebra for their consideration.

Advice from Industry

In the IBM publication *The Quant Crunch: How Demand for Data Science Skills is Disrupting the Job Market*, the authors challenge higher education to respond to the "demand for a new breed of professionals skilled in data analytics, machine learning, and artificial intelligence" [3, p. 3]. To address the skills gap created by an explosion of careers involving analysis of data, a strong foundation in linear algebra is paramount. Students preparing for jobs in Silicon Valley recognize that a proof-based second course in linear algebra is highly valued by potential employers, as evidenced, for example, by the 700 plus students a year registering for such classes at UC Berkeley. The LACSG 2.0 team met with industry representatives from Amazon, Google, Juniper, Schweitzer, Nordstrom, Boeing, and Microsoft from the Seattle-Portland area in the Fall of 2019 and heard from this group that students need a thorough grounding in both abstract and numerical linear algebra.

Our industry experts suggested giving students more pathways into diverse mathematical ideas, including linear algebra. They pointed out that decisions about what and how to teach mathematics need to focus more on how ideas are currently used in industry and not necessarily on how they were developed. For example, many students are still taught Cramer's rule but not LU decompositions, even though the far more efficient LU decomposition is used extensively to solve systems of equations, and Cramer's Rule almost never is. The experts we spoke with saw the ability to experiment and play with data and ideas as an essential skill. Students need to be able to formulate their own observations and explore their validity—and linear algebra provides a beautiful framework for this type of mathematical exploration. They suggested that discussion of data structures is also important. How do we manage the trade-off between run time and storage costs? Scalability is an important consideration. Can an algorithm that works on a small scale be implemented on a larger scale? Linear algebraic ideas, such as the notion of linear independence, help us avoid redundancy. Eigentheory is much richer than the "fixed direction" of a linear transformation and has a wide variety of uses in practice such as compressing data or identifying the essential components of the system under study. Matrix multiplication and factorizations are also used to manipulate and understand data and signals. The critical thinking skills developed in proof writing help individuals in industry assess the validity of algorithms and ideas, work with abstract variables, and understand what different components in a system are contributing to its behavior. The advice we received from our industry experts certainly supports a review of the mathematical curriculum and where linear algebra fits in the broader scheme.

Although calculus is currently the mathematical starting point for almost all STEM students, much of the mathematics currently used in industry has its foundation in linear algebra. Offering linear algebra to incoming freshmen, with a second course offered to juniors or seniors, would benefit students who will use linear algebraic ideas in their careers and elsewhere in their studies. An introductory linear algebra course provides foundational material for many mathematics courses, as well as programs addressing *The Quant Crunch*. Although many universities use calculus as a prerequisite for linear algebra, this is rarely based on any specific mathematical need; most often, it is an expression of a desire for mathematical maturity or the use of calculus to filter enrollment for other mathematics courses. We suggest that linear algebra serves to foster mathematical maturity at least as well as calculus. Indeed, in some ways, it is a better vehicle because it starts with simple arithmetic (in the form of row reduction) and builds to very high-level abstractions (high-dimensional vector spaces, null spaces, linear independence, coordinate free linear operators, etc.). Hence it provides an appropriate foundation for courses such as discrete math, graph theory, combinatorics, and many areas of data analytics. Reviewing the list of topics and level of sophistication asked for by our industry experts, LACSG 2.0 has observed that second (and even third) courses in linear algebra are needed. Quantitative understanding is gaining importance in many disciplines outside of mathematics. All of them will benefit from additional theoretical and numerical understanding of linear algebra.

Research-based Insights on Teaching and Learning in Linear Algebra

An increasing number of research studies regarding teaching and learning in linear algebra have been conducted since the publication of the recommendations in [1]. These studies range from fine-grained investigations of students' cognition to instructional interventions and curriculum development based on research about how students learn linear algebraic concepts (e.g., [4]; [5]) and have helpful implications for teaching linear algebra concepts at the undergraduate level. As noted by Schoenfeld [6], the pure purpose of mathematics education research as a scientific discipline is "to understand the nature of mathematical thinking, teaching, and learning," and the applied purpose is "to use such understandings to improve mathematics instruction" (p. 641). In this section, we highlight a small subset of this research corpus to encourage the reader to explore how research findings in this area can be leveraged towards improving the educational experiences and successes of linear algebra students in the 21st century.

The early work on teaching linear algebra raised awareness of the complexity involved in students developing a conceptual understanding of elementary notions. For example, Dorier, Robert, Robinet, and Rogalski [7] noted that in France, "The general attitude of teachers consists more often of a compromise: there is less and less emphasis on the most formal part of the teaching (especially at the beginning), and most of the evaluation deals with the algorithmic tasks connected with the reduction of matrices of linear operators" (p. 28). As a result, students may develop computational skills but lack conceptual understandings of core elementary linear algebra notions (e.g., linear dependence and subspaces). One rather straightforward recommendation, therefore, is to not teach purely algorithmically. Furthermore, Dorier and colleagues found that students' difficulty with the formalism of vector space theory was not solely an issue with formalism in general but with its specific role in linear algebra. They recommended the consideration of what they call the meta-lever, in which instructors consider what their students know mathematically to use as a lever towards deepening not only their mathematical understanding but also their notion of the nature of mathematics itself. Other early studies indicated that instructors may inadvertently blend various contexts, modes of description, and notation and should be aware that it is not trivial for some students to recognize the same concept in different contexts or notations (e.g., [8]). For example, instructors might seamlessly treat a vector in $\mathbf{R}^2$ as both a point and an arrow (two different geometric modes of description) or treat a vector as an n-tuple and as an element of a vector space denoted with a symbol such as $\mathbf{v}$ (algebraic mode and abstract mode, respectively).

Since the publication of these studies, many researchers have examined a wide variety of topics in linear algebra. Stewart, Andrews-Larson, and Zandieh [9] identified more than 50 empirical studies published within a selected list of peer-reviewed mathematics education journals over the period of 2009–2017. Although many of these contemporary research papers investigate and help us understand students' difficulties with learning linear algebra, many others focus on characterizing the ways students make sense of concepts and how their mathematical insights can be leveraged in helping them grow in their linear algebra reasoning.

The role of graphical representations of $\mathbf{R}^2$ and $\mathbf{R}^3$ in teaching and learning linear algebra has also been the focus of investigation for many researchers. For example, recent studies illustrate that using dynamic geometry software to introduce and explore linear algebra concepts can result in a robust understanding of those concepts at the introductory level (e.g., [10]). Focused on vector spaces beyond $\mathbf{R}^2$ and $\mathbf{R}^3$, other researchers caution that an overreliance on geometry may restrict students' ability to abstract linear algebraic concepts and ideas (e.g., [11]). Taken together, the consideration of the role of geometry in linear algebra is a good example of how the course type and population (e.g., introductory versus advanced course, mathematics versus other STEM majors) are factors in considering which research recommendations are most appropriate for a given course.

Research on teaching and learning in linear algebra is a growing area in mathematics education. We recommend that learning about research findings can help linear algebra instructors be mindful of students' thought processes and needs and possible challenges with certain concepts. For example, the notion of solution being the set of all nonzero vectors $\mathbf{x}$ that satisfy a matrix equation is different than what students previously encountered, such as solving for the specific real-valued x that satisfies the equation $cx = d$; thus, solution strategies that were previously successful may not apply in linear algebra. This is apparent when students, for instance, try to "cancel" the vector $\mathbf{v}$ from both sides of the equation when they encounter $A\mathbf{v}= 2\mathbf{v}$ [12]. One teaching suggestion is to emphasize that in the eigen equation, there are multiple objects (matrix, vector, and scalar) to coordinate in different ways (matrix-vector multiplication, scalar-vector multiplication), yet the result of these differing operations is the same object, a vector [6]; this may help combat the understandable instinct to "cancel the vector" from the eigen equation by conceptually reasoning that a matrix cannot equal an eigenvalue.

In conclusion, the various teaching implications reported from these studies, as well as resources and curriculum materials supported by research (e.g., [5]; [13]) can benefit instructors to reflect on their own teaching practices as they prepare students for future job markets. A more extensive list of research papers on the teaching and learning of linear algebra can be found in [9]. Certainly, there is still much to investigate regarding best practices for teaching linear algebra in the 21st century in ways that promote deep understanding for all students. For example, there is much to learn about the role of technology and visualization, as well as how to meet the needs of students in other disciplines such as engineering, physics, or computer science.

First Courses in Linear Algebra

While we are conscious of students' mathematical maturity and preparedness for college courses, we believe that linear algebra can be successfully learned by students with proficiency in high school algebra. Hence, we recommend that linear algebra could be offered profitably without a calculus prerequisite. Burazin, Jungić, and Lovrić [14] stated "We have heard strong views suggesting that perhaps the time has come to abandon calculus and restructure first-year mathematics around a different paradigm, such as mathematical modeling, problem-solving, or mathematical thinking" (p. 67). Indeed, restructuring first year mathematics to give students early exposure to linear algebra should be a high priority for mathematics departments. The models from some countries around the world (e.g., New Zealand and Ireland) show that both calculus and linear algebra are offered in the first year and often in the first semester. We also recommend that future high school mathematics teachers should take at least one linear algebra course.

We recommend that the following topics (see Table 1) should be considered for inclusion in a first course in linear algebra.

Table 1. Topics appropriate for a first course.

- Systems of linear equations.
- Properties of $\mathbf{R}^n$. Linear independence, span, bases, and dimension.
- Matrix algebra. Column space, row space, null space.
- Linear maps. Matrices of a linear map with respect to bases; the advantages of a change of basis that leads to a simplified matrix and simplified description of a linear map.
- Matrix multiplication and composition of linear maps, with motivation and applications.
- Invertible matrices and invertible linear maps.
- Eigenvalues and eigenvectors.
- Determinant of a matrix as the area/volume scaling factor of the linear map described by the matrix.
- The dot product in $\mathbf{R}^n$. Orthogonality, orthonormal bases, Gram-Schmidt process, least squares.
- Symmetric matrices and orthogonal diagonalization. Singular value decomposition.
- Orthogonal and positive definite matrices.

Second Courses in Linear Algebra

A single semester course cannot possibly cover all the important topics in linear algebra. We recommend that students planning to pursue a graduate degree in mathematics or a career in any type of quantitative analysis take at least two courses in linear algebra. Students majoring in computer science, physics, economics, and other subjects using mathematical models may also benefit greatly from two or more courses in linear algebra.

A second course in linear algebra might focus far more on vector spaces, linear maps, and proofs than a typical first course that focuses on $\mathbf{R}^n$, matrices, and computations. The context for the second course should include complex vector spaces (and possibly vector spaces over an arbitrary field) as well as real vector spaces. The following topics (see Table 2) can be included and explored in depth in a second course aimed at math majors and other students who want to learn the powerful tools of linear algebra.

Table 2. Topics appropriate for a second course.

- Abstract vector spaces. Subspaces, linear independence, span, bases, and dimension in the context of abstract vector spaces.
- Linear maps between vector spaces; the null space and range of a linear map. The matrix of a linear map with respect to a basis; rank and nullity of a matrix.
- Invertibility of matrices and linear maps.
- Eigenvalues, eigenvectors, and diagonalization.
- Matrix factorizations.
- Inner product spaces. Orthogonality and orthonormal bases. Gram-Schmidt process for constructing orthonormal bases.
- Orthogonal projections and best approximations. Upper-triangular matrices with respect to an orthonormal basis (Schur's theorem).
- Finite-dimensional spectral theorem. Singular values and singular value decomposition. Low-rank approximation of linear maps. Pseudo-inverses.
- Positive operators. Unitary operators.
- Geometric multiplicity as dim null $(T - \lambda I)$; algebraic multiplicity as dim null $(T - \lambda I)^n$. Generalized eigenvectors. Jordan form.
- Minimal polynomial, characteristic polynomial, and diagonalizability.
- Determinant and trace as product and sum of eigenvalues.
- Tensors and multilinear algebra.

Other second courses may want to focus more on numerical linear algebra. In addition to some of the topics above, the following topics (see Table 3) can be included in a course with an emphasis on numerical linear algebra.

Table 3. A list of topics in a second course focused on numerical linear algebra.

- Matrix decompositions: LU, QR, Cholesky, Schur.
- Computation of singular value decomposition.
- Computation of eigenvalues and eigenvectors.
- Implementation of Gram-Schmidt process.
- Error analysis.
- Solutions of large systems of linear equations. Techniques that work with sparse matrices.
- Iterative and randomized methods in numerical linear algebra.

Undergraduate Research

Students who have taken a second course in linear algebra should be encouraged to consider following up with a research project. Undergraduate research is a high impact educational practice, and many institutions now consider such experiences invaluable for student engagement and retention. Linear algebra provides numerous research topics and problems accessible for undergraduates. For example, the natural correspondence between matrices and graphs gives a range of possible research questions that students can investigate; other topics could include problems on theoretical or numerical aspects of linear algebra. These projects are best done in collaboration. Students can explore small order examples and formulate conjectures for arbitrary order using numerical simulations and then work together on establishing proofs of their conjectures or constructing counterexamples.

Technology

Interactive student experiments, admittedly in low dimensions, can provide insight. For example, in a first course, input a 2×2 matrix and then drag the head of a unit vector around a circle until the vector and the matrix-vector product align. Then you have an eigenvector of the matrix and the associated eigenvalue. In a second course, have the three matrices of the singular value decomposition operate successively on a unit sphere and the three vectors of the standard basis. The matrices' effects, as a rotation, scaling, and rotation, are then very evident. Programs with extensive symbolic, graphical, numerical, and exact routines include GeoGebra, Matlab, Mathematica, Maple, and Sage. These allow students to actively explore and discover patterns, make conjectures, and determine or verify results, quickly and correctly. For example, it takes only basic numerical experimentation to discover if the determinant function is additive and/or multiplicative. Computations such as the factored characteristic polynomial and factored minimal polynomial of a 10×10 matrix become straightforward. Instructors can compute multiple examples in class, modeling the same sorts of explorations. Using software that performs exact rational arithmetic can allow a course to discuss when exact results are possible and when exact results are not possible. Conversely, a course with an applied focus can address topics from numerical linear algebra in the context of a particular software.

Applications

Linear algebraic ideas are fundamental in understanding and solving a plethora of real-world problems, particularly in our increasingly digital world. Additionally, the majority of students populating linear algebra courses (especially first semester courses) are from engineering, science, business, etc., rather than being primarily from mathematics as they were two or three decades ago. Consequently, to better address evolving student needs and increase student engagement, most linear algebra courses should purposefully include more and better applications to provide motivation of, the context for, and examples of the ideas being taught.

Applications are too numerous in type and number to list more than a few. These might include the Google Page Rank Algorithm, linear programming, traffic/network flow, sports bracketology/predictions, predator/prey models, medical imaging, balancing chemical reaction equations, Markov chains, the Leontief Input-Output Model, etc. Of course, which applications are employed will depend on instructor interests and/or on the needs/interests of the students in class.

Applications and examples can introduce, motivate, and provide context for a specific concept or for several ideas simultaneously. To illustrate this, we briefly discuss one such example. A discussion of least squares could ultimately include the properties of vectors, and the specific concepts of orthogonality, orthogonal projections, projecting a vector onto the column space of a matrix, the least squares solution, and the extension of these ideas to functions, etc. Rather than immediately jumping into the discussion and assuming students care about vectors, orthogonality, etc., one could begin by giving the motivating example of trying to fit a straight line to a given set of points [15]. For example, we can attempt to find the straight line $y = mx + b$ that fits the points $(1,1)$, $(2,2)$ and $(3,4)$. This results in the system of equations

$$\begin{aligned} m(1) + b &= 1 \\ m(2) + b &= 2, \\ m(3) + b &= 4 \end{aligned}$$

that is,

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix},$$

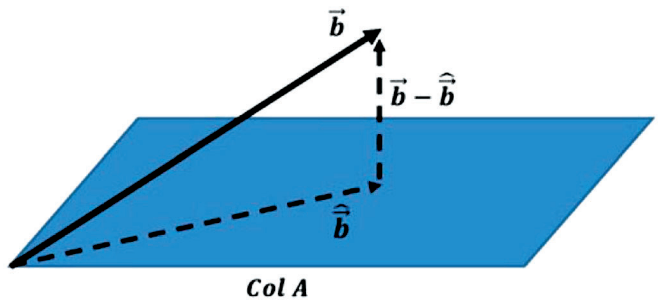

Figure 1.

which we denote A**x** = **b**. That is, we want to build (as a linear combination) the vector $\mathbf{b} = \begin{bmatrix}1\\2\\4\end{bmatrix}$ using the columns of A. Since **b** is not in the column space of A, we do the best we can: we project **b** onto Col A, which leads to the best fit (least squares) solution, that is, the line that best fits the points. So how do we find the vector $\hat{\mathbf{b}}$ in Col A that is "closest" to **b**? (And how do we even define "closest"?) Because the two column vectors are from $\mathbf{R}^3$, this problem is easily visualized: we are trying to determine the position $\hat{\mathbf{b}}$ "on the floor" (in the column space of A) that is closest to the point/vector **b**. So how can we tell which point/vector "on the floor" is closest to **b**? It is the point $\hat{\mathbf{b}}$ directly "underneath" point/vector **b**. What does "underneath" mean? It means we want to find $\hat{\mathbf{b}}$ so that $\mathbf{b} - \hat{\mathbf{b}}$ is perpendicular (in $\mathbf{R}^3$, and orthogonal in general) to Col A, the two-dimensional plane generated by the two columns of A. Without the need to get into all the details at this point, students can already see that there are certain concepts that we will need to understand in order to solve this problem: vectors, orthogonality (including in higher dimensions), projecting one vector onto a collection of other vectors, and so on. Students consequently will have more motivation to learn the ideas they are soon to explore, and they have a problem that is both important and simple to provide context for those ideas.

The Role of Linear Algebra in Other Disciplines

Students majoring in disciplines other than mathematics make up a majority of those taking a first course in linear algebra. Depending upon the clientele, courses offered should find a balance between abstraction, concrete matrix manipulations, and application. Although the range of topics emphasized in different courses varies from discipline to discipline, mathematical reasoning and creative thinking are important components in a linear algebra course. We recommend a set of core topics for a first linear algebra course (see core topics section). Additional topics may be added depending on program needs of students.

Future Work

It has been almost three decades since the publication of the work of the first LACSG [1]. During this time, the tools available to aid teaching and the needs of industry have expanded dramatically. We expect these trends to continue and probably accelerate. By no means does this paper capture everything the linear algebra community will need or hope for in the coming years. It is intended only as a starting point, a vehicle to express our thoughts and vision. We wish to draw some attention to important issues concerning the curriculum and teaching. For example, we hope that this document might serve as a guide for creating high-quality resources and teaching materials designed to facilitate the abstract and conceptual understanding so necessary to the future success of our students. This work will be supplemented by an edited volume to enable the mathematics community to react, elaborate, and generate more ideas that are both practical and thought-provoking. We believe collaborating and working together on instructional matters will have a positive effect on linear algebra students' success.

The LACSG 2.0 committee invites the mathematics community to carefully consider and examine the recommendations with surveys and follow-up studies and is not expecting the faculty to follow a prescribed set of guidelines.

ACKNOWLEDGMENT. This material is based upon work supported by the National Science Foundation under Grant No. DUE-1822247. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

[1] D. Carlson, C. R. Johnson, D. C. Lay, and A. Porter, *The linear algebra curriculum study group recommendations for the first course in linear algebra*, The College Mathematics Journal **24** (1993), no. 1, 41–46.

[2] D. Lay, *Linear Algebra and its Applications*, Pearson, 1994.

[3] S. Miller and D. Hughes, *The quant crunch: How the demand for data science skills is disrupting the job market*, Burning Glass Technologies, 2017.

[4] M.O.J. Thomas and S. Stewart, *Eigenvalues and eigenvectors: Embodied, symbolic and formal thinking*, Mathematics Education Research Journal **23** (2011), 275–296.

[5] H. Salgado and M. Trigueros, *Teaching eigenvalues and eigenvectors using models and APOS Theory*, The Journal of Mathematical Behavior **39** (2015), 100–120.

[6] A. Schoenfeld, *Purposes and methods of research in mathematics education*, Notices of the AMS **47** (2000), no. 6, 641–649.

[7] J. L. Dorier, A. Robert, J. Robinet, and M. Rogalski, *On a research program about the teaching and learning of linear algebra in first year of French Science University*, International Journal of Mathematical Education in Sciences and Technology **31** (2000), no. 1, 27–35.

[8] J. Hillel, *Models of description and the problem of representation in linear algebra*, in: Jean-Luc Dorier (Ed.), On the teaching of linear algebra, Kluwer Academic Publishers, Dordrecht, 2000, 191–207.

[9] S. Stewart, C. Andrews-Larson, and M. Zandieh, *Linear algebra teaching and learning: Themes from recent research and evolving research priorities*, ZDM Mathematics Education **51** (2019), no. 7, 1017–1030, DOI 10.1007/s11858-019-01104-1.

[10] G. Caglayan, *Is it a subspace or not? Making sense of subspaces of vector spaces in a technology-assisted learning environment*, ZDM Mathematics Education **51** (2019), no. 7, 1215–1237.

[11] G. Harel, *Varieties in the use of geometry in the teaching of linear algebra*, ZDM Mathematics Education **51** (2019), no. 7, 1031–1042.

[12] F. Henderson, C. Rasmussen, G. Sweeney, M. Wawro, and M. Zandieh, *Symbol sense in linear algebra*, Proceedings of the 13th Annual Conference on Research in Undergraduate Mathematics Education, Raleigh, NC, 2010. `http://sigmaa.maa.org/rume/crume2010`

[13] M. Wawro, M. Zandieh, C. Rasmussen, and C. Andrews-Larson, *Inquiry oriented linear algebra: Course materials*, 2013. Available at `http://iola.math.vt.edu`. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.

[14] A. Burazin, V. Jungić, and M. Lovrić, *Three Countries, Two Continents: Common Challenges and Opportunities in Teaching First-Year Mathematics,* Notices of the AMS **67** (2020), no. 1, 64–67.

[15] D. Strong, *Motivating examples, meaning and context in teaching linear algebra*, in: S. Stewart, C. Andrews-Larson, A. Berman, and M. Zandieh (Eds.), Challenges and strategies in teaching linear algebra, Springer, 2018, 337–352.

Credits

Figure 1 is courtesy of David Strong.

Photos of Sepideh Stewart, Sheldon Axler, Robert Beezer, Eugene Boman, Minerva Catral, Guershon Harel, and David Strong are courtesy of the authors.

Photo of Judith McDonald is courtesy of Washington State University.

Photo of Megan Wawro is courtesy of Virginia Tech.


Sepideh Stewart


Sheldon Axler


Robert Beezer


Eugene Boman


Minerva Catral


Guershon Harel


Judith McDonald


David Strong


Megan Wawro