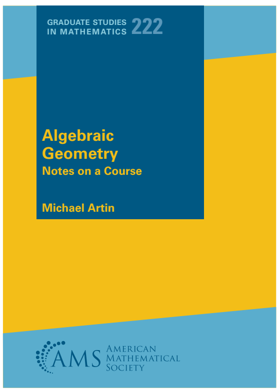
AMS BOOKSHELF

The AMS Book Program serves the mathematical community by publishing books that further mathematical research, awareness, education, and the profession while generating resources that support other Society programs and activities. As a professional society of mathematicians and one of the world's leading publishers of mathematical literature, we publish books that meet the highest standards for their content and production. Visit **bookstore.ams.org** to explore the entire collection of AMS titles.



GSM/222, 2022

Algebraic Geometry

Notes on a Course

By Michael Artin

In his article in "Princeton Companion to Mathematics," Janos Kollar defines algebraic geometry as follows: "Succinctly put, algebraic geometry is the study of geometry using polynomials and the investigation of polynomials using geometry." This attempt to marry algebra and geometry, two quite distant and seemingly unrelated areas of mathematics, led to algebraic geometry, which is a notoriously difficult subject to learn and, therefore, to teach.

One explanation for this difficulty may be that geometric constructions appeal to pictures and images (handled, according to neurologists, by the right brain) whereas algebra involves complicated formal constrictions (handled by the left brain). The struggle between the two approaches is well illustrated by the history of the subject: compare the Italian school (Cremona, Corrado and Beniamino Segre, Castelnuovo, Enriques, Albanese, Bertini, Severi, del Pezzo, among others), which relied heavily on geometric approach and the Grothendieck school (Grothendieck himself, Serre, Deligne, Artin, Tate, Mumford, and many others), which used complicated algebraic constructions. Oscar Zariski is particularly important for his role as a "bridge" between these two schools.

In the last 50 years, there were many attempts to present algebraic geometry to the mathematical community, ranging from books for undergraduates (T. Garrity et al, AMS, 2013) to books for professionals (A. Grothendieck's EGA volumes, IHES, 1960–1967), from books emphasizing geometric aspects (P. Griffiths–J. Harris, Wiley 1978; D. Mumford, Springer, 1976) to books which build algebraic geometry starting with strong algebraic foundations (J. Harris, Springer, 1992; D. Cutkosky, AMS, 2018). Among these books, the most influential ones were those where the authors tried to balance algebra and geometry (I. Shafarevich, Springer, 1974; R. Hartshorne, Springer, 1977). All of these books, and others, have led to spectacular applications of algebraic geometry to many areas of mathematics, including number theory (the proof of Weil's conjectures and Fermat's Last Theorem) and mathematical physics (work by Witten and his school).

The book *Algebraic Geometry: Notes on a Course*, written by Michael Artin, one of the most active and notable modern algebraic geometers, is the latest attempt to reach the right balance between algebra and geometry. It is based on a course for advanced undergraduates and beginning graduate students that the author taught for many years at MIT, and on the feedback he received from his students and colleagues.

The book starts with a chapter about curves on the plane, which provides instructive examples for the material in later chapters. Chapters 2 and 3 introduce algebraic geometry of affine and projective varieties, respectively. Morphisms of algebraic varieties and their geometric properties are presented in Chapter 4. The structure of affine and projective varieties in the (Zariski) topology are described in Chapter 5. In Chapters 6 and 7, sheaves and their cohomology are studied. Finally, in Chapter 8 the author returns to algebraic curves proving the Riemann–Roch Theorem and using it to define the group law on points of an elliptic curve.

Packing a course on algebraic geometry into less than 350 pages requires making several important choices. Artin's choice is to restrict the exposition to the maximal spectrum of a ring and to varieties of complex numbers saying that, in his opinion, "… absorbing schemes and general ground fields won't be too difficult for someone who is familiar with complex varieties." On the other hand, he discusses in detail such crucial notions as integral morphisms, sheaves and their direct and inverse images under a morphism, and the cohomology of sheaves.

The author says in the preface that his goal in teaching algebraic geometry is "to make the development so natural as to seem obvious." The overall impression when reading this book is that he succeeds in reaching his goal.

The AMS Bookshelf was contributed by AMS Book Publisher Sergei Gelfand. For questions or comments, please send email to sxg@ams.org.