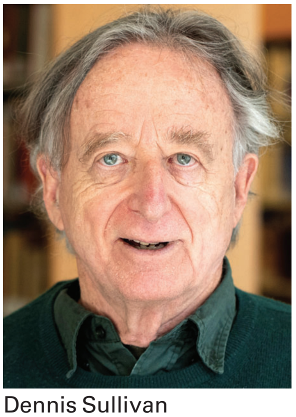
Dennis Sullivan Awarded 2022 Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2022 to **Dennis Parnell Sullivan** of Stony Brook University and the Graduate School and University Center of the City University of New York "for his groundbreaking contributions to topology in its broadest sense, and in particular its algebraic, geometric, and dynamical aspects."



Dennis Sullivan

Citation

Topology was born in the late nineteenth century, as a new, qualitative approach to geometry. In topology a circle and a square are the same, but the surface of the earth and that of a donut are different. Developing a precise language and quantitative tools for measuring the properties of objects that do not change when they are deformed has been invaluable throughout mathematics and beyond, with significant applications in fields ranging from physics to economics to data science.

Dennis Sullivan has repeatedly changed the landscape of topology by introducing new concepts, proving landmark theorems, answering old conjectures, and formulating new problems that have driven the field forward. He has moved from area to area, seemingly effortlessly, using algebraic, analytic, and geometric ideas like a true virtuoso.

His early work was on the classification of manifolds—spaces which cannot be distinguished from Euclidean flat space in the small, but which globally are different (for example, the surface of a sphere is, in the small, roughly a plane). Building on the work of William Browder and Sergei Novikov, he developed an algebraic topological perspective on this problem and invented some brilliant techniques to solve the problems that arise. This included the ideas of "localization of a space at a prime" and "completion of a space at a prime." These are ideas exported from pure algebra that provide a new language for expressing geometric phenomena, which have become tools for resolving multitudes of other problems. Nowadays it is commonplace to work at one prime at a time, using different methods for different primes.

Another of Sullivan's breakthroughs was the study of what is left when all the primes are ignored—known as *rational homotopy theory*. He and Daniel Quillen gave two different complete algebraic descriptions of what is left from a space in this setting. Sullivan's model is based on differential forms—a concept of multivariable calculus, enabling direct connection to geometry and analysis. This made a major part of algebraic topology suitable for calculation and has proven revolutionary. The use of differential forms made it especially relevant to algebraic geometry in combination with Hodge theory, as is shown in Sullivan's work with Pierre Deligne, Phillip Griffiths, and John Morgan.

To understand smooth manifolds, the completions were necessary, and one of the high points of his work here was his proof of the Adams conjecture, independently of Quillen. Sullivan has also drawn attention to the idea of a *homotopy fixed set*, formulating a central conjecture in homotopy and introducing a widely used tool. The original "Sullivan conjecture" was solved many years later by Haynes Miller.

Sullivan went on to tackle a host of topological, dynamical, and analytic problems, always with the idea of a geometric structure on a space playing a central role.

He showed that the topological structure of a manifold of dimension five or more can always be promoted to a *Lipschitz structure*, allowing analytic methods to be brought to bear. His argument uses arithmetic groups to replace Kirby's torus with a hyperbolic manifold immersed in Euclidean space. With Simon Donaldson, he proved such structures need not exist in dimension four.

In dynamics, Sullivan introduced a dictionary between Kleinian groups and iterated rational maps, pivoting on the theory of measurable complex structures. He proved that rational maps have no wandering domains, solving a sixty-year-old conjecture by Fatou, and brilliantly drawing a parallel with Ahlfors' finiteness theorem. He went on to use similar methods to provide a conceptual proof of Feigenbaum's universality for cascades of period doublings, recasting these results as the uniqueness of a smooth structure on a strange attractor. Sullivan's dictionary, his rigidity theorem for Kleinian groups, and his a priori bounds for renormalization are now fundamental principles in conformal dynamics.

In a subsequent return to the development of algebraic structures of manifolds, with Moira Chas, he astonished the field by finding a new invariant of manifolds. With its links to topological field theory, "string topology" has grown quickly into a field of its own.

Dennis Sullivan's insistent probing for fundamental understanding and his capacity to see analogues between diverse areas of mathematics and build bridges between them have forever changed the field.

Biographical Sketch

Note: This biographical information was taken from the Abel Prize website: `https://abelprize.no/page/press-room`

Dennis Parnell Sullivan is an American mathematician most famous for his groundbreaking work on topology and dynamical systems, two fields in which ideas about geometric structure play a central role. A charismatic and lively member of the mathematics community, he has found deep connections between a dazzling variety of areas of mathematics.

Sullivan was born in Port Huron, Michigan, on February 12, 1941. When he was a young child, his family relocated to Houston, Texas. He remained in the city to attend Rice University, initially to study chemistry, but soon switching to mathematics, graduating in 1963.

As a graduate student at Princeton University, Sullivan worked on the classification of manifolds, one of the fundamental questions in topology, building on the work of William Browder, his thesis supervisor, and Sergei Novikov. His 1966 PhD thesis, "Triangulating Homotopy Equivalences," developed techniques and provided insights that helped revolutionize the area. The following year he wrote a paper on the *Hauptvermutung*, an important conjecture in geometric topology, for which he won the 1971 Oswald Veblen Prize for geometry from the American Mathematical Society, the first of many awards he would win in his career.

After his doctorate, Sullivan had fellowships at the University of Warwick, England (1966–1967), the University of California, Berkeley (1967–1969), and the Massachusetts Institute of Technology (1969–1973), where he was a Sloan Fellow. During this time, he gradually changed the way mathematicians thought about algebraic and geometric topology, introducing new ideas and building a new vocabulary. In 1970 he wrote a set of unpublished notes that were widely circulated and considered hugely influential, directly impacting the classification of smooth manifolds and central problems in algebraic topology. Such was the long-standing impact of his ideas, the so-called *MIT Notes* were finally published in 2006.

Sullivan was invited to give a plenary lecture at the 1974 International Congress of Mathematicians, an honor given to the top mathematicians in their fields. He had spent the 1973–1974 academic year at the University of Paris-Orsay in France, and at the end of his stay he was made a permanent professor at the Institut des Hautes Études Scientifiques (IHES) near Paris.

During his time in France, Sullivan made one of his most important breakthroughs, a new way of understanding *rational homotopy theory*, a subfield of algebraic topology. The area had earlier been introduced from an algebraic point of view by Daniel Quillen in 1969, but Sullivan's work used differential forms, an idea from multivariable calculus, which opened the scope of the theory and made calculations much easier.

In 1981, Sullivan was made the Albert Einstein Chair in Science (Mathematics) at the Graduate School and University Center of the City University of New York. He kept his position at IHES and spent the next decade and a half shuttling between Paris and New York, often on the Concorde.

By the late 1970s, Sullivan had begun to work on problems in dynamical systems, the study of a point moving in a geometrical space, a field usually considered far removed from algebraic topology, the area in which he started his career. The ability of computers to iterate functions beyond what was humanly possible had created an explosion of interest in this field, known popularly as "chaos theory," since many of the dynamical systems exhibited chaotic behavior.

One of the best-known images from dynamical systems is a bifurcation diagram, in which a line repeatedly splits into two in an apparently chaotic fashion. The physicist Mitchell Feigenbaum discovered certain ratios in these diagrams that were universal to all systems. In 1988 Sullivan was able to give a conceptual proof of this universality. Another landmark result in this area was to prove, in 1985, that rational maps have no wandering domains.

Sullivan left IHES in 1997 to become professor at the State University of New York, Stony Brook, where he is now Distinguished Professor. Returning to topology, in 1999 Sullivan and Moira Chas discovered a new invariant for a manifold based on loops, which created the field of *string topology*, an area that has grown rapidly in recent years. In 2008, Sullivan also wrote a paper in the *Journal of Topology* with the hedge fund billionaire and philanthropist Jim Simons.

Sullivan's notable awards include the Institut de France's inaugural Êlie Cartan Prize in 1981, the 1994 King Faisal International Prize in Science, the National Medal of Science in 2004, the 2006 Steele Prize for Lifetime Achievement from the AMS (with F. W. Gehring), the 2014 Balzan Prize for Mathematics, and the 2010 Wolf Prize (with Shing-Tung Yau).

He is a member of the US National Academy of Sciences, the New York Academy of Sciences, and the American Academy of Arts and Sciences. Between 1990 and 1993 he was vice president of the AMS. He was named a Fellow of the AMS in 2012.

He has six children: Lori, Amanda, Michael (who is a mathematician), Tom, Ricardo, and Clara.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. It carries a cash award of 7.5 million Norwegian krone (approximately US$861,000). The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized research scientists in the field of mathematics. The Committee is appointed for a period of two years.

Read more about the recipient's life and work, as well as a list of previous recipients of the Abel Prize, at `https://abelprize.no`.

—From a Norwegian Academy of Science and Letters announcement

Credit

Photo of Dennis Sullivan is courtesy of Alex Irklievski.