C. R. Rao (1920–) Celebrates His 101st Birthday

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Introduction

Professor C. R. Rao (Calyampudi Radhakrishna Rao) is a well-known name in statistics, probability, mathematics, information geometry, econometrics, and related fields. He currently lives in Buffalo, New York, and actively participated in an event to mark his 101st birthday that was held online in October 2021 with his former students, long-time friends, and collaborators. His special awards include the US National Medal of Science, the Indian Government's Padma Vibhushan, and the Guy Medal in Gold from the Royal Statistical Society, London. His most recent award is the 2022 IEEE Honorary Membership, which is selected by the Board of Directors of IEEE from among outstanding individuals who have rendered meritorious service to humanity. C. R. Rao's famous results include the Rao–Blackwell Theorem, Fisher–Rao Information, Rao distance, Kagan–Linnik–Rao Theorem, and Cramér–Rao bounds. At the age of 25, he published major results from India before traveling to Cambridge to pursue a PhD under R. A. Fisher. See Table 1 for more detailed highlights of his career.

Another National Medal of Science awardee, Bradley Efron of Stanford University, wrote during C. R. Rao's centenary [1] that "When the fat second edition of Rao's magisterial book on linear statistical inference arrived on my desk (when I was a first-year postdoctoral student), it was a big event in the department (Stanford Statistics), not just for me." And Donald Rubin, Harvard University, writes in [1] that "Despite his dominant reputation, Rao always seemed to be extremely modest and, moreover, helpful to younger colleagues."

Efron further writes in [1] that "the 25-year-old Rao in 1945 [2] introduced differential geometry into statistical inference, opening up the burgeoning field now called information geometry." Rao distances combined with conformal mappings are seen in modern applications such as virtual tourism [4,5]. Shun-ichi Amari, Tokyo University, writes in [1] that "[C. R.] Rao's initiation of information geometry is one of the many achievements for which he was awarded the US National Medal of Science. Information geometry has grown to become an important tool not only in statistics but also in artificial intelligence, data science, signal processing, physics, and many other fields since it elucidates the fundamental structure of the manifold of probabilities."

This tribute to Professor C. R. Rao in the *Notices* is to celebrate his 101st birthday, and it can be treated as a sequel to the two tributes to him by two groups of renowned statisticians and mathematicians published during the centenary. See B. Efron et al. [1] and Prakasa Rao et al. [3].

Communicated by Notices *Associate Editor Scott Sheffield.*

DOI: https://dx.doi.org/10.1090/noti2492

George E. Andrews

It is humbling to participate in the tribute to this truly great man.

The first thing to say as someone who has been on the Penn State faculty since 1964 is that the day in 1988 when Professor Rao joined the Penn State faculty was an academic high point in my 57 years at Penn State. Sadly, Statistics had separated from the Mathematics Department at Penn State in the late 1960s, so Professor Rao and I were never in the same department. However, he served on PhD committees for some of my students and was the first person to welcome me into the National Academy of Sciences. I think of him as a very kind and thoughtful man. My wife remembers a large dinner event where she became ill and was shown great kindness by Professor Rao and his wife.

To conclude this brief note, I had hoped to find some mathematical tie between my work and that of Professor Rao. I was present in Bangalore in 2003 when Professor Rao received the Srinivasa Ramanujan Medal from the Indian National Science Academy, and the majority of my career has been devoted to Ramanujan's mathematics. In actual mathematical terms, the closest I could come is to find a tie to R. A. Fisher, Professor Rao's thesis advisor. In the early 1970s I was editing the collected works of the combinatorialist P. A. MacMahon and contacting all who had cited MacMahon since his death in 1930. Among those was J. H. Bennett, another Fisher student. Bennett clarified his own works and noted that Fisher had used MacMahon's work on Latin squares in an important statistical model in 1934. Indeed, the work Fisher cited was MacMahon's precursor to Pólya enumeration theory which came in the late 1930s.

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G. J. Babu

Prof. C. R. Rao's Move From Pittsburgh to Penn State

In the late 1980s, I had been traveling to University of Pittsburgh in summers to visit and work with Rao and his group. I talked to then department head Thomas P. Hettmansperger in early 1988 about whether Rao might be interested in leaving University of Pittsburgh and would be receptive to an overture from Penn State. This was around the time when Professor Rao lost his closest colleague Professor P. R. Krishnaiah. Delicate efforts by top-level administration helped attract the living legend to Penn State. Professor Rao joined the Department of Statistics at Penn State in fall 1988. Initially Professor C. R. Rao requested

Figure 1. James Rosenberger, John Nash Jr., C. R. Rao, and G. J. Babu. This was taken during the visit by John Nash Jr. in October 2003. As the chair of the John M. Chemerda Lectures in Science, G. J. Babu invited Dr. Nash, 1994 Nobel Prize winner, to give lectures to the Penn State University community. The movie *A Beautiful Mind* was based on his life.

a three-year appointment, thinking he would retire at the age of 70 in September 1990. He stayed on at Penn State until 2001 and retired at age 80 as Professor Emeritus of the Eberly Family Chair in Statistics. He remained in State College until moving to Buffalo, New York, in 2009. On June 12, 2002, Rao was awarded a National Medal of Science by President George W. Bush at the White House for his lifelong achievements.

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Zhidong Bai

At the time of the 101st birthday of Professor C. R. Rao, I recall the happy years together with Dr. Rao and his family. Dr. Rao is one of the greatest statisticians and an easily approachable friend. I have learned a lot from him academically as well as personally. I will recollect some small stories from the years we were living together.

A Chance Meeting with Dr. Rao

The first time I heard the name Professor C. R. Rao was many years ago when I was a student, majoring in statistics. However, our first in-person meeting was 19 years later, in August of 1984 when I was invited, as a scholar, by Professor Paruchuri R. Krishnaiah to visit the Center for Multivariate Analysis at the University of Pittsburgh. I worked with him for a few years thereafter. As a beginning researcher in statistics, I was sincerely scared and timid to approach Professor Rao, as he was such a renowned and well-established statistician in the field around the world. One day, as I was wandering around passing by his office, he saw me and called me into his office. After a brief greeting and introducing ourselves, he raised a question in the area of density estimation of directional data and discussed it

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with me. I told him that Dr. L. C. Zhao was an expert in density estimation and I would discuss the problem with Dr. Zhao first and get back to him the next morning. The next day, we shared our thoughts with him and had further discussions, which led to a joint paper that we published in the *Journal of Multivariate Analysis*. During our discussions, his easygoing and approachable personality, as well as his insightful knowledge in the field, deeply impressed me. With more and more collaboration between us, we got to know each other better and became good friends and partners, especially after Professor P. R. Krishnaiah passed away and Dr. Rao began leading the Center. Our relationship and friendship was extended further after we moved from Pittsburgh to Penn State University in 1987. In everyday life, he was like a kind parent and friend, while academically he was like a great mentor and supervisor. I learned an enormous amount from him in many areas during that time.

Collaboration with Professor Rao

Edgeworth expansion under partial Cramér condition

In 1987, Peter Hall published an Edgeworth expansion for a studentized statistic under minimal moment conditions in the *Annals of Probability*. The studentized statistic is a special case of a function of sample means established by Bhattacharya and Ghosh in the *Annals of Statistics*. We found that the partial derivative of the link function for the studentized statistic with respect to the component x^2 at the mean is zero. Therefore, we decided to extend Hall's result to the general form of functions of sample means. On the other hand, the Cramér condition required by Bhattacharya and Ghosh may be too strong in many real applications, such as the ratio of occurrence to exposure [7]. We proposed to establish the Edgeworth expansion under the condition that the upper limit of the absolute mean of the conditional characteristic function of one component given all others is less than one as the variate t tends to infinity. Dr. Rao named this condition a partial Cramér condition. The name of the condition was very elegant and made the result eminent.

M-estimation

When starting a new topic in my research, I would minimize the assumptions or strengthen the conclusions to the greatest extent possible and then show that my results were no longer improvable. Working with Dr. Rao, I noted that he, instead, was always trying to come up with a new approach or idea to tackle a new problem. For example, in a paper on which I collaborated with Dr. Rao and his graduate student Amy Wu, we initially worked out a new result [8] for M-estimation with convex discrepancy function by creating a stepwise minimization approach so that it covers almost all known results of M-estimation with convex discrepancy in the literature. Later, Dr. Rao motivated us to extend the result to a more general case in which the discrepancy function is a difference of two convex functions, so that the result may cover many robust estimators. The new result [9] was published in the *Handbook of Statistics*.

Birth of GIC

In model selection theory, the AIC is minus two times the logarithm of the maximum likelihood (the information) plus two times the number of parameters (the penalty), while the BIC changes the penalty as $\log n$ times the number of parameters. It was proven that the AIC has a positive probability of overspecification and the BIC was strongly consistent, but sometimes it underspecifies the true model. How to set up information criteria became a hot topic in the area. In late 1985, Professor Krishnaiah presented this problem to Dr. L. C. Zhao and me. We proposed the GIC (general information criterion), which changes the penalty as $C_n/n \to 0$ and $C_n/\log\log n \to \infty$ and we also showed that the GIC is strongly consistent. The publications [10, 11] became highly cited in the literature. However, the criterion was not named in either publication. Later, Dr. Rao, with his deep intuition and insightful understanding, named it GIC and presented it as a tribute in the speech in commemoration of Professor Krishnaiah, and now it is well known as GIC in the literature.

Contributions to image reconstruction

In 1972 Godfrey Newbold Hounsfield invented the CT scanner and was awarded a Nobel Prize in 1979 based on it. The principle of the machine is simple, just an inverse-Fourier transform. Consider a cross-section of a human's body; a two-dimensional distribution density can be easily reconstructed and visualized if its Fourier transform can be observed. Suppose $p(x,y)$ is the density. Then the Fourier transform is $g(t,s) = \int e^{i(tx+sy)} p(x,y)\, dxdy$. Given a direction θ, write $t = \rho\cos\theta$ and $s = \rho\sin\theta$. Make the variable change $\mu = x\cos\theta + y\sin\theta$ and $\nu = -x\sin\theta + y\cos\theta$. Then

$$g(t,s) = h(\rho,\theta) = \int e^{i\rho u} \tilde{p}(u,v)\, du dv = \int e^{i\rho u} \bar{p}_\theta(u)\, du, \quad \bar{p}_\theta(u) = \int \tilde{p}(u,v)\, dv \tag{1}$$

which is the projection of p along θ and $h(\rho,\theta)$ is the Fourier transform of the projection $\bar{p}_\theta$. Because the projection is observable and hence the Fourier transform of $h(\rho,\theta)$ is estimable for each given θ, consequently, the two-dimensional Fourier transform is estimable if the projections can be made along with all directions θ.

In medical science, the heart, especially the shape of the left ventricle, is an important subject for cardiologists. Although the CT scanner is a powerful instrument for observing pathological change in the body, it does not work for moving objects. Thus, Siemens Gammasonics Inc. proposed the problem: how to extend the CT scanner to the left ventricle. Then, the cardiologist Professor P. S. Reddy, along with Professors Krishnaiah and C. R. Rao, recommended the use of two orthogonal X-ray pictures to reconstruct the left ventricle, and obtained a sizable grant of

several hundreds of millions of dollars from Siemens Inc. Professor Reddy provided three hundred thousand dollars after overhead quarterly to the Center. After Dr. L. C. Zhao arrived at the Center, we two, jointly with Professors Krishnaiah and Rao, devoted significant effort to this project. It is hard to say the project is statistical or mathematical. But it is interesting and is related to their academic survival; their stipends were all coming from this grant. This made me realize how broad and inspirational Professor Rao's knowledge is.

The methods commonly used by H. T. Dodge et al. (1960) assumed the cross-section to be close to an ellipse located between two endpoints of each projection profile as an axis of the ellipse. In 1983, S. Eiho et al. (1984) refined the construction of the ventricular cross-section by using three projection profiles under the same assumption of an elliptic shape. These algorithms are simple, but the reconstructed cross-section is far from a real ventricular cross-section, which is usually not elliptical. S. K. Chang and C. K. Chow (1973) proposed another algorithm to specially reconstruct the shape of the ventricle, by assuming the cross-section of a ventricle to be a connected region that is convex and symmetric with respect to the geometrical center of the cross-section. By applying their algorithm, the reconstructed cross-sections can be selected from a few possible solutions. However, a ventricular cross-section is in general neither convex nor symmetric; see D. G. W. Onnasch and P. H. Heintzen (1976) and C. H. Slump and J. J. Gerbands (1982).

In using the data on two orthogonal X-ray pictures taken simultaneously, we developed a strategy to handle the image reconstruction of a sliced object by connecting the midpoints of the slices together. We then developed an algorithm to correctly align the midpoints from slice to slice by comparing the two orthogonal projections. Details of the reconstruction were published in [12] and the consistency of the procedure was theoretically proven in [13] under minor regularity conditions. The algorithm has been tested on computer-synthesized projection data as well as real X-ray pictures with good results.

Root Meaning of English

One day during my student days in Pittsburgh, some graduate students and visiting scholars were discussing canonical correlations. Dr. Rao came in and asked, "what is the meaning of canonical? From what does it come?" Many students and scholars in the meeting thought it came from "cannon," big artillery, so it must mean "important." He said no! Check the word with "ical" removed in the dictionary. We suddenly realized that canon is the basic regulation for religions and understood the meaning of canonical correlations better than ever from that point on.

Here is another story. I wrote a recommendation letter for a Chinese student and asked Dr. Rao for a revision of my English. He changed the word "could" to "would." To me, as I had been taught, both words are in the subjunctive and have the same meaning, as a polite way to request a favor. So I asked Dr. Rao about the difference between the two words. Dr. Rao said, "you are asking the graduate chairman to offer the admission. If you use 'could,' that is to say 'do you have the right to offer the admission?' If you use 'would,' that means 'are you willing to offer the admission?' Of course, there is no doubt he has the right to offer admission, because he is the chairman of the graduate committee!" This made me realize that the challenge in mastering a language may be that a word misusage might lead to a totally different understanding such that "a millimeter miss is as good as a thousand miles," as a Chinese proverb says.

Passionate and Faithful Husband

As we moved together to Penn State and became neighbors, our connection was extended further to daily life. We started to know that Dr. C. R. Rao was not only a good mentor, collaborator, and colleague in the office, but he is also a passionate and faithful husband at home and passionate and considerate to everyone in his family. This explained the full love showing in his wife's eyes whenever she looked at him and when she gratefully talked about her husband. Nearly every evening if you went for a walk, you would see Dr. Rao and his wife walking and talking in the neighborhood or on the trail. As Dr. Rao's amazing companion throughout her life, Ms. Rao was not only a supportive and affectionate wife to her husband but she also showed care and compassion to every student, colleague, and scholar who was working and collaborating with her husband. She sincerely admired their progress and achievements in every aspect.

David Banks

A Few Interactions with Professor C. R. Rao

I first met Professor Rao when I was a graduate student at Virginia Tech. He came to deliver a distinguished talk, and of course the statistics department was excited and delighted by his visit. Being one of the more socially presentable graduate students, I was tasked with picking him up at the Roanoke airport and being his chauffeur and luggage carrier while he was in Blacksburg.

He was exceptionally gracious and kind to me, and we had a number of wonderful conversations as I drove him around. I particularly recall him telling me about what it was like for him to have been a graduate student working with Fisher. He also reminisced about his work for P. C. Mahalanobis when he first came to the Indian Statistical

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Institute and how he had discovered a discrepancy in the way people's weights were being recorded in an Indian province—he joked that his correction averted a famine by adding about 20 pounds to the average weight in that region. But the thing I remember most fondly was that he kept trying to persuade me to give up on statistics and become an engineer. He felt that the future belonged to engineering, and that I was still young enough to jump on that opportunity.

I joined Carnegie Mellon in 1987, and was invited to give a colloquium at the University of Pittsburgh. Professor Rao attended my talk and we spoke afterwards. I reminded him that we had met at Virginia Tech. I am sure he didn't remember me, but he was kind enough to pretend to recall it.

Soon afterwards, Professor Rao joined Pennsylvania State University, and so for many years I would only occasionally see him at a conference, and he was always surrounded by his friends and his students. But in 2004, I was invited to speak at the International Conference on the Future of Statistical Theory, Practice, and Education that he had organized in Hyderabad. And to my delight and surprise, he asked me to give a two-day short course on what was then called data mining to the graduate students at his institute. But what truly astonished me was that, at the age of 85, he sat in on the course too, asking questions, making suggestions, and correcting my mistakes. I like to tell people he was the smartest student I ever had.

In 2020, I was fortunate to be the program chair for the Joint Statistical Meetings, which meant that I could help Arni Rao facilitate an invited session in honor of Professor Rao's 100th birthday. The speakers were Brad Efron, Don Rubin, and Sir David Cox. Since it was virtual, C. R. Rao himself was able to participate and make the closing remarks! It was a wonderful celebration of the last of the giants of statistics and I am proud to have been part of that event.

Herman Chernoff

The first half of the twentieth century saw the development of statistical theory by a series of giants. These included Fisher, Neyman, Wald, and C. R. Rao. Of these giants, only C. R. Rao is still alive, and we hope that he will be with us for many more years and continue to provide us with the benefit of his wisdom.

I claim that one consequence of the work of these men is the ability of the world to support the population of 8 billion people. In recent years the computer has become more important, but the basic ideas exposed by these giants will not be forgotten while statistical theory wrestles with new insights and questions.

Happy birthday CR!

Roy Frieden

I am very appreciative of the work Prof. Rao did in advancing the cause of statistical methods in geometry (Amari). Prof. Rao's invention of what is now called the "Cramér–Rao" or "CR" bound (but of course, should be properly called the "Rao Cramér bound" to express precedence) actually derives the defining expression for the Fisher information. This has had amazing applications to statistical physics. Consider the problem of finding the statistical law $p(x)$, defining any physical process, from its observed data $y = a + x$, with a representing the system "state" such as its mean a, and the x fluctuations from the state a. The C–R expression is that the data fluctuations x obey $x^2 \geq 1/FI$. Outwardly, such a squared-fluctuation error only expresses uncertainty in the observer's knowledge of the system state parameter a. But amazingly, it turns out to also express the uncertainty *in knowledge of the system law* $p(x)$ per se. And so, by the above C–R expression, system laws $p(x)$ that maximize its functional integrals can be accurately derived. This includes quantum laws and classical laws alike.

The following approach uses "functional integrals," called FI and FJ, whose respective values are values I and J

Figure 2. C. R. Rao and Roy Frieden at University of Texas in 2010 during Rao's 90th birthday celebrations.

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of the Fisher information. See references [14,15] for further details.

In particular, let value J be some (any required) fixed, upper bound to I. Philosopher I. Kant famously postulated that such bounds exist for all physically observed laws. In agreement, we find this leads to actual knowledge of the laws. Also, they thereby follow because they are observed! Thus, the laws arise out of man's efforts to gain knowledge. Other creatures do not gain such knowledge, only experiencing the *effects* of the laws.

Integrals FI and FJ are mathematically varied, by infinitesimally changing their integrands, so as to form varied values dI and dJ, respectively. These are called *variations* var. Following Kant's model, the aim is to find the system probability density function (pdf) that *maximizes* the FI, given the *prior knowledge that* FJ *is an upper bound* to the value FI. Could merely such prior knowledge be sufficient to find the pdf? Yes.

The FI is well known to be a concave functional, and our aim is to find the pdf implied by the requirement that FI and FJ obey the variational problem

$$\text{var}(\text{FJ} - \text{FI}) = \text{var}(\text{FJ}) - \text{var}(\text{FI}) = 0. \tag{2}$$

Since the value of functional FJ = J = constant, its var(FJ) = 0. Then the problem statement (2) becomes simply

$$\text{var}(\text{FI}) = 0 - 0 = 0. \tag{3}$$

The solution pdf = $p(x)$, x the independent variables (space, time, …), of course obeys corresponding Euler–Lagrange (E–L) equations, as previously published (e.g., [1–12]), forming known laws of physics $p(x)$.

But historically, in such previous publications, the negative of requirement (2) was addressed: that, instead, the var(FI – FJ) = 0. However, does this problem give a *different answer* for the pdf than does requirement (2)? No, as follows:

Given, again, the requirement that variation(FJ) = 0, this problem becomes

$$\text{var}(\text{FI} - \text{FJ}) = \text{var}(\text{FI}) - \text{var}(\text{FJ}) = \text{var}(\text{FI}) = 0. \tag{4}$$

This is the same as requirement (3). Hence the Euler–Lagrange equations are the same as of those from Eq. (1), where FI and FJ *are switched around*. Then the two problems (2) and (4) give the same solution $p(x)$. So all physical laws so-derived by principle (4) remain valid.

Yasunori Fujikoshi

Statistics and Truth — C. R. Rao

I first met C. R. Rao at the Fourth International Symposium on Multivariate Analysis, held in Dayton, USA, in 1975. It was then that he kindly explained to me and commented upon asymptotic expansion of the distribution of a lambda statistic which appears as the null distribution for various tests, including a MANOVA test. It was noted that an asymptotic expansion of a transformed statistic of it was first obtained by Rao (1948) [16]. Afterwards, G. E. P. Box (1949) gave an asymptotic expansion for a class of statistics including T. It was both a significant surprise and an incredible joy. As is well known, C. R. Rao has made a variety of significant contributions in statistics, including, but not limited to, those in estimation theory, linear models, Rao's score test, multivariate analysis, characterization problems, model selection, and robust statistical inference. In addition to these contributions, he has also done a great service for statistics through his book *Statistics and Truth* published in 1989. I found out about this book at C. R. Rao's 70th birthday celebration meeting, held in Switzerland in 1989, where he personally presented it. The intent of the book was to explain the essence of statistics with real examples, for a broad readership. The second edition was published in 1997. D. R. Cox has recommended this book as follows from the back cover of the book: "The topics discussed in the book range from general philosophical issues, including the nature of creativity, to technical statistical matters. The book is a powerful illustration of the nature of statistical arguments and I can think of no better book to introduce the subject, in particular to a general reader." I think that something of particular note in this book is the following logical equation that was introduced:

Uncertain knowledge
+ Knowledge of the extent of uncertainty in it
= Useable knowledge

This is a new way of thinking, and I believe that this logical equation is exceptionally useful for both understanding and explaining statistics. Incidentally, the Japanese translation of the second edition was published in 1993 by Fujikoshi, Taguri, and Yanai [17], which is some four years earlier than the second edition of the original. This is due to the fact that C. R. Rao kindly presented us a typed version of his manuscript while revising it, when I visited him at Penn State University during the period from October to November in 1991. Related to the above books, I wish to note that C. R. Rao published a Japanese statistical book, *Introduction to Statistical Reasoning by Real Examples* with

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Figure 3. Mrs. and Mr. Fujikoshi with Mrs. and Mr. Rao in 1981.

Taguri, Fujikoshi, and Yanai (2007) [18]. The reason behind the writing of this book goes back to when Yanai and I met C. R. Rao at the statistical conference in Hyderabad, India, in 2004. At that time, we discussed writing about ways of thinking about statistics as well as the essentials of statistics, in a logical order, and in an as accessible and easy-to-understand manner as possible.

Fortunately, I was able to spend a substantial amount of time working with C. R. Rao through various avenues pertaining to our careers in statistics. As an example, C. R. Rao served as the chief editor of *Journal of Multivariate Analysis* from 1988 to 1992, and in that same period I served as an associate editor, per his request. When he stepped down as the chief editor, he kindly sent me the following letter:

"I am writing to thank you for all the help you have given me in processing the papers for the *JMA*. It was a great pleasure working with you." In closing, I would like to pray for Professor C. R. Rao's continued good health.

Kanti V. Mardia

A Very Happy 101st Birthday to You, Professor Rao

To start with let me say Professor Rao has been my role model; indeed, he has influenced many academics and me in particular. I will just record a few remarks from my memory on my meetings with Professor C. R. Rao. Before that, let me start by saying that during 1955–1957 in my MSc at Bombay University, I was taught from his path-breaking book *Advanced Statistical Methods in Biometric Research*, which I enjoyed studying and still refer to (together with his book *Linear Statistical Inference and Its Applications*). In my final MSc examination I was told that my matrix algebra paper was set by C. R. Rao. I got the highest mark in the class which made me so happy.

I never thought that I would have the chance to meet him, but fortunately, I first met him at the Calgary Conference in 1974 (First International Conference on Characterizations of Statistical Distributions). Of course, I was excited to meet a legend and he was so charming that I was totally pleasantly surprised and happy that he was so easy to talk to. Indeed, if I recall correctly, we played cards in one of the evenings. Then I met him several times in the 1990s at Penn State, while I was working with Colin Goodall and visiting him every year from 1991–1995. Once, while walking with him from lunch to the Dept. of Statistics in State College, I asked, "How are you keeping?" He was in his 70s and he replied, "Pain here, pain there, pain everywhere," which aptly summarizes the aging process we all go through. I also remember going to his home with my wife Pavan and enjoying Bhargavi's (Mrs. Rao) cooking during this period.

We had several conversations during this period on directional statistics and shape analysis. On directional statistics he wrote a paper (T. M. Pukkila and C. R. Rao, 1988) in *Information Science*, and we discussed in detail the form of a projected normal distribution. This has now become a popular distribution. More details are given in my joint book (Mardia and Jupp, 2000, *Directional Statistics*). Also, he got interested in shape analysis and reminded me that in the same paper we use triangular shape, since for compositional data the sum of the three angles is fixed.

His interest in shape analysis grew and during this period, we tried to organize a joint workshop at Penn State on shape analysis but there were not enough numbers for it to be viable, so it did not work out. However, he pursued the topic, and he sent his PhD student, Suryawanshi, to my Leeds Annual Statistics Research (LASR) Workshop in 1995. Their subsequent *PNAS* paper (Rao and Suryawanshi, 1996) gave a fresh look at the subject by proposing an alternative approach based on inter-landmark distances using logarithms of distances. Again, more details are given in another joint book (Dryden and Mardia, 2016,

Figure 4. During the Royal Statistical Society's Guy Medal in Gold ceremony for Professor C. R. Rao in London in 2011.

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Statistical Shape Analysis). These indicate how many fields he has influenced.

On his 80th birthday in 2000, we attended a conference arranged to celebrate it in St. Antonio which was well attended. I remember the great conference dinner with so many of his family, friends, collaborators, students, and well-wishers. To continue my profound respect for him, on his 90th birthday in 2010 I attended and presented a paper to another conference in his honor in Hyderabad, and what I remember the most now is Pavan and I walking down C. R. Rao Road (named after him) … a fitting tribute. The last time I met him in person and his full family was in London in 2011 for his RSS Gold Medal's ceremony … it was a great event and I was delighted that finally this honor came to him.

I end my contribution with one of the sentences which he wrote in reviewing my book *The Scientific Foundations of Jainism* in 1992. "The book will be of great value to Jains as well as non-Jains in understanding the emergence of a great religion to help us in seeking the truth and understanding the purpose of life." He has always been after truth, and he is a role model to all of us for the "purpose of life." **Jug Jug Jiyo Professor Rao**

Basilio de Bragança Pereira

Apart from talking about the importance of Prof. Rao's work and dedication to science, in knowing him, I was impressed by how great his human side is. The comments from others that knew him, including in his biography, give you a hint of how nice and helpful he has been to people and scientists in need of help throughout the world. In several visits I made to see him at PSU, he always treated me as a son. People that did not have the opportunity to know him personally should try to know this side of his life. For all of us, it is a great lesson in how we should try to live our lives.

My link to Professor C. R. Rao is an example of the Law of Truly Large Numbers

When studying at Imperial College, I and another Brazilian student at the London School of Economics decided to study C. R. Rao's book *Linear Statistical Inference and Applications* weekly. My brother went to Florida State University to study under Debabrata Basu, the first student of C. R. Rao. A few years later, I was walking near my house and someone was selling old books in the street, and I was able to buy C. R. Rao's *Advanced Statistical Methods in Biomedical Research* (now a rare book). A few weeks later my brother came to Rio de Janeiro and we decided to take a walk. We entered a secondhand book shop, and I was able to buy an Asian edition of C. R. Rao's *Linear Statistical Inference and Applications*. A few years later, in 1999, I went to a conference in Brasilia, and the University of Brasilia had conferred a Dr. Honoris Causa on Professor Rao.

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Figure 5. Basilio, Rao, and C. R. Rao's late wife Mrs. Bhargavi Rao and daughter Dr. Tejaswini Rao in an Indian restaurant, State College, PA, in 2000.

On his return to the USA, he stayed in Rio near my house. He had to take a plane on a Saturday to Carnaval and had to leave the hotel in the morning to travel late at night. Someone asked me to take care of him. He and his wife stayed in my home that day. I and that monster of statistics then decided to ask if he had a student that could write a book based on some of my short notes on neural networks in statistics, which Prof. David Cox had transformed into a book. Three months later he invited me to PSU to give some lectures on the subject. On my return to Brazil, he said he was going to write jointly with me. I then obtained a grant and spent 2003 with him at PSU. I visited him in the years 1999, 2000, 2003, 2009, and 2010.

Our book (Pereira, Rao, and Oliveira) *Statistical Learning with Neural Networks: A Guide for Statisticians and Data Scientists with Python* was published in 2020 [19], and copies of the book arrived in his home a few days before his 100th birthday.

Urmila Pingle

My quest to find the best guide/mentor to steer me through a PhD in population genetics led me inevitably to Dr. Rao. I had a medical degree with very little statistical background but my association with Dr. Helen S. Haldane (wife of Dr. J. B. S. Haldane) in Hyderabad inspired me to take up a project in the population genetics and anthropometrics of tribal communities in Central India. Fortunately for me,

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this field of work was close to Dr. Rao's heart. Seeing the passion I had displayed in initiating this project among the remote tribal communities, he agreed to be my guide. Little did I know what a hard taskmaster he was going to be! He put me through the meticulous process of planning the sample design and later, after having collected population data on multiple genetic and anthropometric variables, he got me to use statistical tools to look for outliers in my data. I remember his stern warning that if I had fudged my data it would be revealed in this statistical scanner. Fortunately, I had little to fear on that score and some outliers that occurred were justifiably removed as these extreme elements would have distorted the multivariate analysis and conclusions arrived therefrom.

In the summer of 1980, I spent some five months in Pittsburgh as the student guest of the Rao family! Dr. Rao had moved to the University of Pittsburgh and he got me a scholarship to visit him. Dr. and Mrs. Rao, with all their generous and warm hospitality, insisted that I stay with them to complete the analysis and writing up of my thesis. I was very homesick having left behind my two-year-old son in India. But it was somewhat mitigated by the warmth of the Rao family, consisting of their daughter Teja and son Veera. All this led me to develop a deep lifelong personal attachment towards the Rao family. They treated me like a close family member and I took part in their day-to-day activities, especially growing vegetables with Dr. Rao in his back garden. Dr. Rao's philosophy of meticulous planning applied not just to statistics but to building scaffolding for his tomato plants in a stage-by-stage manner as they grew. No shortcuts for Dr. Rao. Even during weekends when Dr. Rao took up cleaning his beautiful collection of bronze statues, especially of the god Krishna, he would clean little by little over a length of time in order to do a thorough job and not finish in one go.

The greatness of Dr. Rao lies in his genuine humbleness of character coupled together with a democratic persona. He would give his students breathing space to develop their ideas and not impose his on them. On one occasion, I was stubbornly adhering to using a method Dr. Rao felt was not appropriate for my data. His disapproval was shown not by loudly upbraiding me but only by silence. This made a greater impact on me to think deeply about it and then come around to rejecting the method on my own.

I would like to conclude that my association with Dr. Rao and Mrs. Rao (they were soulmates) was the most transformative experience in my life. I would have been greatly diminished without it. I am also privileged to be his first woman student!

M. B. Rao

Interactions with C. R. Rao

A year ago, I attended a day-long workshop on Data Science conducted by two scientists from Google. They started the workshop with a quotation from C. R. Rao.

> All knowledge is, in final analysis, history. All sciences are, in the abstract, mathematics. All judgements are, in their rationale, statistics.
> —C. R. Rao

As a research scholar at the Indian Statistical Institute (ISI)

In 1966, I was admitted as a research scholar at the Indian Statistical Institute (ISI). Professor C. R. Rao (CRR) was the director of the institute.

At that time, ISI created a 1½-year research course, which was mandatory for all first-year research scholars. I was in the second batch of the course. The course was heavy in mathematics. The motto was that if you are good in mathematics, you are good in everything. I took classes in measure theory, topology, ergodic theory, measure theoretic approach to inference, functional analysis, measures on locally compact groups, advanced probability, discrete dynamic programming, algebraic topology, among others.

The faculty was brimming with young talent comprising Ashok Maitra, Jayanta Ghosh, S. W. Dharmadhikari, and B. Ramachandran, spiced with stalwarts like D. Basu and E. M. Paul. In hindsight, I can see that the introduction of the research course was a bold idea molded and fashioned by the young faculty and shepherded by D. Basu and C. R. Rao.

Several specialties were created from which the research scholars could choose. I chose advanced probability. Professor Rao created biostatistics and statistical genetics specialties.

Visitors kept coming to ISI throughout the year, offering short courses and seminars. C. R. Rao sent circulars to research scholars commanding them to meet the visitors for research topics and ideas.

I never had an opportunity to take classes from C. R. Rao. I was in awe of him. My first contact with him occurred at the end of my first year. Professor J. L. Doob was scheduled to give a series of lectures on probabilistic potential theory. A circular was posted on the bulletin board requiring some first-year and second-year students to attend the lectures, take notes, and create organized lecture notes. My name was on the list.

I completed all my exams. I was feeling homesick. I had never been away for such a long stint. I wanted to go home. I approached the dean of studies for leave. Being on the list

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of note-takers, he advised me to seek permission directly from Professor Rao.

I knocked on CRR's office door.
CRR: Come in!
I entered his office.
CRR: Who are you?
MBR: I am M. B. Rao.
CRR: What are you?
MBR: I am a first-year research scholar. I have completed all my exams.
CRR: What do you want?
MBR: I want to go home for two weeks.
CRR: Go!
MBR: I am one of the note-takers for Doob's lectures.
CRR: Don't go!
MBR: I am feeling homesick.
CRR: Go for a week.

Professor Rao moved to the Delhi ISI a year later. In due course, I completed my research work and submitted my thesis. I wanted to have some work experience abroad. Due to a recession, the job market in the USA was down. I applied for a postdoc position at the University of Sheffield. I needed recommendation letters. I wrote a letter to C. R. Rao asking whether I could include his name as a referee. Along with a cover letter, I sent copies of six papers I wrote. Within a week, the whole package came back. I thought Professor Rao was not willing to write a letter. I opened the envelope. On my cover letter, CRR wrote, "You may!"

I got a position as an assistant professor at Sheffield. In a casual conversation with Joe Gani, my new boss, he mentioned that he got a letter of recommendation from C. R. Rao about me and there was only one line in the letter: "He is suitable for the job!"

Brevity is one of the hallmarks of C. R. Rao!

As a research collaborator

The USA was deeply involved in the Vietnam War in the 70s. It was no match for the Viet Cong guerillas in jungle warfare. Thick foliage in the jungles was providing good cover to the guerillas. In order to negate the advantage, USA had decided to spray 'Agent Orange,' a carcinogenic combination of chemicals, onto the trees. A sizable number of soldiers were entrusted with the job of spraying. The war ended. Troops came home.

The group of soldiers who handled Agent Orange had claimed that the incidence of cancer among them was higher than that of the rest of the population. They demanded compensation. The government needed proof. They monitored a random sample of soldiers for cancer. The data was structurally lifetime data with censoring. The subject could die due to causes other than cancer. In modern terminology, analysis of such data would fall under the umbrella of competing risks. There were no methods available to handle that type of data at that time. The question was to estimate the lifetime distribution of cancer deaths in the environment of competing risks. C. R. Rao was given a research grant to develop methodology for analyzing such data.

He had never worked in survival analysis. He wanted to have a good understanding of the subject. He assembled a study group for this purpose. I was one of the members of the team. We met every weekday for about two months. Every day, someone in the group presented a seminar covering a nucleus of papers on the subject. I benefitted tremendously from such an intensive activity. C. R. Rao's critical insight was a tremendous boon. We were able to come up with a solid method of analyzing this type of data. A paper got written up and published: "Nonparametric Estimation of Specific Occurrence/Exposure Rate in Risk and Survival Analysis," Gutti Jogesh Babu, C. Radhakrishna Rao, and M. Bhaskara Rao, *JASA*, **87**, 1992.

As a collaborator on a book

When I was visiting the University of Pittsburgh on leave from Sheffield, Professor Rao assigned me a class to teach: Advanced Methods in Matrix Analysis and Applications to Statistics. Being an expert in the art of multivariate analysis, he had a very deep knowledge of finite dimensional vector spaces and matrix analysis. I made use of his expertise to fashion my course. The notes that became the backbone of the class were expanded into a book: *Matrix Algebra and Its Applications to Statistics and Econometrics*, C. Radhakrishna Rao and M. Bhaskara Rao, World Scientific, 1998.

As a pioneer in our field, his curiosity in learning never abated. He is always on the lookout for innovative ideas and applications of statistics. He is a prolific contributor. Advancing age has not diminished his zeal for our field.

Arni S. R. Srinivasa Rao

C. R. Rao Was a Brilliant Student from His Younger Days

I came to know the brilliant works of Professor C. R. Rao for the first time through my mathematics teacher, Mr. Perisastri, in the 1990s during my college days in Vizianagaram, Andhra Pradesh, India. My teacher was six years junior to C. R. Rao in their high school through college days in Visakhapatnam, India (50 kilometers from Vizianagaram) during the 1930s and 1940s. They both had common mathematics teachers at Andhra University, Visakhapatnam, who were stalwarts of Indian mathematics around the time of India's independence from British rule. Some

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of these names, as I recollect, are Vommi Ramaswami, Vijay Raghavan, and Meenakshi Sundaram.

My teacher, who made important contributions in number theory and trained me for math olympiads, had great respect for C. R. Rao as the latter had shown brilliance as a student in his high school days and in obtaining his MA in mathematics at Andhra University. We knew about the breakthrough results (mentioned in the introduction) that C. R. Rao obtained before his travel to Cambridge to work under R. A. Fisher. My teacher also used to tell very nice recollections of his teachers mentioned above, and how their teaching style inspired generations of mathematics students at Andhra University. I first heard of the Cramér–Rao bound and Rao–Blackwell Theorem during my college days from my teacher. When I was at ISI Kolkata as a visiting scientist and later as a faculty member I had a great opportunity to know more about the works of C. R. Rao from his long-term friends (J. K. Ghosh, Y. R. Sarma, P. Majumder, etc.) and his PhD students (S. B. Rao, T. J. Rao, A. Sengupta, etc.).

First Meeting Prof. Rao

Although I saw Professor Rao for the first time in 2008–2009 on his lecture tour in India, I did not have close interactions with him until 2010 at his 90th birthday celebration meeting held at ISI Kolkata, India. At that time, I was one of the conveners of those meetings and then I was working as an assistant professor at ISI. All the organizers of the event, students, and faculty members along with C. R. Rao, the late Mrs. Bhargavi Rao, and his daughter Dr. Teja Rao had a cruise trip-cum-dinner on the river Ganges. It was a memorable experience to see him talk to others. During the same visit, he gave an inspiring seminar in the old geology auditorium within the institute to a packed audience consisting of students, faculty, and staff of the institute. It was inspirational for many young colleagues and students to spend time with him on campus. C. R. Rao and his family were given tremendous respect by faculty, staff, and students whenever they would visit the ISI Kolkata campus.

Collaborating on *Handbook of Statistics*

Professor C. R. Rao and I have so far jointly edited eight volumes of the *Handbook of Statistics*. While I was an associate professor at the Medical College of Georgia, news of my research appeared in *Math Digest* of the American Mathematical Society and *AMStat News* of the American Statistical Association in 2015. Professor Rao congratulated me and asked if I could suggest some themes for the *Handbook of Statistics*, a series currently published by Elsevier, which he had been editing for the past 3–4 decades. Working with him gave me a great opportunity to learn the importance of clear communication, how to decide on themes, and how to make things simple in everything we do as academicians. He has been very honest, communicating clearly, committing to his promises, and he is very easy to work with. He respects his family, colleagues, former colleagues, friends, and students. I was very thankful for this opportunity to learn directly from the legend. It also created an opportunity to interact with worldwide researchers in various fields. I was honored to chair a special invited session on C. R. Rao's birthday centenary held at the Joint Statistical Meetings in 2020 (held online only due to COVID-19), where some of the top statisticians gave seminars, and spoke on technicalities of C. R. Rao's contributions [1]. See also related comments by David Banks in this tribute. Very recently, during December 1–2, 2021, I had an opportunity to meet Professor Rao in his residence in Buffalo and he was the same cheerful and respectful person that he always was. He was very well taken care of by his daughter (Dr. Tejaswani Rao) in Buffalo, and his son (Dr. Veerendra Rao) in Pittsburgh, and by other family members. In my recent visit, I also found that he was in good humor. We discussed our ongoing *Handbook* projects as well.

Rao Distance and Conformal Mapping

The idea of Rao distance, published in 1948 [2], has been seen in newer applications in the real world. Steven Krantz and I combined the differential geometric ideas of Rao distances with conformal mappings to demonstrate applications in virtual tourism [4,5]. They show the method of computing distances between two or more points on various geometric structures and provide theoretical background and visualization through their articles.

Let A be a point on a complex plane $\mathbb{C}_1$, and B be a point on a complex plane $\mathbb{C}_2$. Let

$$AB(t) = a < t < b \tag{1}$$

be an *arc* from A to B, and call the *arc* γ. Here $a,b \in \mathbb{R}$. The length $L(\gamma)$ for the *arc* γ can be obtained through the integral

$$L(\gamma) = \int_{\alpha}^{\beta} |AB'[\psi(\tau)]|\psi'(\tau)\, d\tau \tag{2}$$

whenever there exists a complex plane $\mathbb{C}_3$ such that $\mathbb{C}_1 \cap \mathbb{C}_3 \neq \phi$ (empty set) and $A \in \mathbb{C}_1 \cap \mathbb{C}_3$, and $\mathbb{C}_2 \cap \mathbb{C}_3 \neq \phi$ (empty set) and $B \in \mathbb{C}_2 \cap \mathbb{C}_3$. The value of t in (2) can be fit using a parametric function which could be a continuous random variable or through a deterministic model. Such a setting would be helpful in 3D visualizations in virtual tourism projects [4]. Rao distance functions can be made that travel on such *arcs*.

On Complex Plane Bundles

The ideas of differential geometry were extended such that distances between points in a bundle of complex planes could be studied. For example, see [20].

Let us consider a bundle of infinitely many complex planes parallel to each other and call this $\mathbb{B}_{\mathbb{R}}$. Let us keep a complex plane $\mathbb{C}_0$ perpendicular to the bundle $\mathbb{B}_{\mathbb{R}}$, and call this new structure $\mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$. Under certain kinds of randomness $X(z_p)$ introduced at an arbitrary point, say, z_p on a plane $\mathbb{C}_p$ for $\mathbb{C}_p \in \mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$, it was shown in [20] that new kinds of contours passing through $\mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$ could be formed. This led to a new concept called 'multilevel contours.'

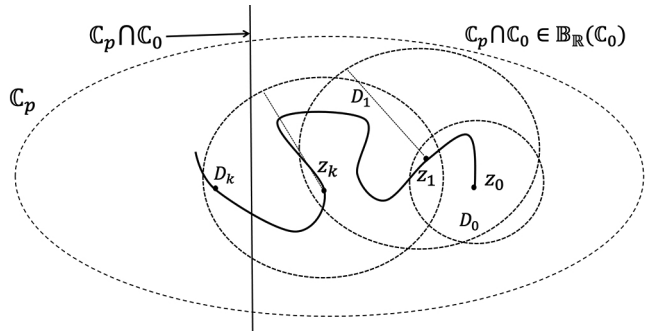

Figure 6. Formation of 'multilevel contours' in a bundle of complex planes.

Theorem [20]**.** A contour formed by the set of points generated by $X(z_p)$ on $\mathbb{C}_p$ for $\mathbb{C}_p \in \mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$ intersecting with the bundle $\mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$ and satisfying a few more properties (explained in [20]) will obey the continuous time Markov property.

Theorem (Spinning of bundles) [20]**.** Suppose the bundle $\mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$ is rotated anti-clockwise such that the line passing through (0,0) of all the planes within $\mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$ forms an angle $\theta(\theta > 0)$ with the y-axis. Suppose the rotation is continued for each $\theta \in (0,360] \subset \mathbb{R}^+$. Then the space created due to such a rotation is in 1-1 correspondence with $\mathbb{B}_{\mathbb{R}}(\mathbb{C}_0)$.

Multilevel contour constructions, apart from showing newer properties on complex planes, are arguably useful in climate analysis to transportation of information from one plane to another plane.

Happy birthday Professor C. R. Rao, and God bless him with continued health and humor!

Figure 7. Arni's visit to Prof. C. R. Rao, Buffalo, New York, during winter 2021 (December 1–2, 2021).

James Rosenberger

My Memories from C. R. Rao's Time at Penn State

Professor C. R. Rao joined Penn State University in 1988, coming from the University of Pittsburgh as the inaugural holder of the Eberly Family Endowed Chair of Statistics. Given that he was almost 68 years old, we thought he may only be active for another couple of years. So, as he reached his 70th birthday, we planned a retirement party for him. It was a joyous occasion, and you can see him here with me as department head, Dean Gregory Geoffroy of the Eberly College of Science at Penn State, and Bill Welch, the mayor of State College. His many friends, family members, and colleagues were also in attendance.

Figure 8. Retirement Celebration with Jim Rosenberger, Dean Gregory Geoffroy, C. R. Rao, and Mayor Bill Welch of State College, PA, in 1990.

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However, his 70th birthday came and went, and the expectation that he would retire was soon waived and forgotten, and he remained active for another decade when he retired from Penn State at the age of 80. During these years, he was a magnet for bringing the world's top statisticians to Penn State, inspiring the department to reach for excellence in hiring faculty and recruiting graduate students. The lectureships he created in honor of Professors Krishnaiah and Khatri provided the platform for many renowned statisticians including every statistician who was a newly elected member of the National Academy of Sciences and other international leaders.

During these years and after retiring, he continued to collaborate and invite visitors to Penn State. His presence provided the catalyst for an intellectual atmosphere in the department that persists today. He was instrumental along with G. Jogesh Babu in bringing John Nash Jr., the Nobel Laureate in Economics from Princeton, to deliver the 2003 Chemerda Lecture Series in Science.

Eberly Professor C. R. also created the *C. R. and Bhargavi Rao Prize* to honor outstanding and influential innovations in the theory and practice of mathematical statistics, international leadership in directing statistical research, and pioneering contributions by a recognized leader in the field. This prize included a lecture and was awarded every two years since 2003 and included Bradley Efron, Jayaram Sethuraman, Lawrence Brown, Peter Bickel, James Berger, Herman Chernoff, Sir David Cox, Donald Rubin, and Grace Wahba.

C. R. was always the perfect departmental citizen. As the inaugural holder of the Eberly Chair in Statistics, he taught both theoretical and applied multivariate courses, served on committees, and participated in the life of the department by hosting and attending social functions that created the collegial atmosphere that characterized our department. He invited visitors from around the world. One example I recall were visitors Professors Yuri Prokhorov and Vassily Sazonov from Moscow State University, when travel from Russia was rare.

It is my pleasure and honor to share these memories of the 15 years when I served as department head along with Eberly Professor C. R. Rao. Congratulations as we celebrate his life during his 101st birthday.

S. R. S. Varadhan

If you were a student of statistics anywhere you would have definitely heard of Dr. Rao when you learned about the Cramér–Rao inequality and the Rao–Blackwell theorem, which are two of the very few important results with an Indian name attached to them. When I started my BSc (Hons) statistics course at Presidency College in Chennai, one of the standard textbooks that we used was his book *Advanced Statistical Methods in Biometric Research*. So the name of Dr. Rao was very familiar to me even as an undergraduate student.

After graduating I applied for and was admitted to ISI as a research scholar. I came to Kolkata (Calcutta then) and met Dr. Rao in person. He was kind and generous. He invited us to a party he had arranged for his son Veera's first birthday and his daughter Teja gave a dance performance. He kept in touch with what we were doing. Although I started with the idea of working in applied statistics, I was influenced by K. R. Parthasarathy and Ranga Rao and joined their group and started working on mathematical issues in probability. He would from time to time ask us what we were doing and encourage us.

The institute had a constant stream of visitors, including Kolmogorov who visited in 1962. I had completed my PhD dissertation and Dr. Rao had asked him to be a reviewer for my thesis and arranged for me to give a seminar describing my work. I was nervous and did not pay attention to the time and went on much longer than the usual one-hour limit. Kolmogorov got up at the end to make some remarks, but people in the audience started to leave. He got angry, threw the chalk down, left the lecture hall, and went back to his room in the guest house. I was really worried and feared that it was the end of my PhD. A group of us ran behind him, caught up with him, and I apologized profusely for not ending on time. His response was: "In Moscow seminars last much longer. I am not angry at you but at the people who left. When Kolmogorov talks people should listen!" Dr. Rao arranged for a group of us to travel with him to visit a few places.

The following year I wanted to visit the US as a postdoc and Varadarajan, who had just returned from the US, suggested that the Courant Institute at NYU would be good for me and wrote to Peter Lax suggesting that they offer me a position. For several months there was no response. I got worried that my plan would not work out and approached Dr. Rao for help. He suggested that I could go to Cornell, and wrote to Jack Wolfowitz. They immediately made me an assistant professor offer by cable and I gladly accepted it. But the very next day I got an offer from Courant and approached Dr. Rao again. I told him I really wanted to go to Courant. He said that it would be OK to explain to Cornell and go to NYU, which is what I did.

Even after I moved to the US he kept in touch with me. I visited ISI-Calcutta when he was still there and ISI-Delhi many times after he moved there. He was always a perfect host. After he moved to Pittsburgh and later to Penn State, I visited him in both places. He nominated me for the Fellowship of the Royal Society. I visited him last September

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in Buffalo and we spent some time together. He is very disciplined and even now he responds quickly to any email correspondence.

I want to join his family, friends, colleagues, and scores of students to wish him a very memorable birthday and many more years of good health.

Yuehua Wu

Memorable Time with Professor C. R. Rao and His Family

I was appreciative and honored to have Professor C. R. Rao, a world distinguished statistician, as my supervisor for my postgraduate degree. To celebrate the 101st birthday of Professor C. R. Rao, the author would like to recollect her memorable time spent with Prof. Rao as his PhD student and contributor to this article.

My PhD Study Under the Supervision of Professor C. R. Rao

Professor C. R. Rao visited China in 1984, before I went to the United States to pursue my PhD program. He was giving a series of talks during his visit. Once I came to know that he was going to give a talk at East China Normal University in Shanghai, I traveled from Nanjing to attend the presentation. The auditorium was full of people that day. Everyone was excited and eager to listen to the talk given by this world esteemed statistician. His voice was graceful and calm but every word in his presentation was inspiring and intuitive. He not only walked us through current hot research topics in statistics, but also provided his invaluable and insightful comments at the same time. His talk led us to a fascinating and promising statistics world. Inspired by his speech, I made up my mind to pursue a PhD degree in statistics. However, I did not dare to dream to be his student then.

Luckily, I was admitted by the University of Pittsburgh for their PhD program in 1985. I was very grateful that I had Professor C. R. Rao as my supervisor.

Professor Rao was very nice and approachable during my study. He was also very supportive and his instructions were also encouraging and inspiring. He provided every student with valuable research opportunities and deep insight into research problems. He constantly directed us to some emerging but also challenging statistical problems and then flexibly let us choose topics we liked to work on. He always encouraged us to attend international conferences and departmental seminar talks to help open our minds to the statistical world.

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Figure 9. Mrs. Rao pointed at Professor Rao's bare foot to explain that Professor Rao did not wear his socks in his photo in the book *Faces of Science* by M. Cook, June 2007. Photo taken in Dr. Rao's home in State College, PA.

Professor Rao and his wife Mrs. Bhargavi Rao were extremely kind and generous. They treated every student and scholar like their family members. During my PhD study, they often invited us to their home and Mrs. Rao used to make us delicious dishes and desserts. As English was my second language, I sometimes could not express myself correctly in my early study. Whenever Professor Rao and Mrs. Rao saw this, they were always willing to help me improve my English. Without a doubt, my PhD study with Dr. Rao was very delightful and unforgettable. Here, I would like to thank Professor Rao for all his guidance and advice during my PhD study, which has benefited me in my career and life remarkably.

My Collaboration with Professor C. R. Rao

I am also filled with gratitude to have been able to continue to collaborate with Professor Rao after I graduated from the University of Pittsburgh. This experience was invaluable in my research career. Our research interests have covered mainly the following areas: (1) model selection; (2) regression clustering; (3) change-point test; (4) M-estimation; (5) multivariate regression analysis. We have coauthored 18 published papers in these areas by now.

During my years of collaboration with Professor Rao, I was greatly impressed that he is not only a super-genius but also has an amazing memory. It seems that he can remember all the material he read before. For example, a few years ago when I was working with him on a challenging statistical problem, and I needed to know what had been done in the literature and all the existing methods and theories for this problem, he gave me the information in detail right away without searching on the computer. Professor Rao has been very supportive and generous to me in my career. I can ask for his advice and help whenever I need

Figure 10. My student Qing Shao, Teja, Dr. Rao, Mrs. Rao, and myself at York University, June 2004.

them. He has also provided me with various opportunities, from writing review papers to giving conference talks, etc., while keeping me aware of cutting-edge statistical problems and current trends in statistics.

In all, Professor Rao is, indeed, my lifetime supervisor in all aspects. It was my great luck and honor to be his student and I am also very thankful for the opportunity to work with him throughout my entire research career. Happy 101st birthday, my lifetime supervisor Professor Rao! I sincerely wish all the best to you and your family.

ACKNOWLEDGMENT. The comments and corrections by the referee (David Banks) helped make the content clearer.

References

[1] B. Efron, S. I. Amari, D. B. Rubin, A. S. R. S. Rao, and D. R. Cox, *C. R. Rao's century*, Significance **17** (2020), 36–38. https://doi.org/10.1111/1740-9713.01424

[2] C. R. Rao, *Information and the accuracy attainable in the estimation of statistical parameters*, Bull. Calcutta Math. Soc. **37** (1945), 81–91. MR15748

[3] B. L. S. Prakasa Rao, R. Carter, F. Nielsen, A. Agresti, A. Ullah, and T. J. Rao, *C. R. Rao's Foundational Contributions to Statistics: In Celebration of His Centennial Year*, AMStat News (2020). https://magazine.amstat.org/blog/2020/09/01/crrao

[4] A. S. R. S. Rao and S. G. Krantz, *Data science for virtual tourism using cutting edge visualizations: information geometry and conformal mapping*, Cell Patterns (2020). https://doi.org/10.1016/j.patter.2020.100067

[5] A. S. R. S. Rao and S. G. Krantz, *Rao distances and conformal mapping*, Information geometry, Handbook of Statist., vol. 45, Elsevier/North-Holland, Amsterdam, 2021, pp. 43–56, DOI 10.1016/bs.host.2021.06.002. MR4382425

[6] Z. D. Bai, C. R. Rao, and L. C. Zhao, *Kernel estimators of density function of directional data*, J. Multivariate Anal. **27** (1988), no. 1, 24–39, DOI 10.1016/0047-259X(88)90113-3. MR971170

[7] Z. D. Bai, P. R. Krishnaiah, and Y. Q. Yin, *Limiting properties of the occurrence/exposure rate and simple risk rate*, Ann. Inst. Statist. Math. **40** (1988), no. 3, 491–505, DOI 10.1007/BF00053061. MR964288

[8] Z. D. Bai, C. R. Rao, and Y. Wu, *M-estimation of multivariate linear regression parameters under a convex discrepancy function*, Statist. Sinica **2** (1992), no. 1, 237–254. MR1152307

[9] Z. D. Bai, C. R. Rao, and Y. H. Wu, *Robust inference in multivariate linear regression using difference of two convex functions as the discrepancy measure*, Robust inference, Handbook of Statist., vol. 15, North-Holland, Amsterdam, 1997, pp. 1–19, DOI 10.1016/S0169-7161(97)15003-9. MR1492706

[10] L. C. Zhao, P. R. Krishnaiah, and Z. D. Bai, *On detection of the number of signals in presence of white noise*, J. Multivariate Anal. **20** (1986), no. 1, 1–25, DOI 10.1016/0047-259X(86)90017-5. MR862239

[11] L. C. Zhao, P. R. Krishnaiah, and Z. D. Bai, *On detection of the number of signals when the noise covariance matrix is arbitrary*, J. Multivariate Anal. **20** (1986), no. 1, 26–49, DOI 10.1016/0047-259X(86)90018-7. MR862240

[12] Z. D. Bai, P. R. Krishnaiah, C. R. Rao, P. S. Reddy, Y. N. Sun, and L. C. Zhao, *Reconstruction of the left ventricle from two orthogonal projections*, Computer Vision, Graphics, and Image Processing **47** (1989), 165–188.

[13] Z. D. Bai, P. R. Krishnaiah, C. R. Rao, Y. N. Sun, and L. C. Zhao, *Reconstruction of the shape and size of objects from two orthogonal projections*, Math. Comput. Modelling **12** (1989), no. 3, 267–275, DOI 10.1016/0895-7177(89)90104-0. MR988077

[14] B. R. Frieden, *Science from Fisher information: A unification*, Cambridge University Press, Cambridge, 2004, DOI 10.1017/CBO9780511616907. MR2069674

[15] B. R. Frieden, *Principle of minimum loss of Fisher information, arising from the Cramer-Rao inequality: its role in evolution of bio-physical laws, complex systems and universes*, Information geometry, Handbook of Statist., vol. 45, Elsevier/North-Holland, Amsterdam, 2021, pp. 117–148, DOI 10.1016/bs.host.2021.07.002. MR4382428

[16] C. R. Rao, *Tests of significance in multivariate analysis*, Biometrika **35** (1948), 58–79, DOI 10.1093/biomet/35.1-2.58. MR25126

[17] Y. Fujikoshi, H. Yanai, and M. Taguri, Japanese Translation of *Statistics and Truth: Putting Chance to Work*, 2nd ed., by C. R. Rao (1989), World Scientific Maruzen Publishing Company, Tokyo, 1993. Its modified version was published in 2010, Chikuma Publishing Company, Tokyo.

[18] M. Taguri, Y. Fujikoshi, H. Yanai, and C. R. Rao, *Introduction to Statistical Reasoning by Real Data*, Kodan Publishing Co., Ltd., Tokyo (in Japanese), 2007.

[19] B. B. Pereira, C. R. Rao, and F. B. Oliveira, *Statistical Learning Using Neural Networks: A Guide for Statisticians and Data Scientists with Python*, Chapman and Hall/CRC, 2020.

[20] A. S. R. S. Rao, *Multilevel Contours on Bundles of Complex Planes*, arXiv e-prints, arXiv:2107.10679 (2021).

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Table 1. Important Milestones and Career Achievements of C. R. Rao

1920: Born in town called Hadagali in India
1937: Completed BA from A.V.N. College, Visakhapatnam
1940: Awarded MA in mathematics, Andhra University, India
1941–45: Published 19 articles before working with R. A. Fisher in Cambridge, UK
1941–79: Served at ISI in various capacities (including professor and director)
1943: Awarded MA in statistics, Calcutta University, India
1945: 11th article [2] later led to famous results such as Cramér–Rao bound, Rao distance, Rao Blackwellization
1947: Foundational paper on Orthogonal Array published from ISI
1948: Foundational paper on Multivariate Analysis of Variance from ISI
1948: Foundational paper on Tests of Significance in Multivariate Analysis from ISI
1948: PhD, Cambridge University, UK, under guidance of R. A. Fisher
1965: Publication of breakthrough book, *Linear Statistical Inference and its Applications* from ISI
1971–76: President, The Indian Econometric Society (TIES), India
1973–75: President, International Biometric Society (IBS)
1976–77: President, Institute of Mathematical Statistics (IMS), USA
1977–79: President, International Statistical Institute (ISI)
1979–88: University Professor, University of Pittsburgh, USA
1988–2001: Eberly Professor, Pennsylvania State University (PSU), USA
1991–2010: Director, Center for Multivariate Analysis, PSU, USA
1995: Fellow of National Academy of Sciences, USA
1996: Appeared in the list of 57 major contributions in probability and statistics during 16th–20th centuries, created at the University of Texas, El Paso, USA
2000: Appeared in the list of 35 major contributions in probability and statistics since 1650, created at the University of Southampton, UK
2001: Awarded Padma Vibhushan, Second Highest Civilian Award, India
2002: National Medal of Science, Highest Science Award, USA
2003: International Mahalanobis Prize
2007: Institute named after C. R. Rao was inaugurated in Hyderabad, India
2010: Guy Medal in Gold, Royal Statistical Society, UK
2010: Research Professor, University of Buffalo, SUNY, USA
2018: 39th honorary doctorate received. Total honorary doctorates received: 39 from 19 countries and 6 continents
2022: IEEE Honorary Membership, elected by the Board of Directors of IEEE from among outstanding individuals (non-members of IEEE)