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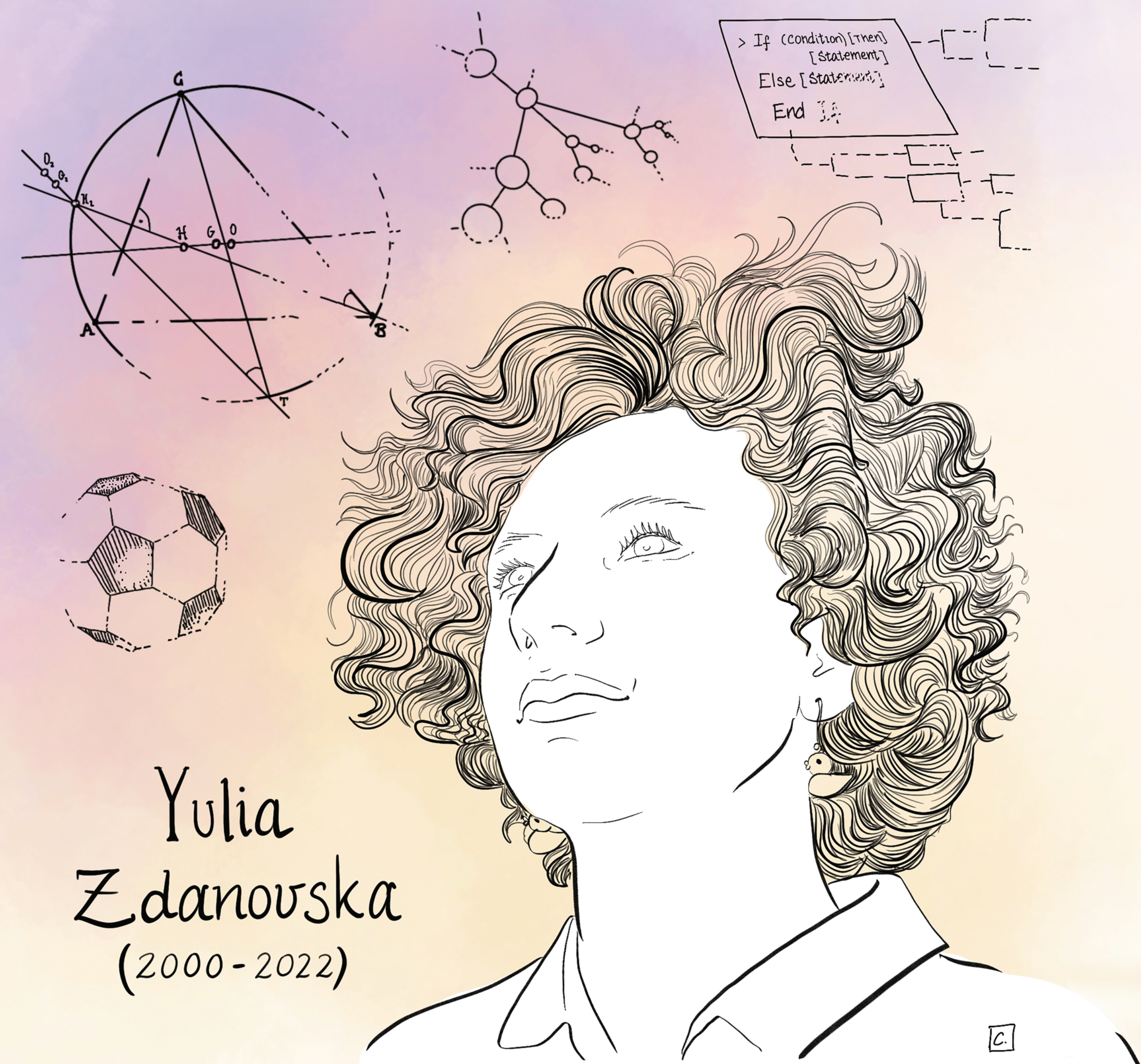

Yulia Zdanovska was a Ukrainian mathematician who died in a fire caused by a Russian missile that hit her residential area in Kharkiv.

American Mathematical Society

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Notices
of the American Mathematical Society

March 2023

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FEATURES

FROM THE AMS SECRETARY

INVITATIONS FROM THE AMS

ERRATUM. In "Isadore M. Singer (1924–2021) In Memoriam Part 2: Personal Recollections" in the November 2022 *Notices*, the affiliation for Nancy Stanton should read "Nancy Stanton is a professor emerita of mathematics at the University of Notre Dame. Her email address is `stanton.1@nd.edu`." Additionally, the correct email address for Lenore Blum is `lblum@cs.cmu.edu`.

Notices

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The American Mathematical Society is committed to promoting and facilitating equity, diversity and inclusion throughout the mathematical sciences. For its own long-term prosperity as well as that of the public at large, our discipline must connect with and appropriately incorporate all sectors of society. We reaffirm the pledge in the AMS Mission Statement to "advance the status of the profession of mathematics, encouraging and facilitating full participation of all individuals," and urge all members to conduct their professional activities with this goal in mind. (*as adopted by the April 2019 Council*)

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LETTERS TO THE EDITOR

A Call to Defend Bodily Autonomy in the Mathematics Community

This statement has been abbreviated; view the full text and a list of signatories here: https://sites.google.com/view/bodily-autonomy-in-math.

The Supreme Court decision overturning Roe v. Wade has a profound impact on the rights of members of our community. We stand with everyone whose rights to medical care including abortions, bodily autonomy, and privacy this reactionary decision attacks.

This decision follows on the heels of escalating attacks on the bodily autonomy of trans people across the US. We are deeply concerned by these rulings' impact on many people in our mathematical community who already face a lack of support for their bodily autonomy, including family planning decisions and access to healthcare, medical leave, and parental support. Moreover, too many departments are hostile to pregnant people, new parents, and trans people, for instance by denying them parental and/or medical leave. This insufficient support and hostility are exacerbated for those in contingent or non-tenure-track positions, including graduate students and staff, and those already oppressed along multiple axes, for example race, gender, sexuality, religious affiliation, ability, or size. These issues are intertwined; departments can and must do more to build structures that support the bodily autonomy and other rights of all of their community members.

In the context of the new attacks on reproductive rights and transgender rights, we call on departments to adopt the following protocol:

1) Two weeks of medical leave, no questions asked, per academic term.
2) Accessible gender neutral bathrooms, with changing stations, as well as rooms to breast/chest feed and pump.
3) Explicit support and information for anyone looking for abortion and/or trans affirming care.
4) Parental leave for all department employees including grad students, postdocs, non-tenure-track faculty, and staff, with clear written policies.
5) A standing fund to support departmental members who are in need of financial support for childcare or other services to attend conferences, which organizations like the NSF will not support. (See Question 6 in this Q&A.)
6) Clear and accessible policies for name changes in all departmental contexts, and support of this process in institutional matters.

When departments are unable to implement these actions without institution-wide approval, we call on them to vocally advocate for these changes within their institutions. The traditional academic trajectory in mathematics is designed to accommodate able-bodied, cishet white men who have familial and spousal support to accommodate their personal needs. It is important for departments to examine how their structures continue to reinforce a culture that punishes child bearers and child rearers, especially in hiring, promotion, and tenure.

Individual mathematicians have a responsibility to act on behalf of the wider community when participating in conferences and visiting colleagues in states that have restricted access to reproductive and gender affirming healthcare. We ask that mathematicians who sign this statement commit to the following actions, as applicable:

1) As a conference organizer, provide a clear statement of support for abortion care and trans rights and a virtual participation option. This is also an important accessibility concern for disabled and immunocompromised people who may face greater risk and/or difficulty in traveling to in-person events. (See the Spectra statement for further suggestions related to conference organization.)
2) As a speaker, strongly advocate that any conference to which you are invited offers an option for virtual participation.
3) As an attendee or when visiting a colleague, donate to a local abortion fund that supports abortion access where you are visiting.

Finally, we are ***not*** asking for a blanket boycott of conferences in states where abortion and medical care for trans people have been criminalized; such boycotts should not

**We invite readers to submit letters to the editor at* notices-letters@ams.org.

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be implemented without an explicit call from local community organizers. No US state is safe for all marginalized people as the US was built on the genocide and enslavement of Black and Indigenous peoples and the legacy of this history is still very much alive today. Conference organizers should be aware and transparent about the risks that participants take on when attending conferences in different locations.

Federico Ardila (San Francisco State University)
Juliette Bruce (Brown University)
Matthew Durham (University of California, Riverside)
Barbara Fantechi (Scuola Internazionale Superiore di Studi Avanzati)
Mark Hagen (University of Bristol)
Denis Hirschfeldt (University of Chicago)
Marissa Loving (University of Wisconsin–Madison)
Seppo Niemi-Colvin (Indiana University Bloomington)
Florencia Orosz Hunziker (University of Denver)
Priyam Patel (University of Utah)
Candice Price (Smith College)
Emily Riehl (Johns Hopkins University)
Noelle Sawyer (Southwestern University)

Letter to the Editor

While I love Boston, I did not attend this year's JMM as, being unmistakably of Jewish appearance, I was concerned for my safety. Antisemitism is on the rise throughout the country, on college campuses in particular, and unfortunately Boston is Ground Zero. Tufts has drawn attention for this since the mid-2010s. Harvard led the country in antisemitic incidents during the 2021–22 academic year with 25. The Wellesley University student newspaper endorsed the Mapping Project, which publicizes the addresses of Jewish organizations in the Boston area, along with those of anyone who has had contact with them, with their intention to ″disrupt″ and ″dismantle.″

Antisemitism differs from other forms of hatred in that there are more calls for violence and that it is found all over the ideological spectrum. There is more excusal, such as celebrities defending other celebrities, than with other forms of hatred. It also leads to people who ignore the plight of the Uyghur Muslims in China (and of the citizens of the 50+ totalitarian regimes in the world) while targeting American Jews, who have no involvement with the Israeli government's policies.

I feel that, if history is an indicator, if the growing antisemitism is not quashed immediately, it will be detrimental to the advancement of mathematics in the United States. Anyone who has studied the history of mathematics knows that while Spain dominated world mathematical activity during the first half of the last millennium, this ceased after 1492 with the Inquisition. The *Notices of the AMS* had a feature a few years ago about Jewish mathematicians in Germany, accompanied by a graph that showed their active years, most of which tragically ended in the early 1940s. While not as pronounced as is medieval Spain and prewar Germany, Jews in the United States still represent a disproportionate percentage of mathematics faculty and I'm sure that everyone wants their Jewish colleague friends to remain safe (remember that antisemites see Jews who are not (yet) observant and/or non-Zionist as Jews).

While to my knowledge all mathematics departments are still safe, many campuses are not and, based upon news reports, antisemitism is increasing by the day. It is the professors who run the universities in this country, not the students. For the sake of mathematics in this country as opposed to a return to 1493 Spain or 1946 Germany, I am requesting that all mathematics professors, both Jewish and non-Jewish, speak out when they observe antisemitic activity on campus. It cannot just be that the administrations around the country disagree with the views of the students but say that they are entitled to their views. As the adage goes, free speech does not entitle one to yell ″fire″ in a crowded theater and neither does it entitle the expression of any antisemitic thoughts that can lead to the physical harm of Jews on campus or worse.

Rabbi Marc E. Jacobson, PhD
(retired; last academic appointment Wharton Dean's Fellow, University of Pennsylvania)

A Beautiful Mind With A Genuine Soul

In loving memory of Prof. James E. Joseph (1937–2022), who passed away peacefully on December 8, 2022. Prof. Joseph was an internationally known topologist. He dedicated his life to mathematics and education.

A beautiful mind with a genuine soul,
Rare a combo but delight to know;
The soul has passed so peacefully,
Leaving those who knew him well
Shocked in grasp, voice that gave
Comfort and counsel now silent!

A student to the core, a teacher to many,
To students, often his silence eloquent,
A listening heart and knowing mind,
Astute, leaves no posturing around.
Saying much with much too little,
Alas, that silence is just silent!

A person who spoke with actions,
James Joseph knew that well.
Dollar bills and checkbook in pocket,
Helping hand for a struggling student

To pay the bills and keep up work,
And to those who make the pavements home.

Topology, and Math in general,
Beautiful mind that rapt them well;
Eager to go in depth seeking elegance
Not knowing limitations, but beauty
With 'limits' and limitless horizon,
Seeking unifying capsule to encompass all.
That fertile mind is no more
To guide, to render, to share!

Field of mathematics lost a genius,
To his students, lost a lasting mentor,
To his friends, lost a genuine hand,
To humanity, lost a shining star.
Now he is the Soul, the Spirit,
The Energy which fills us all.
Goodbye Joe, but no Goodbye at all!

Bhamini M. P. Nayar
Department of Mathematics
Morgan State University
December 11, 2022
This was presented at a small gathering of seven of Prof. Joseph's close friends, paying tribute to him and reminiscing him on December 21, 2022.

A WORD FROM…

Marie-Françoise Roy, Chair of the International Mathematical Union's Committee for Women in Mathematics

The opinions expressed here are not necessarily those of the Notices *or the AMS.*

Photo courtesy of Université de Rennes 1.

Building a Worldwide Community of Women Mathematicians

Women's History Month brings focus to the accomplishments of women and is celebrated in this issue of the *Notices of the AMS*. I am very grateful for the opportunity to share some information about the Committee for Women in Mathematics (CWM),[1] a recently established organization created by the International Mathematical Union (IMU) Executive Committee in 2015. We are a small committee of ten members—eight female and two male—with members from all parts of the world. On this occasion, I'd like to introduce AMS members to one of the main CWM initiatives.

A CWM initiative: the World Meeting for Women in Mathematics $(WM)^2$

One of our main initiatives is the World Meeting for Women in Mathematics $(WM)^2$, which I decided to focus on, rather than trying to give a complete overview of all our activities.

In many ways, $(WM)^2$ is a continuation of the two successful International Congresses of Women Mathematicians (ICWM) held in Hyderabad 2010 and Seoul 2014 on the occasion of the International Congress of Mathematicians (ICM). However, ICWM did not have a very clear status with respect to the ICM. Moreover, the wording of ICWM looked awkward to us as it seemed to say that ICWM was the congress of women, and that women were not fully part of the ICM.

During the first CWM meeting held in Cortona (Italy) in September 2015, which I organized with CWM vice-chair Caroline Series, we decided to create a new event, $(WM)^2$. Our aim was to encourage women in mathematics from all over the world to meet every four years, and to participate fully in the ICM as well.

The plan was to create a regular satellite event of the ICM, taking place the day before the opening of the ICM in the same venue. The program would include a mathematical component, highlighting the diverse contributions by women in mathematics from the part of the world where the ICM was taking place. It would also include discussions on various aspects of the situation of women in mathematics worldwide, and the inauguration of exhibitions and films specially created for the occasion.

The first $(WM)^2$ and its consequences

The first $(WM)^2$ took place in Rio in 2018.[2] The scientific committee, chaired by Georgia Benkart, selected a keynote lecturer (Monique Laurent), a public lecturer (Maria Esteban) and three invited lecturers from Latin America—Alicia Dickenstein (Argentina), Salome Martinez (Chile), and Maria Eulalia Vares (Brazil). The program also included group discussions, posters (both mathematical and on women in mathematics), the world premiere of the film *Journeys of Women in Mathematics*, and a tribute to Maryam Mirzakhani, who passed away on July 14, 2017.

Marie-Françoise Roy is a professor emerita of mathematics at Université de Rennes 1, France, and was the chair of the Committee for Women in Mathematics of the International Mathematical Union from 2016 to December 2022. Her email address is marie-francoise.roy@univ-rennes1.fr.

[1] https://www.mathunion.org/cwm

[2] https://2018.worldwomeninmaths.org/

The *Remember Maryam Mirzakhani* Exhibition[3] created by Thaís Jordão (curator) and Rafael Meireles Barroso (designer), consisting of 18 original posters, was inaugurated at $(WM)^2$, remained open during the ICM 2018, and has been shown in dozens of venues, including in Iran, since then.

The short film *"Journeys of Women in Mathematics"*[4] was created by CWM thanks to the support of the Simons Foundation. The focus of the film is the diversity and common passion of women mathematicians worldwide. The first part of the film, premiered at $(WM)^2$, presented three women active in organizing regional and transnational networks: Carolina Araujo (Rio), a CWM member, the $(WM)^2$ main organizer, and also an ICM lecturer and member of the local ICM organising committee; Neela Nataraj (Mumbai), CWM associate and active member of Indian Women in Mathematics; and Amina Pecha (Yaounde, Cameroon), the founding chair of the Cameroon Women in Mathematics Association. All three were interviewed in their home countries, and attended $(WM)^2$. They appear together in the final version of the film which gives a lively presentation of the atmosphere at $(WM)^2$ and features interviews of six women in mathematics from Latin America: including the three invited lecturers as well as Natalia Garcia (Mexico), Jaqueline Godoy Mesquita (Brazil), and Carolina Neira Jiménez (Colombia).

The $(WM)^2$ 2018 was a wonderful event and gathered around 350 participants, mostly women, from 51 different countries. Many of them were coming from developing countries, thanks to the support of the OpenArms program, with free registration offered by CWM. Twenty Brazilian Black women mathematicians received special support by Serrapilheira (a Brazilian private funding agency) and participated actively in $(WM)^2$ and the ICM.[5] They started a group, Matematicas Negras, which has remained active since.[6]

The book *World Women in Mathematics 2018: Proceedings of the First World Meeting for Women in Mathematics—$(WM)^2$* was published at the end of 2019 as a special volume of the AWM Springer Series, dedicated to the memory of Maryam Mirzakhani.[7]

The May 12 initiative

During the $(WM)^2$ 2018, Ashraf Daneshkhah, on behalf of the Women's Committee of the Iranian Mathematical Society, proposed that May 12, the birthday of Maryam Mirzakhani, become a celebration day for women in mathematics. This proposal was approved by the participants of $(WM)^2$, giving rise to the "May 12 Initiative," led by several organizations for women in mathematics worldwide (European Women in Mathematics, Association for Women in Mathematics, African Women in Mathematics Association, CGD-UMALCA the UMALCA's Gender and Diversity committee, Indian Women in Mathematics, and the Women's Committee of the Iranian Mathematics Society). At the same time, the IMU was involved in an international campaign to have March 14 (Pi day) recognized as International Day of Mathematics by UNESCO, and there was concern that another international initiative involving the IMU would create confusion. For this reason, CWM was not directly involved in the establishment of the May 12 initiative, even though I, and several other CWM members, have been active in the coordination group of May 12 on an individual basis.

The call for action sparked an overwhelming response, resulting in more than 100 events being organized in its inaugural year 2019, showing that the initiative fulfils a strong need.[8]

In 2020, because of the Covid pandemic, the main activity was individual screenings of the one-hour documentary film *Secrets of the Surface, the Mathematical Vision of Maryam Mirzakhani*, organized through an agreement between Zala Films and the May 12 initiative. More than 20,000 requests were received from 131 different countries.[9]

[3] https://www.mathunion.org/cwm/cwm-initiatives/remember-maryam-mirzakhani

[4] *The 20 minute film can be found at* https://www.youtube.com/watch?v=uNJ7riiPHOY, *with nearly 9000 views. The film is also posted on IMAGINARY's platform:* https://imaginary.org/fr/node/1491. *It is also directly available on the CWM website:* https://www.mathunion.org/cwm/cwm-initiatives/journeys-women-mathematics.

[5] *See the documentary* Potência Negra *(in Portuguese):* https://www.youtube.com/watch?v=untetrh5MyM.

[6] *See their anti-racism manifesto on CWM News:* https://www.mathunion.org/cwm/news-and-events/2020-06-14/brazilian-group-black-women-mathematicians-anti-racism-manifesto, *inluding contact email and instagram info.*

[7] *The complete report of $(WM)^2$ 2018 can be found here:* https://www.mathunion.org/fileadmin/CWM/Initiatives/short_report_wm2.pdf.

[8] *"May 12: Celebrating Women in Mathematics—From One Idea to One Hundred Events," published in* Notices of the AMS, *Volume 66, Number 11, 2019.*

[9] *For more information on May 12, in particular the 2021 and 2022 campaign, see* https://may12.womeninmaths.org/.

The second (WM)² and the changes it went through

The second (WM)² was originally planned to take place in St Petersburg on July 5, 2022, the day between the IMU General Assembly and the opening of the ICM.[10] Approved as an official satellite conference by the ICM Local Organizing Committee (LOC), it was granted significant funding, together with meeting rooms (at the Expoforum where the ICM was planned), and technical and logistical assistance. We also had an exhibition project *МАТЕМАТИКА, through a land of mathematics* based on ten interviews with women in mathematics across Russia, led by Olga Paris-Romaskevich. These heroines were researchers, administrators, teachers, with some of them involved in the mathematical olympiad movement. The goal was to sketch a portrait of modern Russian mathematics, through a series of personal stories.[11]

This project was prepared especially for the (WM)² and the exhibition was planned to be inaugurated at the meeting. Funding was available from the ICM LOC for the exhibition space, printing, and setting up of the exhibition, which was to remain open during the ICM 2022.

Following the war of Russia against Ukraine, which started on February 24, 2022, and the IMU's decision to transform ICM 2022 into a virtual event, vICM, with no connection with Russia, (WM)² had to be entirely redesigned. The preparation of the exhibition *МАТЕМАТИКА, through a land of mathematics*, was paused, since its authors did not feel that it could be continued to be prepared in the same spirit as before, indeed presenting joyful stories about Russian mathematics seemed totally out of place at this time.[12]

The date of (WM)² was modified to accommodate the rescheduled Ceremony for the IMU Prizes on July 5, 2022. The second (WM)² became a mostly virtual event taking place on July 1–2, 2022.

The first session of (WM)² 2022 was entirely virtual and took place on July 1, 2022.[13] The first part of the program featured four plenary online lectures by distinguished female mathematicians, selected in 2021 by the scientific committee chaired by CWM member Cheryl Praeger, and presented in the following order: Maryna Viazovska, Natalia Maslova, Mina Aganagic, and Eugenia Malinnikova. In the second part, the panel discussion "Girls and Mathematics: reflections and initiatives" took place with six panelists from all continents. It was devoted to the memory of Yulia Zdanovska. Yulia was a silver medalist at the European Girls' Mathematical Olympiads (EGMO) in 2017, a brilliant young mathematician with a successful future ahead of her. The young woman refused to leave Ukraine amid the war and, working as a volunteer, died in a fire caused by a Russian missile that hit her residential area in the eastern city of Kharkiv. She was 21 years old.

The ICM LOC had made plans to celebrate the 100th birthday of Olga Alexandrovna Ladyzhenskaya (OAL) and establish the OAL Medal in Mathematical Physics, to be awarded without any age or gender restriction. All of these plans collapsed when the war started. Several people worked very hard with CWM to rescue the beautiful idea of celebrating OAL in 2022 and inaugurating a prize named after a woman. The Simons Foundation agreed to fund the prize. Finally, the program of (WM)² on July 2, 2022 was a session organized jointly with the Probability and Mathematical Physics ICM satellite in Helsinki: the OAL Celebration, both live and broadcast.[14]

The program included the world premiere of a film about Olga Alexandrovna Ladyzhenskaya by Ekaterina Eremenko, and the inaugural awarding of the OAL Prize. The presentation of the prize was made by former IMU President Ingrid Daubechies, through a video, and the name the prize winner, Svetlana Jitomirskaya, was announced in a short film. As Ingrid explained, it was a wonderful coincidence that the inaugural OAL Prize, named after a great woman mathematician from Russia was awarded to a woman born in Ukraine, given that the selection process was finished before the war started. The laudatio was given by Artur Avila, while Svetlana's lecture was recorded and became her plenary lecture at the vICM, and the closing vICM lecture.

The remaining activities originally planned for (WM)² 2022, the poster session and the group discussions, were unfortunately cancelled, as was the exhibition *МАТЕМАТИКА, through a land of mathematics*.

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[10] https://2022.worldwomeninmaths.org/

[11] *See the webpage of the project here:* https://matematika.mathematiquesvagabondes.fr/about/.

[12] *See the text of the authors:* https://matematika.mathematiquesvagabondes.fr/.

[13] *See the record here:* https://www.youtube.com/watch?v=CQH5op3T2Z0.

[14] *See the record here:* https://www.youtube.com/watch?v=I-l91hWlD5Q&t=7s.

In spite of the difficult context, $(WM)^2$ was a very successful event. About 1100 people registered for $(WM)^2$ from more than 100 countries. There are plans to publish another proceedings book as a special volume of the AWM Springer Series.

I attended the IMU award ceremony in Helsinki with CWM vice-chair Carolina Araujo on July 5, and we were deeply moved when the Fields Medal was awarded to Maryna Viazovska from Ukraine, who happened to be the first $(WM)^2$ 2022 plenary speaker.

The third $(WM)^2$ in 2026 in Philadelphia (USA)?

CWM hopes to organize the World Meeting for Women in Mathematics—$(WM)^2$—on July 21, 2026, at the Pennsylvania Convention Center, with the full support and logistical help from the LOC of ICM 2026, similar to what was done/planned in 2018/2022 (before the war changed the plans). Preliminary positive contacts have been made with the ICM Organizing Committee.

Reaching gender equality in mathematics all over the world will take tremendous effort, and a lot of time. Working together to support the international community of women in mathematics, we can accelerate this transformation. I hope that the World Meeting for Women in Mathematics will remain instrumental in moving forward the network of women in mathematics all over the world.

Susan Montgomery: A Journey in Noncommutative Algebra

Yevgenia Kashina

Susan Montgomery was born in 1943 and grew up in Lansing, Michigan. She did her undergraduate studies at the University of Michigan where her advisor was J. E. McLaughlin, who inspired her interest in algebra. Having obtained her undergraduate degree in Mathematics in 1965, Susan received an NSF Graduate Fellowship and started her graduate studies at the University of Chicago. In 1969, she defended her Ph.D. thesis titled "The Lie Structure of Simple Rings with Involution of Characteristic 2" under the supervision of I. N. Herstein. She then spent one year at DePaul University. In 1970, Susan joined the faculty at the University of Southern California, where she currently is a professor.

Susan received multiple awards and recognition, including a John S. Guggenheim Memorial Foundation Fellowship in 1984, an Albert S. Raubenheimer Distinguished Faculty Award from the Division of Natural Sciences and Mathematics at USC in 1985, and a Gabilan Distinguished Professorship in Science and Engineering at USC in 2017–2020. In 2012, Susan was selected as an Inaugural Fellow of the AMS and elected as a Fellow of the American Association for the Advancement of Science.

Susan served on many professional society committees, including the AMS Board of Trustees in 1986–1996, the Board on Mathematical Sciences of the National Research Council in 1995–1998 and its Executive Committee in 1997–1998, and the AWM Scientific Advisory Committee in 2015–2017. Furthermore, she was elected as vice president of the AMS for the 2014–2017 term. Moreover, she served as the chair of the Mathematics Department at USC in 1996–1999.

While in her early work Susan studied rings with involutions and group actions on rings, her current research is focused on Hopf algebras and their representations. She has published two books and more than 100 research papers; in addition, she was a coeditor for seven collections of papers on various topics in algebra. Her monograph *Hopf algebras and their actions on rings* became the most cited book on Hopf algebras and quantum groups. Below we discuss Susan's research accomplishments, focusing on several important topics.

1. Group Algebras and Rings with Involution

The first thesis problem on which Susan worked as a graduate student was to determine whether, in characteristic p, left invertible elements of a group algebra are also right invertible. This property was observed by Kaplansky in characteristic 0; his proof followed from results concerning von Neumann algebras. Montgomery was able to give a shorter proof of the characteristic 0 case, based on the properties of C^*-algebras instead (see [Mon69]), but the characteristic p proof eluded her for almost two years. This question still remains open, even though many mathematicians have tried to solve it in the last fifty years.

Susan's second thesis problem was on Lie simplicity of simple rings with involution of characteristic 2. Given an associative ring R, one can introduce a Lie structure on R by defining the Lie product via $[x, y] = xy - yx$ for all x, y in R. This definition immediately raises the question how the ideal structures of R as an associative ring and R as a Lie ring are connected. In particular, if R is simple as an associative ring, what can we say about its Lie simplicity? One of the motivations for such a study, given by Herstein, was to investigate whether the simplicity of four infinite families of simple, finite-dimensional Lie algebras defined as matrices is in fact a consequence of the simplicity of the associative matrix algebra over a field. In the 1950s, there was a series of papers where these questions were considered; in particular, Herstein and Baxter proved several results about the Lie structure of the skew-symmetric elements of a simple ring with involution of

Yevgenia Kashina is a professor of mathematics at DePaul University. Her email address is ykashina@depaul.edu.

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characteristic not equal to 2. The case of characteristic 2 was still unknown at the time; it then became the main topic of Susan's thesis. We will now describe her results on Lie simplicity of simple rings with involution of characteristic 2, which were later published in [Mon70]. Note that, for many of the results discussed below, the rings were not required to have identity; however, for simplicity, we will always assume that R is a ring with identity.

Recall that, if $A \subseteq B$ are additive subgroups of R, we say that A is a Lie ideal of B provided that $[A,B] \subseteq A$, where $[A,B]$ is an additive subgroup of R generated by the commutators $[a,b]$ for all $a \in A$ and $b \in B$. Let R be a simple ring, that is, R has no proper nontrivial two-sided ideals; then its center Z is a field. Assume, in addition, that R has an involution *, which is, by definition, a self-inverse anti-automorphism of R, and that * fixes every element of Z. Let S and K be respectively the sets of symmetric and skew-symmetric elements of R and let $V = \{x + x^* \mid x \in R\}$ be the set of trace elements. It is easy to see that $V \subseteq S$ are additive subgroups of R and K is a Lie subring of R. Herstein and Baxter obtained the following results on characterization of Lie ideals of K and $[K,K]$ in the case of characteristic different from 2:

Theorem 1. *Let R be a simple ring with involution of characteristic not 2 and assume that $\dim_Z R > 16$. Then*

1. *(Herstein) Every Lie ideal of K either is contained in Z or contains $[K,K]$.*
2. *(Baxter) Every proper Lie ideal of $[K,K]$ is contained in Z.*

Note that, since Z is fixed by the involution, the intersection of K and Z is trivial. Thus every proper Lie ideal of $[K,K]$ is trivial and therefore the latter part of the theorem implies that $[K,K]$ is a simple Lie ring. Moreover, since the 4×4 matrices over a field, with the transposition used for the involution, do not behave well, the condition $\dim_Z R > 16$ is necessary.

In her thesis, Montgomery investigated the case of characteristic 2. It turned out that, in contrast to the case of characteristic not 2, the Lie ideals were now characterized in terms of $[V,V]$, and not $[K,K]$. Moreover, Montgomery showed that in this situation $[V,V] = [[S,S],S]$ and therefore the principal difference is that previously the main object of study was the Lie square of K, whereas in characteristic 2 the main object becomes the Lie cube of S. Montgomery's results can be summarized in the following theorem:

Theorem 2. *Assume that R is a simple ring with involution of characteristic 2 and that $\dim_Z R > 144$. Then*

1. *Every Lie ideal of V either is contained in Z or contains $[V,V]$.*
2. *Every Lie ideal of S either is contained in Z or contains $[V,V]$.*
3. *Every proper Lie ideal of $[V,V]$ is contained in Z.*

In light of the Lie square of V being equal to the Lie cube of S, the latter part of the theorem implies the following Lie simplicity result:

$$\frac{[[S,S],S]}{[[S,S],S] \cap Z} \quad \text{is a simple Lie ring.}$$

The methods used to prove the theorem involved applications of several results on polynomial identities; the degree of one of these identities was 144 which caused the condition on the dimension of R over Z. Then, in joint work with Lanski, Montgomery used similar methods to extend her results to prime rings of characteristic 2.

Montgomery spent most of the next few years studying rings with involution, in particular when properties of the ring were inherited by the symmetric elements. Note that one can introduce a Jordan structure on R by defining a new product via $x \circ y = xy + yx$ for all x, y in R; then the set S of symmetric elements becomes its Jordan subring. Using this observation, Montgomery obtained many important results describing the connection between the ideal structure of R as an associative ring and R as a Jordan ring, as well as the ideal structure of the Jordan ring S.

2. Fixed Rings of Automorphism Groups of a Ring

Figure 1. Susan Montgomery at USC in 1971.

Since the techniques used by Montgomery in her studies of rings with involution relied on polynomial identities, she was led to investigate the question of when having a subring satisfying a polynomial identity implies that the entire ring satisfies a polynomial identity. Recall that a ring satisfies a polynomial identity and is called a PI-ring if there is a polynomial with integral coefficients in noncommuting variables which vanishes under the substitutions from the ring; PI-rings generalize the class of rings which are finitely generated as modules over their center. In [Mon74], Montgomery proved several results for the subrings being centralizers and, as an application, gave an affirmative answer to a question raised by Bjork; namely, she showed that under certain conditions the ring itself satisfies a polynomial identity if the fixed subring of an automorphism group satisfies a polynomial identity. A natural direction from this work was studying fixed rings R^G of automorphism groups G of a ring, in particular, the connection between the structure of the ring and its fixed subring. Another important problem is to investigate the relationship between the fixed ring R^G and the skew group ring $R * G$.

Here, we say that a group G acts via automorphisms on a ring R if there exists a homomorphism from G to $\mathrm{Aut}(R)$, the group of automorphisms of R. Often this homomorphism is going to be injective, so that G could be considered as a subgroup of $\mathrm{Aut}(R)$. For any $g \in G$, the action of g on R will be denoted by $r \mapsto g \cdot r$. An automorphism g of R is called inner if there exists a unit in R such that g acts on R via conjugation with this unit; otherwise g is called outer. A subgroup G of $\mathrm{Aut}(R)$ is called inner if every element of G is inner and it is called outer if its only inner element is 1. If G is finite, we say that R has no $|G|$-torsion provided that $|G|r = 0$ implies $r = 0$.

Recall that the fixed subring R^G under the action of G is

$$R^G = \{r \in R \mid g \cdot r = r, \text{ for all } g \in G\}.$$

The fixed ring R^G is guaranteed to be nontrivial under the following sufficient conditions: Either R has no $|G|$-torsion and R is not nilpotent, as proved by Bergman and Isaacs in 1973, or R has no nilpotent elements, as shown by Kharchenko in 1975. For this reason, most of the following results will, implicitly or explicitly, rely on one of these hypotheses.

Another important ring to study in the situation of a group acting on a ring is the skew group ring $R * G$, which extends the semidirect product for groups: a free R-module with basis $\{g \in G\}$ with multiplication defined via $(rg)(sh) = r(g \cdot s)gh$ for r, s in R and g, h in G. Looking closely at this product formula, we can notice that the action of G is responsible for interchanging g and s via $gs = (g \cdot s)g$. This construction can be extended to a more general notion of the crossed product $R *_\sigma G$ with elements of the form $r\bar{g}$, where elements from G and R commute by the same rule as before, but $\bar{g}\bar{h} = \sigma(g, h)\overline{gh}$ for a 2-cocycle $\sigma : G \times G \to U$, where U is a group of units of R. If the 2-cocycle σ is trivial then $R *_\sigma G = R * G$, a skew group ring, and if the group action is trivial then $R *_\sigma G = R^t[G]$, a twisted group ring. Furthermore, if $R = k$ is a field and G acts trivially on k, then $k * G = kG$ is called a group algebra.

Note that in the original definitions one has a right group action, denoted by $r \mapsto r^g$, and the skew group multiplication defined via $(rg)(sh) = rs^{g^{-1}}gh$. We chose the current notation so that it matches the (left) Hopf algebra action and smash product defined in Section 3.

This raises an important question: Considering the three rings, R, R^G, and $R * G$, if one of them is prime, semiprime, semisimple Artinian, primitive, semiprimitive, or satisfies polynomial identities, can we say the same about the other two and under what conditions? And what is the relationship between Jacobson radicals of these rings? Montgomery provided answers to many of these questions and her results and techniques motivated the work of other researchers.

All of the notions mentioned above are basic properties of noncommutative rings that one wants to understand to get an initial picture of the ring's structure. Some of them reduce to well-known properties when the ring is commutative, for example, a commutative ring is primitive if and only if it is a field. Or, when a ring is Artinian (that is, satisfies the descending chain condition on ideals, which generalizes the notion of finite-dimensional algebras), the conditions of being simple, prime, and primitive are equivalent. A ring is called semiprimitive (or Jacobson semisimple) if its Jacobson radical is zero. A ring is semiprimitive if it is semisimple, that is, semisimple as a module over itself; when a ring is Artinian these two notions coincide. A ring is called prime provided that the zero ideal is a prime ideal (that is, if a product of two ideals is zero then one of them is zero) and is called semiprime provided that it has no nontrivial nilpotent ideals (in other words, if a square of an ideal is zero then the ideal itself is zero).

In 1980, Montgomery published a Springer Lecture Notes volume titled *Fixed Rings of Finite Automorphism Groups of Associative Rings* [Mon80], in which she summarized the progress made in the field in the 1970s. We will now discuss some of her most important results in this area.

In 1973, Susan visited Israel, where she started a lifelong collaboration with Miriam Cohen. In their first paper, they proved that, assuming that a ring R has no nilpotent ideals and no $|G|$-torsion, R is semisimple Artinian if R^G is semisimple Artinian. The converse of this result was shown by Levitzki in 1935, under the assumption of $|G|^{-1}$ being an element of R (see [Mon80, Theorem 1.15]). In 1976, Montgomery proved a more general result, completely describing the relation between Jacobson radicals of the ring and its fixed subring: Assuming that $|G|$ is a bijection on R (that is, if $|G|R = R$ and there is no $|G|$-torsion), $J(R^G) = J(R)^G$, that is, the Jacobson radical of R^G is the intersection of R^G with the Jacobson radical of R. Note that the assumption of $|G|$ being a bijection can be replaced by the hypothesis of $|G|^{-1}$ being an element of R, as explained in [Mon80, Theorem 1.14].

As mentioned before, the above results relied on R having no $|G|$-torsion. Montgomery's next goal was to establish a connection between the ring structures of R, R^G, and $R * G$ with no assumptions about $|G|$ acting on R. It turns out that if R is simple with 1 or a direct sum of simple rings and if G is outer, the assumption of no $|G|$-torsion can often be dropped. But if R is semiprime, G being outer is not enough, so further restrictions are necessary. In 1975, Kharchenko used the Martindale ring of quotients $Q_0(R)$ to generalize the definition of an inner automorphism of a ring, in a way restricting the definition of an outer automorphism (see [Mon80, page 42]):

Definition 3. 1. *An automorphism g is X-inner if there exists a nonzero x in $Q_0(R)$ such that $x(g \cdot r) = rx$ for all r in R.*

2. *A subgroup G of* Aut(R) *is called X-inner, if every element of G is X-inner, and it is called X-outer if its only X-inner element is* 1.

Note that if G is X-outer then it is always outer, but the converse is not always true.

Kharchenko used this definition to prove that R^G is prime if R is prime and G is X-outer. Montgomery then further developed the method of X-inner automorphisms and investigated the connection between R, R^G, and $R * G$ (see [Mon80, Theorems 3.17, 5.3, 6.9, Corollary 6.10]):

Theorem 4. *Let R be a ring with a finite group G of automorphisms.*

1. *Assuming $R * G$ is semiprime, R^G is also semiprime. Moreover, R is Goldie if and only if R^G is Goldie.*
2. *Assuming R is semiprime and G is X-outer, both $R * G$ and R^G and are semiprime. If, in addition, R^G is Goldie, then R and R^G have the same PI degree.*
3. *Assuming R is prime and G is X-outer, $R * G$ is also prime and R and R^G have the same PI degree.*

The above results were then extended by Montgomery in her joint work with Fisher (see [Mon80, Corollary 3.18]):

Theorem 5. *Let R be a semiprime ring and let G be X-outer.*

1. *If R is simple then $R * G$ is simple.*
2. *If R is primitive and G is finite then $R * G$ is primitive.*
3. *If R is semiprimitive then $R * G$ is semiprimitive.*

Furthermore, Fisher and Montgomery used the method of X-inner automorphisms to prove a "Maschke-type" theorem, effectively answering the question of when the skew group ring is semiprime. Recall that, a classical theorem due to Maschke states that, for a field G, the group algebra kG is semisimple if and only if the characteristic of k does not divide $|G|$. Generalizations of this fundamental result, often referred to as "Maschke-type" theorems, are extremely important; the Fisher–Montgomery theorem provides such generalization from group algebras to skew group rings (see [Mon80, Theorem 7.4]):

Theorem 6 (Fisher–Montgomery). *If G is finite and R is semiprime with no $|G|$-torsion then $R * G$ is semiprime.*

The question about the semiprimeness of a skew group ring naturally led to the one about the semiprimeness of a crossed product. In [MP78], Montgomery and Passman obtained necessary and sufficient conditions for the crossed product to be prime or semiprime, assuming that the ring itself is prime. As a consequence, they proved that in characteristic 0, if R is prime, then $R *_\sigma G$ is semiprime. Because X-inner automorphisms played a very important role in describing the above-mentioned conditions, Montgomery and Passman followed this paper with a series of joint works in which they studied X-inner automorphisms of various rings including group rings and crossed products.

In related work [Mon81], Montgomery studied the connection between the prime ideals of R and R^G, by passing through the skew group ring $R * G$. In order to formalize this correspondence, she introduced certain equivalence relations on $Spec(R)$ and $Spec(R^G)$, the sets of prime ideals of R and R^G, and proved that the sets of equivalence classes are homeomorphic with respect to the quotient Zariski topology.

In a different direction, joint with Small, Montgomery extended Noether's classical theorem on affine rings of invariants from the commutative to non-commutative case:

Theorem 7. [MS81, Theorems 1 and 2] *Let R be a Noetherian ring which is affine (that is, finitely generated) over a commutative Noetherian ring C and let G be a finite group of C-automorphisms of R. Then R^G is affine over C provided that one of the following conditions holds:*

1. $|G|^{-1} \in R$.
2. *R is a domain satisfying a polynomial identity and R^G is Noetherian.*

The authors also provided examples when these results fail if either R is not a domain and $|G|R = 0$ or R is not Noetherian.

3. Hopf Algebras

3.1. Group actions, group gradings, and module algebras.

Since group algebras provide the first example of Hopf algebras, in the beginning of the 1980s, Susan Montgomery got interested in these kinds of algebras and started to work on the topic with Miriam Cohen. As for an action of a group on a ring, one can define an action of a group G on an algebra A over a field k as a homomorphism from G to $\mathrm{Aut}_k(A)$, the group of k-linear automorphisms of A. Having such an action of G on A is equivalent to A being a module algebra over the group algebra kG, and this notion can be generalized to the notion of an H-module algebra over any Hopf algebra H. In their first paper on the subject [CM84], Cohen and Montgomery pointed out that, for a finite group G, a grading of A by G is equivalent to A being a $(kG)^*$-module algebra, where $(kG)^*$ is the Hopf algebra dual to kG; in this case it is possible to define a smash product $A\#(kG)^*$, which we will discuss later. Similar to how the connection between the structures of R, R^G, and $R * G$ were studied in the previous section, Cohen and Montgomery investigated the relationship between A, A_1 (the identity component of the graded algebra A), and $A\#(kG)^*$, in particular, they obtained results about the Jacobson radical, prime ideals, and semiprimeness. Furthermore, the authors proved a Maschke-type theorem, analogous to the original Fisher–Montgomery theorem, but for group gradings instead of group actions (see

[CM84, Theorem 2.9]). The most important results of the paper are the following duality theorems relating group actions and group gradings:

Theorem 8. [CM84, Theorems 3.2 and 3.5] *Let G be a group of order n and A be an algebra.*

1. *If G acts on A, then $A * G$ is naturally G-graded and $(A * G)\#(kG)^* \cong M_n(A)$.*
2. *If A is G-graded, then $A\#(kG)^*$ has a natural G-action and $(A\#(kG)^*) * G \cong M_n(A)$.*

The ideas of duality were inspired partly by results in von Neumann algebras and C^*-algebras. As an application, the authors showed that the graded Jacobson radical $J_G(A)$ is always contained in the usual Jacobson radical $J(A)$, proving a conjecture of Bergman on radicals of graded rings.

Having proved the duality theorems, Cohen and Montgomery asked the natural question whether the analogs of the duality theorems hold not only for group algebras and their duals, but also for other finite-dimensional Hopf algebras. In a series of papers, joint with Blattner and Cohen, Montgomery extended these ideas in several directions. Since then, Susan has worked almost exclusively on topics related to Hopf algebras. In 1992, Susan Montgomery was the Principle Lecturer at the Conference Board of the Mathematical Sciences conference on Hopf Algebras and, in 1993, she published the CBMS monograph [Mon93] *Hopf algebras and their actions on rings*.

Before describing Montgomery's results further, we will discuss the motivation behind these generalizations. A group algebra kG, in addition to being an algebra, has a coalgebra structure; in particular, there is a k-linear map $\Delta : kG \to kG \otimes kG$ called comultiplication and defined via $\Delta(g) = g \otimes g$ for $g \in G$ and extended linearly. This additional structure, used implicitly, allows us to define a kG-action on a tensor product of two kG-modules via

$$g \cdot (v \otimes w) = (g \cdot v) \otimes (g \cdot w) = \Delta(g) \cdot (v \otimes w),$$

that is, we see that a tensor product of two representations is again a representation. This property is not true anymore if the group algebra is replaced by an arbitrary algebra, but it can be extended from group algebras to Hopf algebras H, using the comultiplication map $\Delta : H \to H \otimes H$. In this case the H-module structure is defined on the tensor product of two H-modules via

$$h \cdot (v \otimes w) = \Delta(h) \cdot (v \otimes w) = \sum (h_{(1)} \cdot v) \otimes (h_{(2)} \cdot w)$$

using the so-called Sweedler notation for the coproduct to write $\Delta(h) = \sum h_{(1)} \otimes h_{(2)} \in H \otimes H$ for $h \in H$.

For an algebra A over a field k, we can treat its multiplication and unit as k-linear maps $m : A \otimes A \to A$ and $u : k \to A$. When A is finite-dimensional, the notion of an algebra can be dualized in the following way: Consider the dual vector space $C = A^*$ of k-linear maps from A to k. This dual vector space has two k-linear maps $\Delta = m^* : C \to C \otimes C$ and $\varepsilon = u^* : C \to k$; these maps, called comultiplication and counit give C the structure of a coalgebra. We call H a bialgebra if it is both an algebra and a coalgebra and these two structures satisfy a compatibility condition, namely that comultiplication and counit are algebra maps. A bialgebra H becomes a Hopf algebra if in addition it has a map $S : H \to H$, which is called an antipode and satisfies requirements that generalize the ones for the inverse map in groups.

Historically, the first Hopf algebras studied were cocommutative, that is, the ones where for every element h, the coproduct $\Delta(h) = \sum h_{(2)} \otimes h_{(1)}$. Over an algebraically closed base field of characteristic 0, the only finite-dimensional cocommutative Hopf algebras are group algebras. Other examples of cocommutative Hopf algebras include universal enveloping algebras of Lie algebras and restricted Lie algebras. For any Hopf algebra H one can consider all elements g such that $\Delta(g) = g \otimes g$ and $\varepsilon(g) = 1$, called group-like elements of H. They are linearly independent, form a group, denoted by $G(H)$, and generate a cocommutative Hopf subalgebra $kG(H)$ of H.

When A is both an algebra and an H-module over a Hopf algebra H, we say that A is an H-module algebra, or, equivalently, that A is an algebra in the category of left H-modules ${}_H\mathcal{M}$, if its multiplication and unit are H-module maps, that is, $h \cdot (ab) = \sum (h_{(1)} \cdot a)(h_{(2)} \cdot b)$ and $h \cdot 1_A = \varepsilon(h) 1_A$. Similarly to skew group rings arising when groups act on rings as automorphisms, for an H-module algebra A the smash product algebra $A\#H$ is defined to be $A \otimes H$ as a vector space, but with multiplication

$$(a\#h)(a'\#h') = \sum a(h_{(1)} \cdot a')\#h_{(2)}h'$$

where the elements of $A\#H$ are denoted by $a\#h$.

For example, for an algebra $A = A_1 \oplus A_g$, graded by the group $G = \langle g \rangle \cong \mathbb{Z}_2$, the $(kG)^*$-action on A is defined via $p_1 \cdot a = a_1$ and $p_g \cdot a = a_g$, where $\{p_1, p_g\}$ is the dual basis of $(kG)^*$ and $a = a_1 + a_g$ for $a_1 \in A_1$ and $a_2 \in A_2$. Since $\Delta(p_x) = \sum_{yz=x} p_y \otimes p_z$, the multiplication in $A\#(kG)^*$ is determined by $(1\#p_1)(a\#1) = a_1\#p_1 + a_g\#p_g$ and $(1\#p_g)(a\#1) = a_1\#p_g + a_g\#p_1$.

If H is a finite-dimensional Hopf algebra, then the dual vector space H^* is also a Hopf algebra. In particular, the multiplication in H^* is the map dual to the comultiplication in H, and vice versa, as mentioned before. When H is not finite-dimensional, H^* is not a Hopf algebra anymore, but one can use the so-called finite dual H^0 instead. In 1985, Susan Montgomery, in collaboration with her husband Bob Blattner, extended the results of [CM84] from group algebras to infinite-dimensional Hopf algebras H with bijective antipode, where H^* is replaced by a Hopf subalgebra U of the Hopf algebra H^0, and proved that,

Figure 2. Susan Montgomery and Bob Blattner visiting Munich in 1994.

under certain conditions,

$$(A\#H)\#U \cong A \otimes (H\#U)$$

(see [BM85, Theorem 2.1]). In order to obtain these results, Bob and Susan combined their perspectives from functional analysis and noncommutative algebra. The authors then discussed several applications of this theorem, in particular, in the case when H is a universal enveloping algebra of a finite-dimensional Lie algebra or a group algebra of a residually k-linear FC-group. Furthermore, when H is a finite-dimensional Hopf algebra of dimension n, this result yields the generalization of the duality results in Theorem 8 (see [BM85, Corollary 2.7]):

$$(A\#H)\#H^* \cong A \otimes (H\#H^*) \cong A \otimes M_n(k) \cong M_n(A).$$

3.2. **Crossed products.** In the same way as smash products generalize skew group rings, the notion of a crossed product can be extended to the case when groups are replaced by Hopf algebras; such a crossed product of an algebra A with a Hopf algebra H is denoted by $A\#_\sigma H$, where $\sigma : H \otimes H \to A$ is an invertible cocycle. These general crossed products were introduced independently in 1986 by Blattner, Cohen, and Montgomery, and by Doi and Takeuchi (see [Mon93, Chapter 7]). Next, in a 1989 joint paper with Blattner, Montgomery continued her studies of crossed products and further extended the results of the previous papers. In particular, they generalized the duality theorems from smash to crossed products (see [Mon93, Theorem 9.4.17]):

Theorem 9. *Let H be a Hopf algebra of dimension n and $A\#_\sigma H$ be a crossed product. Then*

$$(A\#_\sigma H)\#H^* \cong M_n(A).$$

Another direction was to extend the Maschke-type results to crossed products $A\#_\sigma H$ for semisimple Hopf algebras H. All previous results were obtained either under the assumption that H is a group algebra or its dual or by imposing additional conditions on H. Blattner and Montgomery proved a Maschke-type theorem by restricting the action instead (see [Mon93, Theorem 7.4.7]):

Theorem 10. *Let H be a semisimple Hopf algebra, A be a semiprime algebra, and $A\#_\sigma H$ be a crossed product. Then $A\#_\sigma H$ is semiprime if the action of H is inner.*

Many ideas and results discussed in this section, as well as in Section 2, motivated Montgomery, together with Linchenko and Small, to investigate two related questions about H-module algebras and smash products. In the first question they asked whether, for a semisimple Hopf algebra H and an H-module algebra R, the Jacobson radical is stable under the action of H. As it was mentioned in the beginning of Subsection 3.1, for $H = (kG)^*$, it is equivalent to the question of Bergman about the graded Jacobson radical and it was answered positively in [CM84]. In 2001, Linchenko showed that it is true when R is finite-dimensional and the base field has either characteristic 0 or characteristic $p > \dim R$ with additional condition of H being cosemisimple (that is, H^* being semisimple). In [LMS05, Theorem 3.8], Linchenko, Montgomery, and Small proved that the answer is positive for any infinite-dimensional PI-algebra R which is either affine or algebraic over the base field of characteristic 0. In the case of positive characteristic they showed that the Jacobson radical is H-stable under the additional assumptions of H being cosemisimiple and the characteristic being large enough compared to the dimension of H and the degree of the polynomial identity satisfied by R. Note that, by the Larson-Radford theorem, in characteristic 0 semisimplicity and cosemisimplicity are equivalent.

The second question addressed in [LMS05] was whether $A = R\#H$ is semiprime provided that H is semisimple and R is H-semiprime. This open question was asked by Cohen and Fischman in 1984, under the stronger hypothesis of R being semiprime, and the positive answer would generalize the results of Fisher and Montgomery for $H = kG$ and Cohen and Montgomery for $H = (kG)^*$. In [LMS05, Theorems 2.8 and 2.11], Linchenko, Montgomery, and Small proved that these two questions are connected: First, given a finite-dimensional Hopf algebra H, they established two conditions equivalent to the one that the Jacobson radical of every H-module algebra is H-stable. Then the authors proved that if the first question is answered positively for all H-module algebras R, then the prime radical of every H-module algebra is H-stable and $R'\#H^*$ is semiprime for all H^*-semiprime H^*-module algebras R'. In conclusion, they showed that $A = R\#H$ is semiprimitive provided that H is semisimple, R is an H-semiprime H-module algebra satisfying a polynomial identity, and the base field has characteristic 0 (in the case of positive characteristic

Figure 3. Susan Montgomery with her mathematical siblings Gail Letzter, Daniel Farkas, Lance Small, and Lynne Small in Torrey Pines in 2014.

some extra hypotheses on k and H were needed); in particular, under the above assumption, the second question was answered positively, since semiprimitiveness implies semiprimeness.

3.3. **Coalgebras and comodules.** In the same fashion how dualizing the multiplication and unit of an algebra A, as k-linear maps led to the concept of a coalgebra C, the notion of a C-comodule is dual to the one of an A-module. That is, treating the A-action on a left A-module M as a k-linear map $\cdot : A \otimes M \to M$, one can define a left C-comodule M via the coaction $\rho : M \to C \otimes M$. The category of left C-comodules is denoted by ${}^{C}\mathcal{M}$, while the category of right C-comodules is denoted by $\mathcal{M}^{C}$. If C is finite-dimensional, then left C-comodules are exactly right C^*-modules; furthermore, for a group G, a vector space is a kG-comodule if and only if it is G-graded. Additional background on coalgebras and comodules can be found in [Mon93, Chapter 5].

Note that, in the infinite-dimensional case, the dual vector space A^* of an algebra A is not always a coalgebra, since $(A \otimes A)^*$ is larger than $A^* \otimes A^*$, and therefore there is no one-to-one correspondence between the theories of algebras and coalgebras. Nevertheless, by the fundamental theorem of coalgebras, any finite subset of elements of a coalgebra is contained in a finite-dimensional subcoalgebra, and, thus, every simple coalgebra is finite-dimensional. This fact led Montgomery, as well as the other researchers in the area, to working on the extension of the results from the theory of finite-dimensional algebras to general coalgebras.

In [Mon95] Montgomery used the classical Brauer theorem about the indecomposable finite-dimensional algebras to prove the decomposition theorem for coalgebras. She considered the quiver Γ_C whose vertices are simple subcoalgebras of coalgebra C, showed that it is isomorphic to the so-called Ext quiver whose vertices are the isomorphism classes of simple (right) C-comodules, and proved that any coalgebra can be decomposed as a direct sum of indecomposable components, each of which corresponds to a connected component of Γ_C. Montgomery then applied these results about coalgebras to prove that every pointed Hopf algebra, that is, the one for which every simple subcoalgebra is one-dimensional, can be decomposed as a crossed product. It was shown independently by Cartier and Gabriel and by Kostant in the early 1960's that a pointed cocommutative Hopf algebra is a skew group ring of its group of group-like elements over the irreducible component of the identity element. For an arbitrary pointed Hopf algebra H, Montgomery showed that $H_{(1)}$, the indecomposable component containing 1, is a Hopf subalgebra of H, the group of group-like elements $G = G(H)$ acts on $H_{(1)}$ via conjugation, and the group of group-like elements of $H_{(1)}$, $N = G(H_{(1)})$, is normal in G. She then proved that H is isomorphic to the Hopf algebra $H_{(1)} \#_\sigma k(G/N)$, which has a structure of a crossed product with a certain cocycle σ as an algebra and a structure of a tensor product as a coalgebra.

Then, in [CM97], Chin and Montgomery constructed, for a given coalgebra C, an associated basic coalgebra B for which every simple subcoalgebra is the dual of some finite-dimensional division algebra and proved that categories of C-comodules and B-comodules are equivalent, that is, that C and B are Morita–Takeuchi equivalent. In particular, when the base field k is algebraically closed, every finite-dimensional division algebra and, therefore, every simple subcoalgebra of B is one-dimensional, implying that B is pointed. The authors then applied their results to path coalgebras, and showed that, over an algebraically closed base field, any coalgebra is equivalent to a large subcoalgebra of a path algebra of the Ext quiver.

3.4. **Extensions.** As crossed products, considered in Subsection 3.2, play a fundamental role in the theory of extensions, the next direction of Montgomery's research was to study certain types of these extensions. In the joint paper with Blattner from 1989, she started working on Hopf Galois extensions, which were first introduced in 1969 by Chase and Sweedler for commutative algebras; the general definition was given by Kreimer and Takeuchi in 1980 (see [Mon93, Chapter 8]).

Recall that for an H-module algebra A, the algebra of H-invariant elements A^H is defined via

$$A^H = \{a \in B \mid h \cdot a = \varepsilon(h)a \text{ for all } h \in H\},$$

extending the notion of a fixed subring R^G from Section 2. Dualizing, if A is a (right) H-comodule algebra with coaction $\rho : A \to A \otimes H$ (that is, an algebra in the category $\mathcal{M}^H$), then one can define A^{coH}, its algebra of

H-coinvariant elements, via

$$A^{coH} = \{a \in A \mid \rho(a) = a \otimes 1\}.$$

Using this terminology, $R \subset A$ is called a (right) H-extension if A is a right H-comodule algebra with $R = A^{coH}$ and the H-extension $R \subset A$ is called (right) H-Galois provided that the canonical map $\beta : A \otimes_R A \to A \otimes H$ defined by $\beta(a \otimes b) = (a \otimes 1)\rho(b)$ is bijective. This definition extends the notion of the classical Galois extensions as follows: Let $k \subset E$ be fields, G be a finite group acting as automorphisms of E fixing k, and $F = E^G$ be the set of elements fixed by G. Since E is a kG-module algebra, it becomes a $(kG)^*$-comodule algebra with $E^{co(kG)^*} = F$. Then E/F is a classical Galois field extension with Galois group G if and only if $F \subset E$ is $(kG)^*$-Galois.

Important examples of H-Galois extensions include $R \subseteq R\#_\sigma H$, where $R\#_\sigma H$ is a right H-comodule via coaction $\rho = \mathrm{id}\otimes\Delta$, but not every H-Galois extension can be written as a crossed product. In fact, combining the results of Doi and Takeuchi and of Blattner and Montgomery, one can show that for an H-extension $R \subset A$, the algebra A is isomorphic to $R\#_\sigma H$ if and only if $R \subset A$ is H-Galois with the so-called normal basis property (see [Mon93, Corollary 8.2.5]).

While group crossed products are transitive in the sense that if $R *_\sigma G$ is a crossed product, N is a normal subgroup of G, and $L = G/N$, then there exists a cocycle $\tau : L \times L \to R *_\sigma N$ such that $R *_\sigma G \cong (R *_\sigma N) *_\tau L$, Hopf crossed products are not transitive in general. In order to state the transitivity problem, we first consider an exact sequence of Hopf algebras $K \hookrightarrow H \twoheadrightarrow \overline{H}$, where K is a normal Hopf subalgebra of H and $\overline{H} = H/I$ is the quotient Hopf algebra for the Hopf ideal $I = HK^+ = K^+H$ of H, with $K^+ = \mathrm{Ker}(\varepsilon) \cap K$. This sequence is called an extension of $\overline{H}$ by K; note, however, that not every quotient Hopf algebra arises from a normal Hopf subalgebra. Then, by an example of Schneider, it is not true in general that a crossed product $R\#_\sigma H$, with H being an extension of $\overline{H}$ by K, can always be written as $(R\#_\sigma K)\,\#_\tau \overline{H}$ for some cocycle τ.

One of the advantages of studying Hopf Galois extensions rather than crossed products is that, unlike Hopf crossed products, faithfully flat Hopf Galois extensions are transitive, which enables the use of inductive arguments. This was proved by Montgomery and Schneider in their joint paper [MS99]:

Theorem 11 (Transitivity). *Let $R \subset A$ be a faithfully flat H-Galois extension, H be an extension of $\overline{H}$ by K, and define $B = \rho^{-1}(A \otimes K)$. Then*

1. *$R \subset B$ is faithfully flat K-Galois.*
2. *$B \subset A$ is faithfully flat $\overline{H}$-Galois.*

The main focus of [MS99] was, however, to study prime ideal structure in faithfully flat H-Galois extensions $R \subset A$ for a finite-dimensional Hopf algebra H. First, since there is no H-action on R in this situation, the authors introduced the notion of an H-stable ideal I of R as the one satisfying $AI = IA$; when $A = R\#H$, it coincides with the usual notion of H-stable ideals. Since H is finite-dimensional, A becomes an H^*-module algebra, and the authors applied the Morita equivalence to obtain a bijective correspondence first between the set of H-prime ideals of R, H-Spec(R), and the set of H^*-prime ideals of A, H^*-Spec(A), and then between the sets of H-equivalence classes of Spec(R) and of H^*-equivalence classes of Spec(A). In addition, they proved that H-Spec(R) can be identified with H_0-Spec(R), where H_0 is the coradical of H, that is, the sum of its simple subcoalgebras.

Next they defined the version of the Krull relations, extending the basic relations that hold between the prime ideals of a ring R and the group crossed product $R *_\sigma G$, such as lying over, incomparability, and going up. A Hopf algebra H satisfies one of these six Krull relations (three basic and three dual) if for all faithfully flat H-Galois extensions $R \subset A$ a certain relation between the prime ideals of R and A holds. For example, H has incomparability if for all faithfully flat H-Galois extensions $R \subset A$ and any $P_2 \subset P_1$ in Spec(A) the condition $P_2 \neq P_1$ implies $P_2 \cap R \neq P_1 \cap R$. Furthermore, Montgomery and Schneider analyzed the conditions under which a Hopf algebra satisfies each of these Krull relations; they proved that in order to see whether H has the given Krull relation, it suffices to check the special case of smash product extensions $R \subset R\#H$ for all H-prime H-module algebras R. The strongest results, when H satisfies all six Krull relations, were obtained with the help of the transitivity theorem either when H is solvable and cosolvable (where H being (co)solvable means that it has a normal series in which all of the quotients are (co)commutative) or when H is semisolvable (that is, H has a normal series in which the quotients are either commutative or cocommutative) and (co)semisimple.

In addition, the authors extended the notion of equivalent prime ideals in R^G from [Mon81] to the case of an H-module algebra A with $R = A^H$, satisfying certain conditions, and showed that the sets of equivalence classes of Spec(A) and Spec(R) are in one-to-one correspondence.

Another type of extensions studied by Montgomery, jointly with Fischman and Schneider, were β-Frobenius extensions of subalgebras of a Hopf algebra. One of the equivalent definitions for a finite-dimensional algebra A to be a Frobenius algebra is that A should be isomorphic to A^* as right A-modules. In the beginning of 1960s, this concept was generalized twice: First Kasch introduced the notion of Frobenius extensions of rings, and then Nakayama and Tsuzuku defined β-Frobenius extensions, also called Frobenius extensions of the second type.

One of the main theorems proved by Fischman, Montgomery, and Schneider in [FMS97], states that, under suitable conditions, the property of being a β-Frobenius extension is inherited by the subalgebras of coinvariants. One of these conditions was that a certain Hopf algebra extension was of the so-called (right) integral type; such extensions include Hopf algebra extensions $U \subset W$ when either U is a normal Hopf subalgebra of finite index or W is finite-dimensional. The authors also showed that, if U and W are pointed and U is of finite index, then such an extension is always of integral type.

The general theory, developed in this paper, was then applied to the situation when $B \subset A$ and H are Hopf algebras with bijective antipodes and there exists a Hopf surjection $\pi : A \to H$ which is still surjective when restricted to B. Then A and B become right H-comodules with H-coaction $(\mathrm{id} \otimes \pi) \circ \Delta$ and one can consider the subalgebras of coinvariants, $R = A^{coH}$ and $S = B^{coH}$. Then the general results of the paper imply, under the assumptions that $B \subset A$ is a faithfully flat extension of right integral type and $R \subset A$ and $S \subset B$ are faithfully flat H-Galois extensions, that $S \subset R$ is β-Frobenius. In particular, these results are applicable in the following two important special cases:

1. When $A = R\#_\sigma H$ and $B = S\#_\sigma H$ are crossed products for an invertible cocyle $\sigma : H \otimes H \to S$.
2. When $S \subset R$ are Yetter–Drinfeld Hopf algebras, that is, Hopf algebras in the category ${}^H_H YD$ of all Yetter–Drinfeld modules over H, and $B = S \star H$ and $A = R \star H$ are their Radford biproducts, as described in [Mon93, Section 10.6].

As a corollary, the authors showed that the finite-dimensional Yetter–Drinfeld Hopf algebras themselves are Frobenius algebras and proved a Maschke-type result in this case.

3.5. Representations and Kaplansky's Conjectures.

As it was mentioned before, for a Hopf algebra H, the tensor product of two left H-modules is again an H-module. Moreover, for any left H-module V, its dual V^* is also a left H-module via $(h \cdot f)(v) = f(Sh \cdot v)$, where $h \in H$, $f \in V^*$, and $v \in V$. Therefore the category of finite-dimensional H-modules is a rigid monoidal category, and this fact establishes a strong connection between Hopf algebra theory and category theory. Many of the results about Hopf algebras and their representations were later extended to tensor categories, which makes the study of the representation theory of Hopf algebras very important.

In 1975, Kaplansky stated ten conjectures about Hopf algebras, most of which were motivated by corresponding properties of groups and played a fundamental role in the development of Hopf algebra theory. One of them, the sixth conjecture, which will be referred to as simply the Kaplansky Conjecture, appears to be particularly important and is still open. This conjecture suggests that the classical Frobenius theorem for groups extends to semisimple Hopf algebras: Roughly, it states that, for a semisimple Hopf algebra H over an algebraically closed field, the dimension of any irreducible representation of H (that is, a simple H-module) divides the dimension of H. It is true for all known examples of such algebras, including group algebras and their duals, and often the results about semisimple Hopf algebras are proved under the assumption that the Kaplansky Conjecture holds for them. In recent years this conjecture was shown to be true in many special situations, and one of the first results obtained in this direction was by Montgomery and Witherspoon in [MW98]. The authors first established a one-to-one correspondence between irreducible representations of a crossed product $A\#_\sigma kG$ and irreducible representations of certain twisted group algebras of subgroups of G. This so-called Clifford correspondence allowed them to show that for a finite-dimensional algebra A over a field k and a group G, with characteristic of k not dividing the order of G, if the dimension of any irreducible A-module divides the dimension of A then the same is true for $B = A\#_\sigma kG$ or $A\#_\sigma(kG)^*$. The proof of the latter result involves the application of the duality results in Theorem 9. Then the authors showed that, over an algebraically closed field, the above divisibility property still holds if the group algebra or its dual is replaced by a lower or upper semisolvable semisimple Hopf algebra; in particular, every lower or upper semisolvable semisimple Hopf algebra, over an algebraically closed field of characteristic not dividing its dimension, satisfies the Kaplansky Conjecture. Here, the notion of lower semisolvablity coincides with the notion of semisolvablity from [MS99] (see Subsection 3.4), and upper semisolvable Hopf algebras are defined similarly, using the series of quotients instead of normal Hopf subalgebras.

Finally, Montgomery and Witherspoon proved that every semisimple Hopf algebra of prime power dimension over an algebraically closed field of characteristic 0 is both lower and upper solvable and cosolvable and, therefore, satisfies the Kaplansky Conjecture.

4. Frobenius–Schur Indicators

In 2000, Susan Montgomery and her student Vitaly Linchenko showed that the standard trichotomy for group representations given by Frobenius–Schur indicators (real, non real but real valued characters, and totally non real) can be extended to Hopf algebras. This seminal paper played a role of fundamental importance as it laid the groundwork for the theory of Frobenius–Schur indicators and made a huge impact on the field, laying out the directions for further development: First, Kashina, Sommerhäuser, and Zhu developed the theory of higher

Frobenius–Schur indicators, which was then extended to the case of semisimple quasi-Hopf algebras by Mason and Ng and to the case of spherical fusion categories by Ng and Schauenburg (see, for example, [KSZ06] and [NS07]). Frobenius–Schur indicators became a very important categorical invariant and had a lot of different applications ranging from the proof of the analogue of Cauchy's theorem for Hopf algebras to finding the dimensions of simple modules to classification results for Hopf algebras and fusion categories. Montgomery continued studying these indicators in a series of papers with Guralnick, Iovanov, Jedwab, Kashina, Mason, Ng, Vega, and Witherspoon (see [KMM02], [GM09], [JM09], and [IMM14]).

Recall that in group theory the n-th Frobenius–Schur indicator of the character χ of a finite group G is defined as

$$\nu_n(\chi) = \frac{1}{|G|}\sum_{g\in G}\chi(g^n).$$

When $n = 2$, this definition yields the classical Frobenius–Schur indicator, which was used by Frobenius and Schur in the beginning of the 20th century to determine whether an irreducible complex representation of a finite group can be realized by matrices with real entries. Note that, when the characteristic of the base field k does not divide $|G|$, the element $\Lambda = \frac{1}{|G|}\sum_{g\in G} g$ becomes the so-called normalized integral for the Hopf algebra $H = kG$. Linchenko and Mongomery realized that, when the group algebra is replaced by an arbitrary semisimple Hopf algebra, the n-th Frobenius–Schur indicator of the character χ can be defined as $\nu_n(\chi) = \chi(\Lambda^{[n]})$ for the n-th Hopf power of the normalized integral Λ; when $n = 2$ these indicators are just referred to as Frobenius–Schur indicators, while higher Frobenius–Schur indicators stand for the case of $n > 2$. The authors proved the following generalization of the classical Frobenius–Schur theorem:

Theorem 12. [LM00, Theorem 3.1] *Let H be a semisimple Hopf algebra over an algebraically closed field of characteristic not 2 and let χ be the character of an irreducible representation of H corresponding to the simple H-module V; in the case of positive characteristic assume that H is also cosemisimple.*

1. *The Frobenius–Schur indicator $\nu_2(\chi)$ takes on only the values* 0, 1, *or* -1.
2. $\nu_2(\chi) \neq 0$ *if and only if V is selfdual. Moreover,* $\nu_2(\chi) = 1$ *(respectively,* -1*) if and only if V admits a symmetric (respectively, skew-symmetric) nondegenerate bilinear H-invariant form.*
3. *The trace of the antipode is* $\mathrm{Tr}\, S = \sum \nu_2(\chi)\chi(1)$*, where the summation is taken over all irreducible representations of H.*

Since Frobenius–Schur indicators proved to be a very useful invariant of Hopf algebras, Montgomery, together with Kashina and Mason in [KMM02], obtained an explicit formula for the indicator of an important class of semisimple Hopf algebras, which are called cocentral abelian extensions and include Drinfeld doubles of group algebras. The authors used this formula to compute the Frobenius–Schur indicators for various examples; in particular they established conditions under which all irreducible representations of the smash product $(kG)^*\#kL$, where G and L are groups, have positive Frobenius–Schur indicators. As a consequence, they showed that the indicator is always positive for Drinfeld doubles of symmetric groups as well as of generalized dihedral groups and their direct products. In group theory, the groups which admit only irreducible representations with positive indicators are of particular interest and are called totally orthogonal. In a joint paper with Guralnick [GM09], Montgomery introduced the same terminology for Hopf algebras and proved that the Drinfeld double of any finite real reflection group is totally orthogonal. In addition, the authors showed that Frobenius–Schur indicators of irreducible representations, taking on only the values 0, 1, or -1, can be defined even in positive characteristic for involutory non-semismple Hopf algebras. Then, together with Jedwab in [JM09], Montgomery studied Frobenius–Schur indicators for bismash products $(kC_n)^*\#kS_{n-1}$ and $(kS_{n-1})^*\#kC_n$, for the cyclic group C_n and the symmetric group S_{n-1}, where k is an algebraically closed field of characteristic 0. The authors showed that $(kC_n)^*\#kS_{n-1}$ is totally orthogonal, while $(kS_{n-1})^*\#kC_n$ admits irreducible representations with indicator 0 as well as 1. In the next paper with Jedwab, Montgomery studied the representations of general bismash products $(kG)^*\#kL$ over algebraically closed fields of positive characteristic not equal to 2 and defined Brauer characters for such bismash products. They proved the bismash product analog of the theorem of Thompson on lifting Frobenius–Schur indicators of group representations from characteristic p to characteristic 0 and deduced that $(kG)^*\#kL$ is totally orthogonal if $(\mathbb{C}G)^*\#\mathbb{C}L$ is totally orthogonal.

While higher Frobenius–Schur indicators of group representations are always integers, this does not hold for Hopf algebras. Since there is a relation between group representations and representations of Drinfeld doubles of groups, an open question was whether this integrality property still holds for these Drinfeld doubles. This turned out to be an interesting group-theoretic question, which was studied by Montgomery together with Iovanov and Mason in [IMM14]. They showed that, while the n-th indicators are integers for $n = 2, 3, 4, 6$ as well as for many important families of groups, such as symmetric groups, alternating groups, the projective special liner groups $PSL_2(q)$, and the sporadic simple groups M_{11} and M_{12}, it is false in general.

We will now discuss some significant applications of Frobenius–Schur indicators in Hopf algebra theory and beyond, for which Montgomery's results played an instrumental role.

In [KMM02, Section 7], Montgomery with her coathors used the Frobenius–Schur indicators for classification purposes as an invariant that helps distinguishing between nonisomorphic semisimple Hopf algebras, for example, by showing that one Hopf algebra has a two-dimensional irreducible representation with indicator -1, and the other one does not. Moreover, since (higher) Frobenius–Schur indicators are categorical invariants, they can be used to show that Hopf algebras are not twist-equivalent to each other, that is, the corresponding representation categories are not equivalent. For example, this line of argument was later used by Kashina to show that all nonisomorphic Hopf algebras from a certain family of dimension 32 are also not twist-equivalent.

In [KSZ06, Section 3.4], Kashina, Sommerhäuser, and Zhu used the (higher) Frobenius–Schur indicators to prove the Hopf algebra analogue of Cauchy's theorem. The original theorem for finite groups states that for every prime divisor p of the order of the group there exists an element of order p and can be reformulated by saying that every prime divisor of the order of the group also divides its exponent. The authors showed that if a prime p does not divide the exponent of a semisimple Hopf algebra H, then the p-th Hopf power of the normalized integral Λ equals Λ itself. Then for the regular representation χ_R, in which H acts on itself via left multiplication, the p-th Frobenius–Schur indicator equals

$$\nu_p(\chi_R) = \chi_R(\Lambda^{[p]}) = \chi_R(\Lambda) = 1.$$

On the other hand, by another formula for indicators, established by the authors, if p divides $\dim H$ then

$$\nu_p(\chi_R) = (\dim H)^{p-1} \equiv 0 \pmod{p}.$$

This establishes the Hopf algebra analogue of Cauchy's theorem: Every prime divisor of the dimension of H divides its exponent.

As it was mentioned in the beginning of this section, Ng and Schauenburg extended the notion of (higher) Frobenius–Schur indicators to category theory. These generalizations were essential to the proofs of several important results, such as the categorical version of Cauchy's theorem for integral fusion categories by Ng and Schauenburg in [NS07] and for spherical fusion categories by Bruillard, Ng, Rowell, and Wang in [BNRW16]. The latter group of authors applied Cauchy's theorem to prove the rank-finiteness theorem, which was a longstanding conjecture of the fourth author. This theorem states that, up to equivalence, there are finitely many modular categories of given finite rank. As modular categories contribute to the mathematical foundation of topological quantum computation and are closely related to conformal field theories and topological quantum field theories, Frobenius–Schur indicators played an important role for the further development of these areas.

5. Concluding Remarks

Susan's research on Hopf algebras and their representations, as well as her work on group and Hopf algebra actions on rings, established the framework and motivated further studies in these areas. She gave numerous invited talks at conferences, seminars, and colloquia around the world, including the AMS invited addresses at the Joint Mathematics Meetings in 1984, the Joint meeting of the AMS and the Israel Mathematical Union in 1995, and the Spring Southeastern sectional meeting in 2005, as well as the plenary lecture at the Canadian Mathematical Society summer meeting in 2009. In 2011, Susan was selected to deliver the Emmy Noether Lecture at the Joint Mathematics Meetings.

Susan Montgomery was a co-organizer of more than 35 conferences, workshops, and special sessions on ring theory, Lie algebras, Hopf algebras, tensor categories and related topics, as well as the noncommutative algebra year at MSRI in 1999–2000. Two Hopf algebra conferences were held in Susan's honor: one at the University of Southern California in 2009 and another one as a special session at the Joint Mathematics Meetings in San Diego in 2018.

Susan served as an editor for numerous mathematical journals, including the *Proceedings of the AMS*, *Advances in Mathematics*, and *Journal of Algebra*.

Susan's impact on noncommutative algebra is not limited to her own research accomplishments and editorial work. She had, and continues to have, a broad influence on the field through mentorship and collaboration with young mathematicians. Thirteen graduate students received their Ph.D. degree under Susan's supervision, and she currently has one Ph.D. student. Many experts in the areas of Hopf algebras, quantum groups, and category theory spent their postdoctoral years at USC working with her. Susan is a wonderful mentor and advisor; she cares deeply about her graduate students and provides a lot of guidance and encouragement throughout their academic careers. I was very fortunate to have Susan as my graduate advisor and to be able to collaborate with her. More than twenty years after graduation, I still feel her strong support.

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Yevgenia Kashina

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Caroline Series and Hyperbolic Geometry

John R. Parker and Ser Peow Tan

Figure 1. Caroline Series.

1. Introduction

Caroline Series has made important contributions to hyperbolic geometry and symbolic dynamics, and her elegant and clear explanations of the connections between these two topics together with carefully chosen examples that explore the beautiful geometry underlying many spaces of Kleinian groups have influenced a generation of hyperbolic geometers and dynamicists.

Caroline obtained her BA at the University of Oxford in 1972 and her doctorate at Harvard University in 1976 under the supervision of George Mackey. She spent the first couple of years of her career at Berkeley and Cambridge before settling in Warwick in 1978, where she stayed until her retirement in 2014. Her stint at Berkeley led to an important but short-lived collaboration with Rufus Bowen which ended abruptly when Bowen passed away unexpectedly in July 1978. Her work on the connections between symbolic dynamics and hyperbolic geometry done during the early period of her career led to an invitation to speak at the ICM at Berkeley in 1986. She has received numerous awards and honors for her work including the Senior Anne Bennett Prize (LMS, 2014), Junior Whitehead Prize (LMS, 1987), Fellow of the Royal Society (2016), and the David Crighton Medal (IMA-LMS, 2021). She has honorary degrees from Duke and St Andrews Universities and is a Fellow of the AMS. She served as president of the LMS (London Mathematical Society) from 2017 to 2019.

The 1990s and 2000s saw intense activity in the fields of hyperbolic geometry, hyperbolic 3-manifolds, and Kleinian groups following the revolutionary ideas and conjectures of William Thurston. Many important conjectures like Thurston's geometrization conjecture, Marden's tameness conjecture, the virtual Haken conjecture, the ending lamination conjecture, and the Ahlfors measure conjecture were solved during this period. During her tenure at Warwick, working alongside David Epstein, Caroline played a pivotal role in helping to make the Mathematics Institute there a leading international center for hyperbolic geometry, organizing many large conferences and programs attended by leading researchers in the field. She also played host and mentor to a large number of young international visitors, particularly researchers from Japan. We give below a description of some of her considerable contributions to the field.

2. Symbolic Dynamics and Fuchsian Groups

Caroline wrote her thesis on Mackey's theory of virtual groups, which involves the ergodic theory of groups acting measurably on measure spaces; in particular two groups G, G' acting on measure spaces X, X' and preserving measure classes μ, μ' are said to be orbit equivalent if there is a measure class preserving map T from X to X' which sends the orbits of G to the orbits of G'. Note that G, G' are not necessarily isomorphic, nor does T necessarily preserve any order in individual orbits. By a remarkable result of H. Dye, all finite measure preserving actions of $\mathbb{Z}$ are orbit equivalent; such actions are called hyperfinite. Caroline's first contributions involved various results in this area; see for example [Ser78].

John R. Parker is a professor of mathematics at Durham University. His email address is j.r.parker@durham.ac.uk.

Ser Peow Tan is a professor of mathematics at the National University of Singapore. His email address is mattansp@nus.edu.sg.

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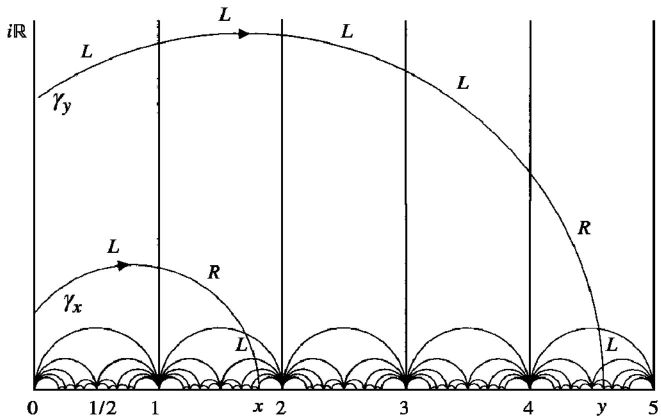

Figure 2. The cutting sequence of $x = 9/5 = [1; 1, 4]$ and $y = 23/5 = [4; 1, 1, 2]$ across the Farey tessellation.

Starting in the late 1970s, Caroline wrote a number of important papers elucidating the connections between hyperbolic geometry and symbolic dynamics. We start with a simple but very illuminating result concerning continued fractions and cutting sequences of geodesics.

2.1. Continued fractions and cutting sequences. Recall that the upper half plane

$$\mathbb{H}^2 = \{z \in \mathbb{C} \mid \Im z > 0\}$$

and the unit disk

$$\mathbb{D} = \{z \in \mathbb{C} \mid |z| < 1\}$$

both serve as models for the hyperbolic plane with ideal boundary $\partial\mathbb{H}^2 = \mathbb{R} \cup \{\infty\}$ and $\partial\mathbb{D} = S^1$ respectively. In an elegant paper [Ser85b], Caroline explained the beautiful relationship between the continued fraction of $x \in \mathbb{R}$ and the manner in which the hyperbolic geodesic joining some point on the positive imaginary axis to x cuts across the Farey triangulation of $\mathbb{H}^2$. That there seemed to be a close connection between continued fractions and cutting sequences of geodesics across the classical fundamental domain of the modular group had been known for some time, but details remained elusive. Caroline was the first to give a precise relation using the left/right cutting sequences explained next.

The Farey triangulation $\mathcal{F}$ of $\mathbb{H}^2$ is a tessellation by triangles each of whose vertices lie in $\mathbb{Q} \cup \{\infty\}$. Its edges are the hyperbolic geodesics joining Farey neighbors, where two fractions $\frac{p}{q}, \frac{m}{n} \in \mathbb{Q} \cup \infty$ in reduced form are called Farey neighbors if $|pn - qm| = 1$, with the convention that $\frac{1}{0} = \infty$. (Recall that a geodesic in $\mathbb{H}^2$ is a semicircle with center on $\mathbb{R}$ or a vertical line.) The tessellation $\mathcal{F}$ can be built up recursively using "Farey addition" as follows; for simplicity we restrict to the right half plane. Start from an initial triangle with vertices at $\infty = \frac{1}{0}, \frac{0}{1}$ and $\frac{1}{1}$ and note that each pair of vertices are neighbors. Two Farey neighbors $\frac{p}{q}$ and $\frac{m}{n}$ spawn a new vertex $\frac{p+m}{q+n}$ which is a neighbor of both. Thus, for example, neighbors $\frac{1}{0}, \frac{1}{1}$ spawn $\frac{2}{1}$, while $\frac{0}{1}, \frac{1}{1}$ spawn $\frac{1}{2}$. Joining each pair with a hyperbolic geodesic defines new triangles with vertices $\{\frac{1}{0}, \frac{1}{1}, \frac{2}{1}\}$ and $\{\frac{0}{1}, \frac{1}{1}, \frac{1}{2}\}$. Repeating this procedure generates $\mathcal{F}$.

Now let $x > 0$ and consider the oriented geodesic γ_x from a point on the positive imaginary axis to $x > 0$. Its left/right cutting sequence across $\mathcal{F}$ is defined as follows: if γ_x cuts a triangle of $\mathcal{F}$ with single vertex on the left it is recorded as a left cut L, while if the single vertex is on the right, it is a right cut R; see Figure 2. The cutting sequence for γ_x is then $L^{n_0}R^{n_1}L^{n_2}\dots$, with the convention that the initial term L^0 is omitted if $0 < x < 1$.

Now suppose $x > 0$ is expressed by the continued fraction

$$x = [a_0; a_1, a_2, \dots] := a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cdots}}$$

where $a_0 \in \mathbb{N} \cup \{0\}$, $a_i \in \mathbb{N}$ for $i \geq 1$. Then

Theorem 2.1. *(Series* [Ser85b] *1985) For $x > 0$, its continued fraction is $[a_0; a_1, a_2, \dots]$ if and only if the geodesic γ_x ending at x has left/right cutting sequence $L^{a_0}R^{a_1}L^{a_2}\dots$.*

Notice that if $x \in \mathbb{Q}$ then γ_x ends at a vertex of the tessellation so that the label of the final cut is ambiguous, corresponding exactly to the ambiguity in the final terms $[\dots a_n, 1]$ or $[\dots, a_n + 1]$ of finite continued fractions. Left/right cutting sequences are now widely used.

2.2. Special tessellations and symbolic dynamics. This theme of using special tessellations of $\mathbb{H}^2$ or $\mathbb{D}$ arising from Fuchsian groups with special fundamental domains, and the associated cutting sequences, to describe the dynamical behavior of the geodesic flow occurs in several of Caroline's other works. In particular, it features in an earlier well-known paper [BS79] with Bowen which explained the connection between Fuchsian groups and symbolic dynamics. Here it is convenient to use the model $\mathbb{D}$ with ideal boundary $\partial\mathbb{D} \cong S^1$ and isometry group $\mathrm{Isom}^+(\mathbb{D}) \cong \mathrm{PSL}(2, \mathbb{R})$.

Recall that if Γ is a finitely generated Fuchsian group, then $\Gamma < \mathrm{Isom}^+(\mathbb{D})$ is discrete and $\mathbb{D}/\Gamma$ is a hyperbolic surface, possibly with cusps, boundary components, and cone points.

A piecewise differentiable map $f : S^1 \to S^1$ is called *Markov* if there is a finite partition of S^1 into intervals I_j such that $f(I_j)$ is a union of intervals I_k. This means that the associated symbolic dynamics is of finite type; that is, an infinite sequence $I_{i_1}, I_{i_2}, I_{i_3}, \dots$ of labels records the intervals through which an orbit of f passes if and only if each pair of adjacent symbols in the sequence is permissible. Furthermore, the map f is orbit equivalent to the action of the Fuchsian group Γ on S^1 if, except for a finite number of pairs of points, $x, y \in S^1$, $x = gy$ with $g \in \Gamma$ if and only if there exists $n, m \geq 0$ such that $f^m(x) = f^n(y)$.

Theorem 2.2. *(Bowen, Series* [BS79]*) For any finitely generated Fuchsian group Γ, there exists a Markov map of S^1 which is orbit equivalent to the action of Γ on its limit set $\Lambda \subset S^1$. This map admits a unique finite invariant measure, which, in the case of a group for which $\Lambda = S^1$, is equivalent to the Lebesgue measure.*

For any surface Σ of fixed topological type, the proof involves the clever idea of first considering a particular Fuchsian group Γ whose quotient surface $\mathbb{D}/\Gamma$ is homeomorphic to Σ, together with a *special* type of fundamental domain D with corresponding tessellation of $\mathbb{D}$ (described below). This is used to construct the partition of S^1 and the Markov map of the theorem. Then using the theory of quasiconformal deformations, the result can be extended to a general Fuchsian group whose quotient is homeomorphic to Σ.

A fundamental domain D in $\mathbb{D}$ is *special* if it is a finite-sided convex polygon whose sides, when extended, are contained in the tessellation of $\mathbb{D}$ arising from the translates of D in $\mathbb{D}$. The sides should be matched in pairs by isometries which form a generating set for Γ. From this one constructs a partition of $S^1 = \partial\mathbb{D}$ with the requisite Markov property, and such that moreover each interval in the Markov partition is associated to a specific generator of Γ. [1]

Continuing along the same lines, in [Ser81], Caroline explicitly constructs a symbolic dynamics for the geodesic flow on any hyperbolic surface by associating to each geodesic a doubly infinite sequence in the two-sided extension of the above Markov shift. Roughly speaking, this is done by mapping the two endpoints of a lifted geodesic γ to the semi-infinite sequences associated to it by the Bowen–Series map constructed above. By making precise the connection between this coding and the cutting sequence obtained by reading off the labels of sides of D traversed by γ in its passage across $\mathbb{D}$, she elucidated the connection between the Artin and Koebe–Morse methods for obtaining symbolic representations of geodesic flows on surfaces of constant negative curvature; see also [Ser86]. This circle of ideas is now a standard starting point for many results on orbit counting and symbolic dynamics. Later, she made many applications of this coding to Fuchsian groups, for example to the word problem [BS87], to random walks [Ser83], to group cohomology [BS84], and to simple curves on surfaces, of which more below.

[1] *In* [BS79] *it was required that the sides ∂D of the fundamental domain were part of the isometric circles of the corresponding side pairings of D, but in fact this condition is not necessary.*

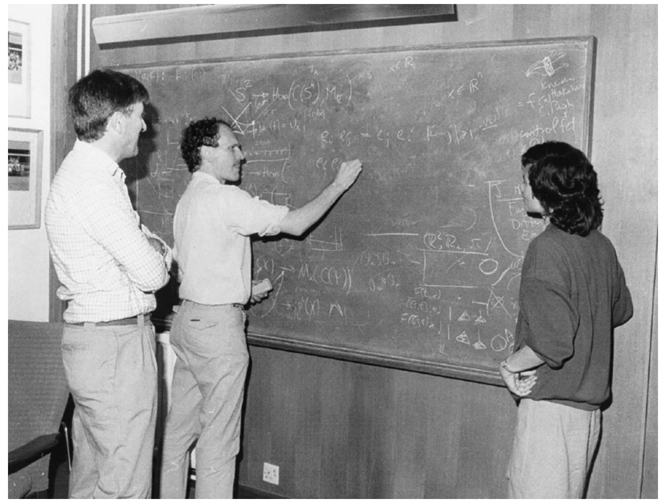

Figure 3. Caroline with colleagues David Fowler (left) and David (Dai) Evans in the maths common room at Warwick, 1980.

3. Curves on Surfaces

In the 1980s, together with Joan Birman, Caroline wrote a series of papers studying curves on hyperbolic surfaces, in particular their self-intersection numbers. These used many of the ideas she had used in her earlier work on cutting sequences of geodesics and the associated symbolic coding. The most influential of these was a paper containing a remarkable theorem about the sparsity of the set of points lying on complete geodesics with bounded self-intersection on a hyperbolic surface of finite type, that is, of finite genus with finitely many cusps or boundary components.

3.1. Geodesics with bounded self-intersections. Let Σ be a closed hyperbolic surface of finite type and let G_k be the set of all complete geodesics on Σ which have at most k self-intersections. In particular, G_0 is the set of complete simple geodesics on Σ.

Theorem 3.1. *(Birman, Series,* [BS85] *1985) Let $k \geq 0$. The set S_k of points on Σ lying on any geodesic $\gamma \in G_k$ is a set of Hausdorff dimension* 1 *and is nowhere dense.*

This result contrasts with the case of a Euclidean torus E where every point on E is contained in some simple geodesic. It also contrasts with the case of a generic geodesic on Σ which is typically dense on Σ. The set S_0 is now commonly referred to as the Birman–Series set.

The main idea behind the proof is very simple and elegant. We give a brief sketch for the case of simple geodesics on a closed hyperbolic surface Σ.

Choose a finite-sided convex polygonal fundamental domain D for Σ with m sides $s_1, \dots, s_m$ labelled on the inside. The images of D under $G = \pi_1(\Sigma)$ define a tessellation $\mathcal{T}$ of $\mathbb{H}^2$ and the G action carries the labelling to each copy of D in $\mathcal{T}$. Suppose γ is a simple geodesic

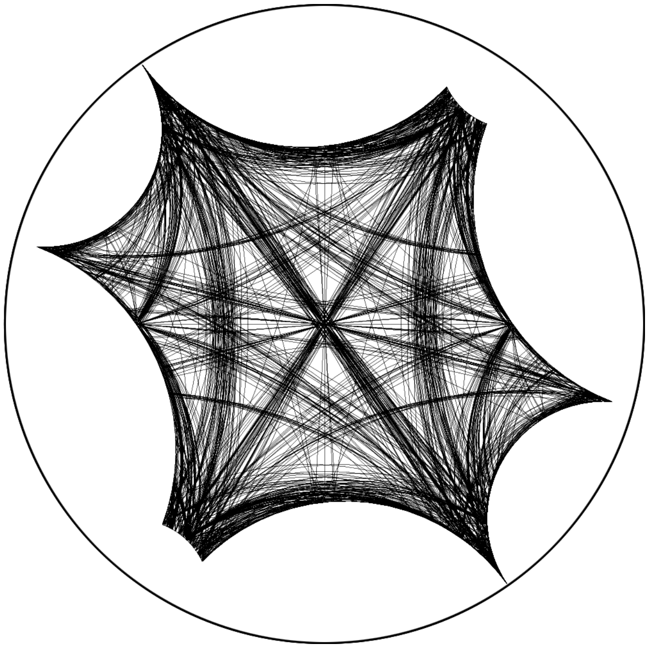

Figure 4. The first 300 or so simple curves ordered by length on a genus two surface Σ. Graphics by Peter Buser and Hugo Parlier from their paper "Quantifying the sparseness of simple geodesics on hyperbolic surfaces."

(closed or otherwise) on Σ. Since D glues up to reconstitute Σ, the lifts of γ to $\mathbb{H}^2$ appear as a collection of disjoint strands σ_i across D. Starting with the strand σ_0 and working backward and forward, we can trace the sequence of sides $\ldots a_{-1}a_0a_1 \ldots, a_i \in \{s_i\}$ in which γ cuts D, numbered so that σ_0 is the strand between labels a_0 and a_1. This is called the cutting sequence of γ.

Now suppose that γ and γ' are two such geodesics whose cutting sequences $\ldots a_{-1}a_0a_1 \ldots$ and $\ldots a'_{-1}a'_0a'_1 \ldots$ are such that $a_i = a'_i$ for $-N \leq i \leq N$. Lifting γ and γ' to $\mathbb{H}^2$ in such a way that σ_0, σ'_0 are the strands between labels a_0, a_1 and a'_0, a'_1, we see that the initial segments $\sigma_{-N}, \sigma'_{-N}$ of the lifts $\tilde{\gamma}$ and $\tilde{\gamma}'$ begin at points x, x' on the same side of the same copy of D, so that x, x' are a bounded distance apart, and likewise for the final points of the segments σ_N, σ'_N. It follows from the properties of hyperbolic geodesics that the segments σ_0, σ'_0 between the cuts labelled a_0, a_1 are exponentially close, that is, they lie in some strip connecting two sides of D of width less than e^{-cN} for some constant $c > 0$.

Requiring that a geodesic γ is simple is extremely restrictive because its segments across D must all be disjoint. These segments can be isotoped so that each crossing of a side of D is at the midpoint of the side, resulting in a graph Γ whose vertices are the midpoints of sides of D and whose edges are arcs between these vertices. Moreover all its edges are disjoint. Clearly there are only finitely many such graphs. Now given the number n_{ij} of arcs between the sides labelled s_i and s_j, one can reconstruct up to isotopy the segment of $\tilde{\gamma}$ between the initial and final points x, y. If γ and γ' share the same Nth forward and backward cutting sequences, then $n_{ij} = n'_{ij}$ for all i, j and moreover $\Sigma_{i,j} n_{ij} = 2N$. It follows that there are only $P(N)$ possibilities for the cutting sequences, where P is a polynomial of degree equal to the number of edges of Γ. Hence the projections of the segments s_0, s'_0 onto D lie in a collection of strips of width bounded by e^{-cN} for some $c > 0$, and the number of such strips is bounded by $P(N)$. Letting $N \to \infty$ gives the result.

Notice that this also gives a polynomial upper bound for the number of simple curves of length bounded by N; this type of bound was vastly improved by Mirzakhani; see the next section.

The proof can be adapted easily to the case of geodesics with bounded intersection, and for surfaces with cusps and/or geodesic boundary. The Birman-Series set S_0 contains a wealth of information and has been intensively studied. An illustration of a subset of S_0 on a perturbation of the Bolza surface (the genus two surface with maximal symmetry group) is given in Figure 4.

3.2. Identities and other applications. The above result has many consequences, and also raises many questions about geodesics of bounded intersection on Σ. It has led to an active field of research and some surprising applications. In his Warwick PhD thesis, Greg McShane studied the nature of the Birman–Series set for a hyperbolic once-punctured torus, and in particular its intersection with the horocycle of length one about the cusp. This is a nowhere dense subset of zero measure on the horocycle, and by studying the complementary gaps, he obtained the following remarkable identity:

Theorem 3.2 (McShane, 1991). *Let T be a complete hyperbolic once-punctured torus. Then*

$$\sum_{\gamma} \frac{1}{1 + e^{l(\gamma)}} = \frac{1}{2}$$

where the sum is over the set of simple closed geodesics on T and $l(\gamma)$ denotes the length of γ.

This was subsequently generalized first by McShane himself to general surfaces with cusps [McS98] and then by Maryam Mirzakhani [Mir07] to surfaces with geodesic boundary, where, in a tour de force, she used her formulae to determine the volumes of the moduli spaces of surfaces with geodesic boundary. There is now a large body of work on various other generalizations and applications by Bowditch, Akiyoshi–Miyachi–Sakuma, Tan–Wong–Zhang, Labourie–McShane, and several others which trace their inspiration to the original Birman–Series result.

In another direction, as noted above, the Birman–Series result raises the question of the growth rate of geodesics in G_k with length less than L as $L \to \infty$. This has also led to a large body of work, notably by Rivin, Mirzakhani ($k = 0$), Erlandsson–Souto ($k > 0$), Sapir, and Erlandsson–Parlier–Souto.

4. Pleating Rays

In the 1990s, Caroline shifted her attention to Kleinian groups[2] and hyperbolic three manifolds. Together with Linda Keen, she invented the theory of pleating rays. These rays (or their higher-dimensional analogues pleating varieties) foliate spaces of various parametrized families of Kleinian groups. Initially inspired by the computer graphics of Mumford and Wright which showed that the limit sets of many two generator groups consist of chains of overlapping or tangent circles such as are clearly visible in Figure 6, the theory was initially worked out with Keen in the case of two special one complex dimensional families, the Maskit and Riley slices of Schottky space, described in more detail below.

4.1. Background. As Keen and Series discovered, the key to understanding these Mumford–Wright graphics is to pass to the hyperbolic convex hull $\mathcal{C}$ of the limit set Λ of a Kleinian group G in hyperbolic 3-space. Thurston described the boundary $\partial\mathcal{C}$ as a pleated surface: this is a surface made up of pieces of hyperbolic planes, meeting along hyperbolic lines called bending lines; see Figure 5. This surface itself carries an intrinsic hyperbolic structure so that $\partial\mathcal{C}/G$ is a collection of hyperbolic surfaces. Also recall that G acts discontinuously on Ω, the complement of the limit set Λ in $\mathbb{C} \cup \infty$ and that the quotient Ω/G is a collection of Riemann surfaces. It can be shown that the surfaces $\partial\mathcal{C}/G$ are homeomorphic to the Riemann surfaces Ω/G, although with different hyperbolic structures.

Using this circle of ideas, Keen and Series were able to relate circle patterns in Λ to the shape of $\partial\mathcal{C}/G$. In the upper half space model of hyperbolic space $\mathbb{H}^3$, a plane is a vertical half plane or a hemisphere with center on the boundary $\partial\mathbb{H}^3$, identified in this model with $\mathbb{C} \cup \{\infty\}$. Likewise a hyperbolic geodesic or line is a vertical line or semicircle with center on $\mathbb{C} \cup \{\infty\}$. Thus a circle or arc of a circle in Λ gives rise to a plane or part plane in $\partial\mathcal{C}$, likewise two circles which overlap give rise to intersecting planes which meet along a bending line in $\partial\mathcal{C}$. The bending lines project to disjoint geodesics on $\partial\mathcal{C}/G$ and thus are either simple closed curves or, in the case of non-closed curves, what Thurston called a (measured) geodesic lamination. Such laminations can be organized as a space of projective measured laminations, denoted $\mathcal{PML}$; for present purposes it suffices to note that $\mathcal{PML}$ contains simple closed curves or multi-curves as a dense subset. Such laminations are called rational for reasons which will be clearer in the next section.

[2] *A Kleinian group is a discrete group of orientation-preserving isometries of hyperbolic 3-space* $\mathbb{H}^3$, *identified with* $\mathrm{PSL}(2, \mathbb{C})$.

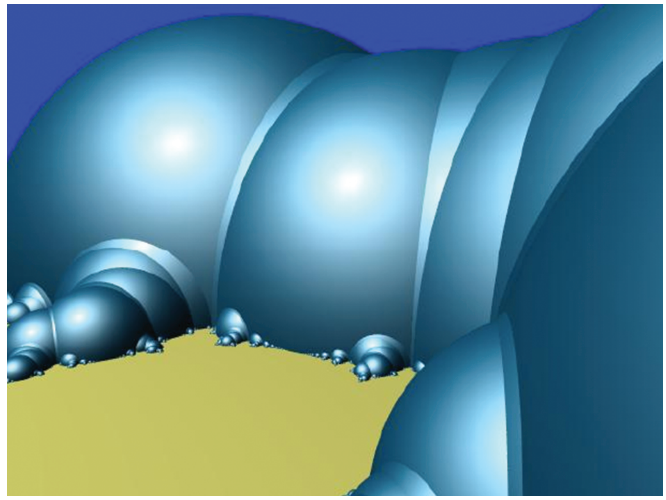

Figure 5. The boundary of the hyperbolic convex hull of a Kleinian group limit set is a pleated surface made up of pieces of hyperbolic planes meeting along hyperbolic geodesics called bending lines.

A pleating ray as introduced by Keen and Series is by definition the locus in some parameter space of Kleinian groups on which the bending lamination is a fixed element in $\mathcal{PML}$. It is called rational if the corresponding lamination is rational, that is, comes from bending lines which project to simple closed curve or multi-curves on $\partial\mathcal{C}/G$. In the case of "rational rays" the limit set of the group is formed by chains of overlapping circles such as are apparent, for example, in Figure 6. Movement along the ray corresponds to keeping the same combinatorial pattern while varying the radii and angles of overlap between the circles. A key point for both theory and computations is that on a rational ray, the traces of the corresponding group element or elements are real valued.

Using some of the powerful ideas of Thurston theory, Keen and Series were able to show the rational rays have many remarkable properties. In spaces involving one complex parameter such as the Maskit and Riley slices described below, they were able to compute the rational rays explicitly showing that they fill out the entire space of discrete free groups in the given family densely. This gave in particular an algorithmic way in which to locate the boundary between discrete and non-discrete groups.

This should all become clearer when we look at two particular examples explained in the following two sections.

4.2. The Maskit slice. Let

$$G_\mu = \left\langle X := \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, Y_\mu := -i\begin{pmatrix} \mu & 1 \\ 1 & 0 \end{pmatrix} \right\rangle, \quad \mu \in \mathbb{C}.$$

The Maskit slice is the set

$$\mathcal{M} = \{\mu \in \mathbb{C} \mid G_\mu \text{ is discrete and free}\}.$$

This set has two components which are symmetric under reflection across the real axis, both of which by results of

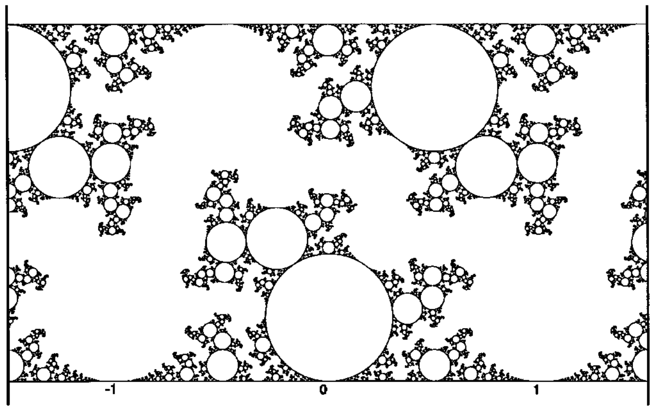

Figure 6. Limit set of a group on the 3/10 ray in the Maskit slice, $\mu = 0.5 + 1.6583124i$. The disks bounded by full circles in Λ_μ all project to the triply punctured sphere boundary while the overlapping circles give rise to part hyperbolic planes. The lines along which they intersect project to the 3/10 curve on $T_{1,1}$.

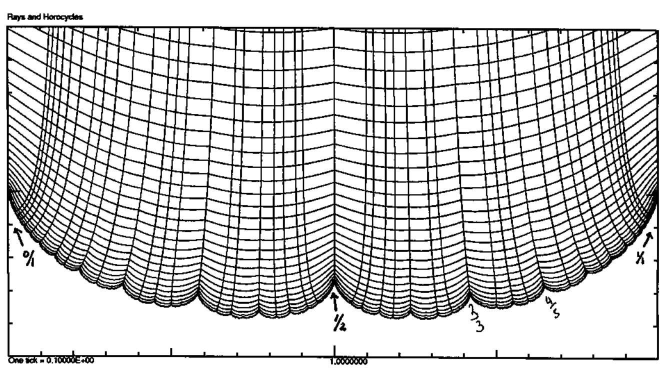

Figure 7. The Maskit slice. The pleating rays are the approximately vertical lines. The approximately horizontal lines represent laminations with the same length, appropriately normalized.

Bers can be identified with the Teichmüller space $\mathcal{T}(T_{1,1})$ of the punctured torus $T_{1,1}$. We consider the upper component $\mathcal{M}^+$; see Figure 7.

For $\mu \in \mathcal{M}^+$, let Ω_μ be the complement in $\mathbb{C} \cup \{\infty\}$ of the limit set Λ_μ of G_μ. As the reader can discern from Figure 6 with a bit of effort, Ω_μ consists of a simply connected component Ω_0 which is invariant under the G action, together with a collection of round disks each of which is fixed by a subgroup of G_μ and which together form a single G-orbit under the action. The component Ω_0/G_μ of Ω_μ/G_μ is a once-punctured torus $T_{1,1}$, while all the other disk components all project to a single triply punctured sphere. Since a hyperbolic triply punctured sphere is rigid, its geometry is fixed independent of μ.

Now the component Ω_0 faces a component B_0 of the convex hull boundary $\mathcal{C}(\Lambda_\mu) \subset \mathbb{H}^3$. As discussed in the previous section, B_0/G_μ is homeomorphic to Ω_0/G_μ hence is itself a hyperbolic once-punctured torus with the hyperbolic structure induced from the ambient space $\mathbb{H}^3$. Thus the possible bending lines on the pleated surface B_0/G_μ are the possible geodesic laminations on $T_{1,1}$. With respect to a particular generating set, a simple closed curve on T winds around p times in one direction and q in the other, so can be indexed by a rational number p/q. Thus the set of simple closed curves is identified with the rational numbers $\mathbb{Q} \cup \infty$. Thurston's theory shows that the space of projective measured laminations $\mathcal{PML}$ on $T_{1,1}$ can be identified with the closure $\overline{\mathbb{Q} \cup \infty}$, that is, the extended real line $\mathbb{R} \cup \infty$.

Following the discussion in the previous section, the λ pleating ray $\mathcal{P}_\lambda$ is the set of $\mu \in \mathbb{C}$ for which B_0 is bent along the lamination λ. When λ is a rational number p/q, the lamination is a simple closed geodesic which after suitable normalisation can be represented by a special word $W_{p/q}$ in X and Y_μ. In particular, $\infty = 1/0$ corresponds to $X = W_{1/0}$ and $0/1$ corresponds to $Y_\mu = W_{0/1}$. On the p/q pleating ray $\mathcal{P}_{p/q}$ the trace of $W_{p/q}$ is necessarily real valued, but the converse is not true. (The real locus of $\mathrm{tr}(W_{p/q})$ has other branches that correspond to groups where the geodesic represented by $W_{p/q}$ lies in the interior of the convex core.)

In [KS93], Keen and Series explicitly describe the rational pleating rays for $\mathcal{M}$ for this setting. Each rational ray is a connected component of the set of μ for which $\mathrm{tr}\,(W_{p/q}) \in [2, \infty)$ which, remarkably, contains no singularities. Moreover by analyzing the leading terms of $\mathrm{tr}(W_{p/q})$ as a polynomial in μ, combined with some new results on the behavior of $\mathcal{C}$ as μ varies, they show that for real λ the pleating ray $\mathcal{P}_\lambda$ is asymptotic to the line $\mathrm{Re}(\mu) = 2\lambda$.

The rational pleating rays are clearly pairwise disjoint. They show further that they are dense in $\mathcal{M}$ and are interpolated by 'irrational rays' corresponding to non-closed bending laminations which share similar properties to the rational ones. Each rational ray ends in a unique point on $\partial\mathcal{M}$; these are the cusp groups described by Bers and Maskit. Minsky's famous ending lamination theorem for T can be viewed as the statement that the irrational rays also have a unique ending point, from which it follows that $\partial\mathcal{M}$ is a Jordan curve.

4.3. The Riley slice. Let

$$H_\rho = \left\langle A := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B_\rho := \begin{pmatrix} 1 & 0 \\ \rho & 1 \end{pmatrix} \right\rangle, \quad \rho \in \mathbb{C}.$$

The Riley slice is the set

$$\mathcal{R} = \{\rho \in \mathbb{C} \mid H_\rho \text{ is discrete and free}\}.$$

This slice was studied by Keen and Series in [KS94]. For $\rho \in \mathcal{R}$, the complement Ω_ρ of the limit set Λ_ρ of H_ρ has just one connected component which is however no longer simply connected. The quotient Ω_ρ/H_ρ is a four-times-punctured sphere $S_{0,4}$. As in the previous section,

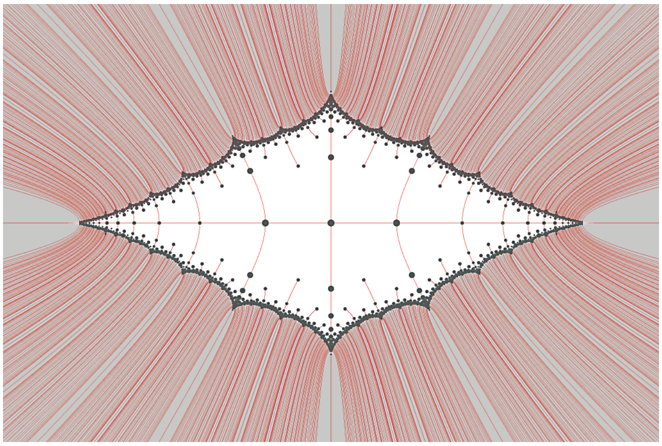

Figure 8. The Riley slice with extended pleating rays.

this means that the convex hull boundary $\partial\mathcal{C}(\Lambda_\rho)/H_\rho$ is a pleated surface homeomorphic to $S_{0,4}$. However in this case, since Ω_ρ is no longer simply connected, there is a simple closed loop on Ω_ρ/H_ρ which bounds a disk which is homotopically trivial in the hyperbolic 3-manifold $\mathbb{H}^3/H_\rho$. Such a loop cannot be realized as a bending line of $\partial\mathcal{C}(\Lambda_\rho)/H_\rho$.

By realizing $S_{0,4}$ as a quotient of the plane $\mathbb{R}^2$ punctured at lattice points $\mathbb{Z}^2$, it is not hard to see that simple curves on $S_{0,4}$ are in bijective correspondence to lines of rational slope on the plane. Taking into account the homotopically trivial missing loop which we label $1/0 = \infty$, one sees that the possible bending lines are indexed by $p/q \in \mathbb{Q}/2\mathbb{Z}$, and moreover each possible bending line curve corresponds to a specific element $V_{p/q} \in H_\rho$. The pleating rays $\mathcal{P}_{p/q}$ turn out to have two symmetrical connected components each contained in the set of ρ for which $\mathrm{tr}\,(V_{p/q}) \in (-\infty, -2]$.[3] Moreover, again by analyzing the top terms of $\mathrm{tr}(V_{p/q})$, Keen and Series show that for real λ the components of the pleating ray $\mathcal{P}_\lambda$ are asymptotic to the lines $te^{\pm i\pi(1-\lambda)}$ for $t > 0$.

A similar analysis for a family of groups corresponding to a hyperbolic 3-manifold which is a handlebody with a certain 3-fold symmetry was carried out more recently by Series, Yamashita and the second author [STY17]. The corresponding pleating rays are illustrated on the cover of the December 2016 AMS Notices.

The family of groups H_ρ had been investigated by Robert Riley in the 1970s. He showed that each two-bridge knot or link complement corresponds to a point in the complement of the Riley slice, thus for example, the figure eight knot complement corresponds to $\rho = e^{i\pi/3}$. With the aid of elaborate computer computations, he also identified families of discrete groups, which he called Heckoid groups, in the complement of $\mathcal{R}$. These groups generalize the Hecke groups where $\mathrm{tr}\,(V_{0/1}) = \mathrm{tr}(AB_\rho) = -2\cos(2\pi/m)$ or $\mathrm{tr}(V_{1/1}) = \mathrm{tr}(AB_\rho^{-1}) = -2\cos(2\pi/m)$.

Following a suggestion of Sakuma, and as explained by Caroline in her 2019 Presidential Lecture, it is conjectured that the pleating rays may be analytically continued along arcs where $\mathrm{tr}\,(V_{p/q}) \in (-2, 2]$. These extensions are the black arcs outside the colored region in Figure 8. The groups along these arcs are holonomies of hyperbolic cone manifolds. The points marked by black dots correspond to the Heckoid groups for which $\mathrm{tr}\,(V_{p/q}) = -2\cos(2\pi/m)$ for m an integer at least 3, while the rays end in the knot and link complement groups occurring when $V_{p/q}$ is the identity for which $\mathrm{tr}\,(V_{p/q}) = 2$.

As a consequence of unpublished results of Ian Agol, recently completed by Sakuma and several collaborators including the first author, the Heckoid groups and knot or link complement groups are the only discrete H_ρ in the complement of $\mathcal{R}$. It follows from the above that all such groups should be located on the extended pleating rays. The pleating rays for $p/q = \pm 1/m$ or $\pm(m-1)/m$ may be extended to points on the real axis corresponding to Hecke groups (or the origin when $m = 1$ or 2), and the remaining rays may be extended as far as points away from the real axis corresponding to knot or link complement groups.

4.4. Pleating varieties. Subsequent to these results on the Maskit slice and the Riley slice, Caroline went on to generalize pleating rays to families of Kleinian groups depending on more parameters, thus relating to higher genus surfaces.

In one direction, Caroline considered Maskit embeddings for higher complexity surfaces. She proved general results on the top terms of the trace polynomials, first for the case of twice-punctured torus, jointly with Keen and the first author [KPS99], and then for all surfaces, in joint work with her student Sara Maloni [MS10]. Caroline then gave formulae for the asymptotic directions of pleating varieties in the case of the twice-punctured torus, the general case being worked out by Maloni in her thesis.

In another direction, one can consider pleating varieties for families of quasifuchsian groups G, that is, groups for which the limit set is a Jordan curve. Thus, Ω is separated into two simply connected invariant components and Ω/G consists of two Riemann surfaces $\Sigma^\pm$ each of which is homeomorphic to the same surface Σ. The corresponding parameter space is $\mathcal{T}(\Sigma)\times\mathcal{T}(\Sigma)$, where $\mathcal{T}(\Sigma)$ is the Teichmüller space of Σ.

In this case one has to consider two geodesic laminations, μ and ν, one on Σ^+ and one on Σ^-, and to analyze the set of those groups for which μ and ν can be the bending lines on the two components of $\partial\mathcal{C}/G$. Of particular interest is the set points where this locus meets the Fuchsian locus $\mathcal{F}$, that is, the set of groups for which the limit set is a round circle. Note that $\mathcal{F}$ can be identified with the Teichmüller space $\mathcal{T}(\Sigma)$.

[3] *The original paper contained an error pointed out and corrected by Y. Komori.*

Keen and Series made an initial analysis of this situation in the case of once-punctured tori and showed that all possible pairs of distinct projective laminations can occur. In [PS95], she and the first author looked in detail at the case in which the bending laminations are a pair of closed curves which intersect exactly once, deriving exact formulae for the relationship between the bending angle between planes in $\partial\mathcal{C}/G$ and the geodesic length of the bending curve. These formulae were used to show that, if the two curves represent generators A, B of G, then the corresponding pleating variety meets $\mathcal{T}(T_{1,1})$ exactly in the set of rectangular tori.

The case of twice-punctured tori was studied in various papers with Raquel Díaz, Linda Keen, and the first author. Caroline then showed in [Ser05] that for any surface, the pleating variety associated to the pair of laminations (μ, ν) meets the Fuchsian locus $\mathcal{F}$ exactly on Kerckhoff's line of mimima for (μ, ν) in $\mathcal{T}(\Sigma)$. This line is defined as follows: for a given $t \in (0,1)$ there is a unique point in $\mathcal{T}(\Sigma)$ at which the length of $t\mu + (1-t)\nu$ is minimized. The line of minima is the collection of all such points for $t \in (0,1)$. In particular, the set of rectangular tori in $\mathcal{T}(T_{1,1})$ is exactly the line of minima for the generating curves A, B above.

In an important paper with Young Choi [CS06], she went on to extend the one-dimensional result on non-singularity of pleating varieties by proving that for general Kleinian groups the complex lengths of the bending lines are local holomorphic parameters for the ambient parameter space.

Caroline has a number of other more general results on hyperbolic 3-manifolds and Teichmüller theory, for example her variant on Wolpert's well-known derivative formula, work with Choi and Kasra Rafi on the relationship between lines of minima and Teichmüller geodesics, and with Mahan Mj on the behavior of individual limits under geometric and algebraic convergence of groups.

5. Exposition and Service to Community

An important contribution of Caroline's to the field has been her expository work which has brought the fascination of the subject to many young mathematicians and the broader public. Her paper [Ser85a] on the geometry of the Markoff numbers was a very attractive and elegant exposition of the intimate connections between number theory, Diophantine approximation, hyperbolic geometry, and the modular hyperbolic punctured torus which has been read by generations of students.

One cannot discuss Caroline's work without mention of her aptly named and lavishly illustrated book *Indra's Pearls* written jointly with David Mumford and David Wright. This widely praised book describes everything that is needed to understand and draw limit sets of two-generator Kleinian groups, including basic computer

Figure 9. Caroline with Yair Minsky and Makoto Sakuma at her Spaces of Kleinian Groups programme at the Isaac Newton Institute, 2003.

algorithms needed to make the pictures. Written so that very little technical knowledge is required and beautifully illustrated with quirky figures, precise geometric drawings, and intricate computer-generated figures, its appeal extends not just to experts in the field, but also to undergraduates and amateur mathematicians. In particular it explains and illustrates the many extraordinarily striking and intricate patterns that arise in these limit sets, such as those shown in Figures 6 and 10, and it touches on some of the combinatorial ideas describing the possible cutting sequences for simple geodesics used in her subsequent work with Linda Keen.

Another important contribution of Caroline is her substantial service to the mathematical community at large. She has an immensely influential track record in opening up the subject to underrepresented groups, particularly for women. She was a founding member of European Women in Mathematics (EWM) in 1986 and has held senior roles in the EMS Women in Math Committee and the IMU Committee for Women in Mathematics. Together with other senior women, such as her long-term collaborators Joan Birman and Linda Keen, the effect that Caroline has had on improving gender equality in mathematics has been of great benefit to us all.

On the broader front, Caroline has made many significant contributions to organizations that support the mathematical community. She has given many years of dedicated service to the London Mathematical Society, the European Math Society, and the IMU. Within the LMS she has held several important roles, most notably, as already mentioned, serving as its president from 2017 to 2019. She twice served on mathematics panels for the UK Research Excellence Framework (REF). She has also been significantly involved in the Isaac Newton Institute, in

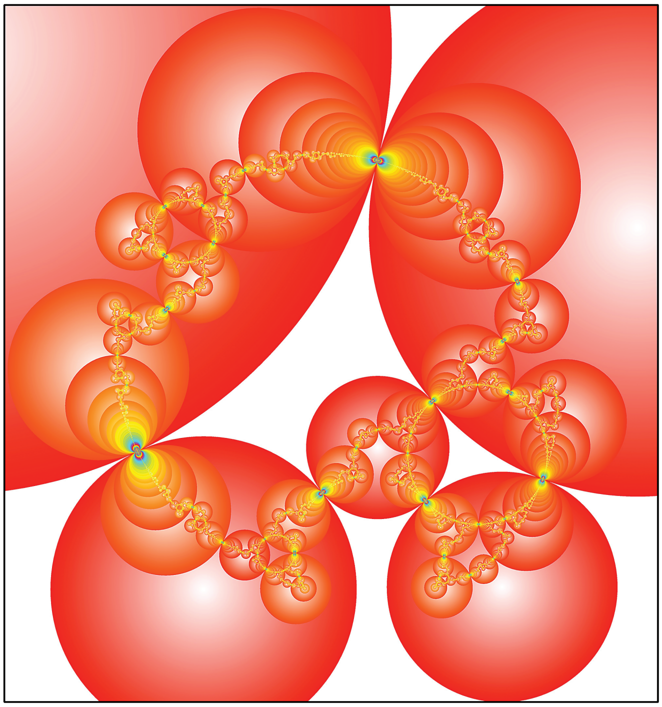

Figure 10. Computer rendering by David Wright of the limit set of a two generator Kleinian group based on an original hand drawing by Fricke and Klein; see Fig. 156 in *Vorlesungen über die Theorie der automorphen Functionen*, Vol. 1, Leipzig 1897.

recognition of which she was recently (2022) made an Honorary Fellow of the Institute.

6. Conclusion

Caroline has made valuable and fundamental contributions to hyperbolic geometry, in particular, the geometry of hyperbolic surfaces and three manifolds. She has introduced important ideas, elucidating and clarifying connections between different aspects of hyperbolic geometry and symbolic dynamics. However, her contributions to the field go far beyond the mathematical aspects. She has been an influential role model who has fostered and nurtured a generation of young mathematicians who have gone on to make significant contributions in their own right. She has been particularly supportive of women mathematicians and other mathematicians from disadvantaged groups, both from official and unofficial positions. She has given numerous public talks which have contributed to the general public's understanding of the work of research mathematicians. She has also organized and co-organized several major conferences, workshops, and meetings in her field, bringing together leading experts and generating a very vibrant and collegial atmosphere for research. She personally ensured that all participants, whatever their background, feel welcomed and part of the proceedings. In particular, she has been instrumental in fostering connections between European, American, and Asian researchers, strengthening in particular the Europe/Asia mathematical ties. Her efforts have helped create an inclusive and supportive atmosphere in the field, making it a very congenial and pleasant area to work in.

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John R. Parker

Ser Peow Tan

Credits

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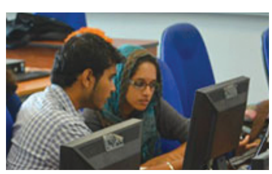

Mary Wheat Gray: "I Never Give Up"

Amanda L. Golbeck and Madhu Mazumdar

Introduction. Mathematician Mary Wheat Gray received a law degree, taught herself statistics, and became the "mother of us all" [5] for her work to advance women's and human rights. With the new Mary and Alfie Gray Award for Social Justice, established by the Association for Women in Mathematics, we decided it was time to take a more personal and reflective dive into Dr. Gray's life and career. We interviewed her to see what more we could learn from her story.

Mary Gray is a mathematician, attorney, and statistician. She is highly decorated, the most recent of her many awards being the Elizabeth L. Scott Award from the Committee of Presidents of Statistical Societies (2012) and the American Statistical Association Karl E. Peace Award for Outstanding Statistical Contributions for the Betterment of Society (2017). American University recently promoted her to the prestigious rank of Distinguished Professor (2019).

Known for crashing a meeting of the American Mathematical Society after confirming in the by-laws that these were open meetings, Mary was told she was not welcome. She was told this was because there was a gentleman's agreement that the meetings were closed. She famously replied, "I'm not a gentleman; I'm staying" [17].

In 1971, the Association for Women in Mathematics (AWM) was founded, and Gray became the first chairperson [4]. The AWM is now "the leading national society for women in the mathematical sciences" and plays "a critical role in increasing the presence and visibility of women in the mathematical sciences" [5], [3].

Between 2000 and 2018, Mary was elected as a Fellow of six organizations: Washington Academy of Sciences, Association for Women in Science, American Association for the Advancement of Science, American Mathematical Society, American Statistical Association, and Association for Women in Mathematics. All of these Fellow awards are highly coveted and rarely given. For example, the American Statistical Association awards this highest honor to only up to 0.3% of its members. Mary's ASA Fellow citation read: "For outstanding contributions in encouraging women and minorities in the study and practice of

Amanda L. Golbeck is a professor of biostatistics and associate dean for academic affairs at Fay W. Boozman College of Public Health at the University of Arkansas for Medical Sciences; and professor emerita in the Department of Mathematics and Statistics at San Diego State University. Her email address is agolbeck@sdsu.edu.

Madhu Mazumdar is a professor of biostatistics in the Department of Population Health Science and Policy, and director of the Institute for Healthcare Delivery Science at the Icahn School of Medicine at Mount Sinai. Her email address is madhu.mazumdar@mountsinai.org.

Opening images: Amanda Golbeck with Mary Gray at the Joint Statistical Meetings in Baltimore, 2017; and Madhu Mazumdar with Mary Gray at the Joint Statistical Meetings in Washington, DC, 2022.

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statistics and mathematics; for leadership in the applications of statistics in the field of human rights and litigation; and for statistical services to developing countries." Mary was also elected to the International Statistical Institute.

Early life. Mary Wheat Gray (born April 8, 1938) was born and raised in the busy, small town of Hastings, Nebraska. It is a place where people come to experience the magic of "one of the greatest wildlife spectacles on the continent," where annually 80% of the world's sandhill cranes and 250 species of water fowl migrate [16] . . . and where mathematicians come to model it.

Mary has deep roots in Nebraska. One grandparent emigrated there from Germany, and three others were born there. Her father, Neil Claude Wheat, was manager of a trucking firm, after earlier holding jobs as a policeman, truck driver, and mechanic. Her mother, Lillie Alves, was a cafeteria worker, after earlier being a schoolteacher and homemaker.

Mary's parents had strong values for education and encouraged her to learn. At home, they taught Mary history, sparking a lifelong interest in the subject. Also, her father began asking her mental arithmetic questions when she was only five years old, sparking her love for mathematics. It is no surprise that the subjects Mary enjoyed most while attending Hastings Senior High School were mathematics, history, and physics.

In high school, girls had to take sewing instead of mechanical drawing, so Mary did experience some gender discrimination. She said, "I've always, when I taught a lot of calculus, blamed the fact that I couldn't draw very good diagrams on the board to the fact that they never let me take mechanical drawing. Of course, I never learned how to sew very well either. I took all the math that was available. My parents were supportive. All the teachers were supportive. So aside from not learning how to draw good pictures, I did fine [in high school]."

Hastings is also a college town, being the home to Hastings College. It is a highly ranked college: in the *U.S. News and World Report*'s 2022 Best College rankings, it ranks 17 out of about 90 Midwestern colleges. With a good college right on her doorstep, Mary decided to go to school there, and her talent for mathematics was recognized.

Mary graduated *summa cum laude* from Hastings College in 1959, with majors in mathematics and physics. Hastings College mathematics faculty member Dr. Jim Standley suggested she continue her studies in mathematics in a graduate program. She decided this would be her path. Many years later, Hastings College did not forget her. In 1996, they gave her a Doctor of Humane Letters, *honoris causa*.

Later, Mary received the same award (Doctor of Humane Letters, *honoris causa*) from Mount Holyoke College in Massachusetts (2008). In addition, she received the Doctor of Laws, *honoris causa*, from the University of Nebraska–Lincoln (1993).

Early international experience. But first, Mary decided to broaden her horizons. Colleges like Hastings promote international study experiences, because they aim to provide broad intellectual experiences for their students. Mary applied to the Fulbright U.S. Student Program for study in Germany, which filled a "gap" year between her undergraduate and graduate study. This Fulbright program aims to develop cross-cultural academic connections that expand the perspectives of participants [19].

The Fulbright experience was a big move for Mary, because previously her only international experience had been a one-time crossing over the border to Canada from Detroit. The experience was what she called "a great revelation," and it was the beginning of her international orientation which has led both to her pleasure and to her impressive body of international work that we will describe later in this article.

Mary used her Fulbright fellowship to study mathematics at the J.W. Goethe Universität Frankfurt. She received some good mathematics instruction there. A bonus was that National Medal of Science winner Saunders Mac Lane was in residence, and the leading mathematician at the university in Frankfurt was a woman. Mary reflected: "certainly I picked up other stuff—like the German version of the history of WW2—but the grant was to study math and that's what I did."

This experience centrally located Mary in Europe, and she got to travel a lot. The university she was studying at had an office that facilitated travel within Germany and all-around Europe. They wanted to show off Germany and Europe, and she was able to use their services. Mary met a lot of German students, and other people on her travels. Mary summed up her experience by reflecting that the Fulbright was also a positive experience "from the point of view of learning about other people and how to get around on your own and all kinds of things." She told us: "It changed my entire perspective and made me very anxious to travel and to meet people in all sorts of places from then on. My life was definitely changed."

Not many people in STEM fields apply for such experiences between undergraduate and graduate study (a "gap year"). STEM students don't apply, according to Mary, because they "are afraid they'll get behind and lose a year. And all too frequently, mathematicians don't have another language besides their native language, or if so, it may not be one that's usable under a Fulbright grant." Most of the people who study on student Fulbrights are in the humanities or the arts. So the Fulbright is an important opportunity for STEM students. Because it was unusual that Mary came from the Midwest and wanted to study mathematics, it was not that hard for her to get a Fulbright grant.

Mary said that now she encourages students to "think a little bit larger," and doesn't necessarily think that they have to immediately go to graduate school. Mary said she intends to encourage more students to pursue this type of opportunity.

Higher mathematics education. Mary decided to do her graduate work at the University of Kansas. She earned an MA (1962) and a PhD there (1964), both in mathematics. Mary financed her studies with National Science Foundation and National Defense Education Act fellowships, employment as an assistant instructor at the university, and summer employment as a physicist at the National Bureau of Standards. Her dissertation was on *Radical Subcategories* [13]. It was supervised by William Raymond Scott, an expert in group theory. Scott had 21 PhD students. Mary was the most prolific of them, having 34 PhD students of her own. Mary published this work on radical subcategories a few years after receiving her PhD.

Students in the doctoral program in mathematics at the University of Kansas had to demonstrate proficiency in two languages other than English. Mary declared, "Having just gotten off the boat, so to speak, from Germany, my German was pretty good. But I then met my future husband, and we both decided to take Russian, and we learned enough Russian that we could handle Russian mathematics and maybe a little bit more." Mary told us that her language advantage helped her to get along with everyone, including the faculty.

But all was not ideal in graduate school. At the time, Mary was the only woman in the PhD program at the University of Kansas. The only other woman to get a PhD there in mathematics had done so 30 years prior. That woman was working in the department, but teaching in a "rather subsidiary role." There were quite a few women in the beginning graduate courses, in the master's degree program, and pursuing mathematics education, but there were no other women in the advanced mathematics courses that Mary took.

In graduate school, Mary experienced her first discrimination for being a woman in mathematics. In her words: "I had never really encountered much discrimination at all as an undergraduate, nor in Germany. It was a revelation when I got to graduate school in Kansas, on the other hand..." Mary explained: "On my first day in graduate school, I had a course in a subject which I had not had as an undergraduate. At the end of class, when I was walking out, the instructor said to me, 'You know, I don't know what you're doing here. You're occupying a perfectly good position that could be filled by a man.' I was very taken aback, having come from my undergraduate school and from a year in Germany where I hadn't seen any of this."

Mary agreed that this attitude was common at the time. The expectation was that women would be in the master's, not doctoral, programs. "Maybe to work very hard, but not have much insight into mathematics as a subject matter," Mary explained.

Married women then were expected to stay at home and take care of the family, so seeing a woman studying to be a professional was, as she put it, "totally beyond their general comprehension." Mary continued to explain: "Since there was only one of me, I wasn't threatening anybody... matter of fact, it's only when they see people taking their jobs that people get worried about women or get worried about people from different backgrounds. As long as they're just cooking dinner and cleaning the house, you know, they're fine."

Then Mary described what she did about her early experiences involving discrimination: "You know, my motto is: Don't get mad, just get even. I decided to do better than everybody else in the class. The most interesting part is that many, many years later, when I would encounter this instructor in a joint mathematics meeting or something like that, he would always act as if he'd been my mentor and had contributed to my great success... And, you know, I could have said you're a jerk <laughs> but I figured, well, he'll have more women students and maybe he'll learn and treat them a little bit better." Mary continued: "While not overly enthusiastic, I was always very pleasant and encouraging to the men."

Professorships. Mary met the love of her life, mathematician Alfred Gray (1939–1998), when both were in graduate school in mathematics at the University of Kansas. Alfie moved to the University of California, Los Angeles to complete his PhD in differential geometry.

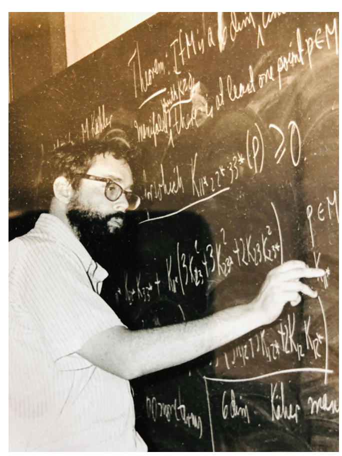

Figure 1. Alfred Gray at work.

Mary and Alfie married after graduating and became part of a dual-career couple, with the need to find two academic positions in the same geographic location. Their long-term strategy was to find a place "where there were two reasonable universities... We decided long ago we would probably never stay married if we were in the same department," Mary explained.

The couple's first move was to Berkeley, in 1964. Alfie took a post doc position at the University of California, Berkeley. He spent four years there. Mary taught a course in the Berkeley mathematics department in the summer (1965). Then Mary moved to a faculty position at the California State University, Hayward, which is now California State University, East Bay. Mary was on the faculty in the mathematics department at Hayward for three years (1965–68).

Their second move was to the Washington DC area, in 1968. Mary took a faculty position at American University and Alfie took a faculty position at the University of Maryland, College Park. The couple lived between their two places of employment so as to split the commute between them. Because they liked to travel, the idea of living in DC was attractive to them because there were lots of good universities and lots of nonstop flights to Europe.

Mary spent the rest of her career at American. She rose through the ranks to become a tenured full professor in the Department of Mathematics, Statistics, and Computer Science, which became the Department of Mathematics and Statistics. She was appointed chair of the department in 1977. Mary had this to say about her career opportunities: "I generally have felt that I got the position I was asking for in most cases, although sometimes it was a battle... sometimes I had to go out of the way to have this happen. Sometimes I haven't gotten what I thought I should get because I was a woman. For example, it was only recently a few years ago that my university made me what's known as a distinguished professor, which is a very rare category. There were only six or eight in the university as a whole, out of 800 faculty, something like that."

Alfie spent the rest of his career at the University of Maryland. He made broad contributions to differential equations and complex variables, and specific contributions to classifications of types of geometrical structures. Mary reflected on Alfie: "He had an extraordinary ability to spend a period of time with differential geometers, or potential differential geometers, engage in joint work and establish ongoing productive collaborations, particularly with mathematicians in Spain, Italy, and Eastern Europe, when such engagement was not easy. His ability to learn languages was remarkable. At a memorial conference after his untimely death in Bilbao—where he tackled even Euskara—I learned the extent of his productive mentoring and our shared commitment to extending the inclusivity of mathematics." Alfie died from a heart attack on October 27, 1998 at age 59 while working at 4 a.m. with students in a computer lab at a college in Bilbao, Spain. Two years later, there was a conference on differential geometry held in Bilbao in his memory [6]. Mary added: "Alfie's too-short career was marked with our common commitment to mathematics and social justice."

Mary reflected on the topic of awards going to women. She gave an example where, in an academic unit, about half of the awards went to women. But the people who presided at the awards ceremony—the people who introduced the program, called out the winners, and handed out the awards—were not half women. "They could have had a lot of different people handing out the awards," Mary observed. "There should be just as many senior women as senior men speaking. Women are perfectly capable of handing out an award," Mary insisted. The lesson is that we need to be vigilant about correcting these inequities. We need to speak out about them, if we are to change them, even at the risk of embarrassing people. And we need to speak out about them repeatedly... we need to keep reminding people, because "they don't remember," Mary said.

Law school. After a fruitful career start in mathematics, Mary went back to school to study law, and she studied law through the 1970s. She used her faculty benefits to pay part of her law school tuition and fees. Her benefits included payment for two courses per semester.

On why she went to law school, Mary said: "Before taking on the world, I discovered that sometimes social justice begins at home." She had gotten a statement about her work-related retirement benefits. From it, she learned that a man of the same age and with the same amount of accumulated funds would get 15% more per month than she would at retirement. "To my indignant complaint to the insurance company about discrimination in employee benefits on the basis of sex being illegal, the agent asserted that this was not discrimination on the basis of sex but rather on the basis of longevity." Mary continued: "So, then I asked for a guarantee of longer life, and he accused me of just not understanding statistics. But I worked very hard with attorneys who were handling the case putting together data to back up the notion that in fact what they were doing was discriminatory. Then the insurance company accused me of not understanding the law."

"I fixed that one by going to law school," Mary said [11]. She graduated with her Juris Doctorate (JD) *summa cum laude* from Washington College of Law in 1979. She is a member of the District of Columbia Bar, the US Supreme Court Bar, and the Federal District Court Bar, District of Columbia. This means she is able to practice law in the District of Columbia and before the Supreme Court.

Learning statistics. Mary learned statistics less formally by attending various seminar series and asking others for guidance on books she was reading. When asked about her education in statistics, Mary explained that she just "fell into it." People kept approaching her and asking for her help on things that involved basic statistics:

> There were a couple of questions—way back in the late seventies or early eighties... in the early feminist movement—that involved selection of White House Fellows. A question was: Why weren't more women chosen to be White House Fellows? Someone wanted to look at the probability of the selection turning out the way it was, given what was known about the people who had been applicants for the position. They were looking for

a mathematician and a statistician, anybody they could find who could calculate a probability and was a welcome co-conspirator in whatever it was they were doing.

I started on things like that. One thing led to another, and before long I was consulted on various things. There was another question about how these people were conducting the draft lottery. I decided that I better do some in-depth background checking on the issue, because my background wasn't that strong. That's the nice part about Washington DC. There's an endless series of talks going on. The Census Bureau has a series. The Bureau of Labor Statistics has a series. A couple of the other agencies have various series going on. There are not-for-profit companies. So, you can always find something to go to, and you can always find somebody to help you out. If you need to pick a textbook and have no idea, there's somebody—well, usually three or four people who will tell you different things, but at least there's somebody—to guide you along. I don't think I could have done [the advocacy work] without the facilities that Washington presented. Not to say there might be other places, either in a city with many agencies or in a place with a large research university, but it's not something that you pick up that easily on your own."

Today many people understand the value that statistical and mathematical knowledge can bring to the justice system, as evidenced by a number of commentaries and books on the subject (e.g., [7]).

Which books or articles have been especially helpful to Mary as an attorney who regularly uses statistics in the practice of law? Actually, Mary recommends "that they sit in on a beginning graduate level stat course in a stat dept (and/or persuade their alma mater to offer such a course). Any attorney will realize that the changing scene of the law is accompanied by a changing scene of which statistics are useful; the subtleties are such that what is in writing at a given time is of limited use—as we tell students, a basic understanding is essential. However, the Finkelstein and Levin text is still good—but scares lawyers (and statisticians too)."

What law text(s) would be helpful to a statistician/expert witness? Mary pointed out that Harvard University, New York University, and Columbia University, among others, have first-year guides for law students that are helpful in explaining the topics and the way law study is organized.

Even well-educated lawyers in our highest courts struggle to fully understand data presented in law cases, and few seek their own further education to fill the gap in their

Figure 2. Katerina Nesvit, data scientist, Marymount University (Arlington, VA), from Kharkiv, Ukraine—in the US for a little over a year—and Kristina Crona, mathematician, American University, originally from Sweden, with Mary Gray.

knowledge. Mary was a true pioneer in this area, and she deserves credit for creating a path that others should follow. Her focus on understanding statistics to provide credible legal evidence has surely influenced a new generation of lawyers who followed a similar path to broaden their education beyond the legal field into statistics. Without more students willing to seek such education, however, legal decisions, even in the highest court, might be made without a full understanding of the evidence [18].

In her eighties and suffering from arthritis and other problems, Mary still finds energy to be an active participant in law and statistics. She regularly pens a column for *Chance*, a nontechnical magazine published by the American Statistical Association and Taylor & Francis Group that highlights the application of sound statistical practice (`https://chance.amstat.org/about/`). Her columns include "The Odds of Justice," "Statistics Go to School and Then to Court," "Who Gets to Vote?", "Whom Shall We Kill? How Shall We Kill Them?", and "Alexa Did It!" These columns discuss complex legal issues involving the use of statistics to make solid arguments. Mary notes: "Believe it or not, these [columns] are very hard to write, because you want to write something that both lawyers and statisticians will be happy with when they read it, and will maybe learn something from as well."

Mary's unique ability to find connections between the seemingly disparate fields of mathematics, law, and statistics undoubtedly impacted her career trajectory and success in facilitating change. With her big-picture perspective and varied skill set, she was able to argue for change in human rights, including women's rights, which we discuss later.

Federal testimony. Mary has testified on a variety of issues. These have included inequity in social security, pension, insurance, and pay; income tax reform; affirmative action; age discrimination; women and the military; and constitutional rights to travel. She has testified in front of a variety of federal committees. These have included US House committees: Ways and Means, Energy and

Commerce, Education and Labor. They have also included US Senate committees: Human Resources and Labor, Commerce, Education and Civil Rights. Mary's efforts were always to change the law to make it more equitable, and statistical evidence has always been the centerpiece of Mary's testimonies.

Mary had this to say: "All of the testimony has involved statistics in one way or another, rather than necessarily anything legal. Usually what we're trying to do with the testimony is get the law to be changed. Take income tax reform as an example. It didn't used to be that you had a way to file a joint return if you had two incomes. And that was costing employed women a lot of money. So, the idea was to change the rules in order to make [tax law] more amenable to two-income families. The affirmative action is, same old, same old, let's train more women and then hire them. The age discrimination, that had to do with the pension issue and the fact that there was a discrimination in pension...The point is that the law needs to be changed because you can show through statistics that the law is not treating people fairly."

Most of the issues that Mary worked on involved gender in some way. Social security and retirement issues involved gender. Mary also worked on civil rights issues that involved race, and on constitutional rights that involved everyone. Which testimony had the most impact? Mary thought that making social security issues less gender discriminatory for two-career couples, divorced spouses, and employed wives ultimately had the most impact, because it affected the biggest portion of the population.

Mentoring. Mary has been a caring mentor to many students, faculty members, and other kinds of people who have crossed paths with her. The American Association for the Advancement of Science recognized her superior mentoring and guidance in 1994 in the form of a Mentor Award for Lifetime Achievement. This award recognizes Mary's "extraordinary leadership to affect the climate and increase the participation in doctoral science education of individuals who are from underrepresented groups" [1].

In 2001, Mary received another national mentoring award: A Presidential Award for Excellence in Science, Engineering, and Mathematics Mentoring established by the White House and administered by the National Science Foundation. This award recognizes that Mary has been generous as a mentor to many individuals—both women and men—in science, technology, engineering, and mathematics (STEM). She has especially encouraged promising women to major or pursue higher degrees in mathematics, guided many as they pursued their degrees, and given useful career advice.

Mary educates and engages her students in important world issues as part of their coursework by involving them in data collection and analysis for her projects. One such student was a faculty member who retired from the law school at American University and entered one of Mary's graduate courses in statistics. Mary recalled: "She said she'd heard all these cases all the years that she'd been a practicing attorney, [but] she never understood what they were talking about [when it came to statistics]. So now she was coming back and taking graduate work in statistics. She did very well. Once she got her hands on statistics, she turned out to be good at it. I got her to do some nice volunteer work where she could use both her legal knowledge and her newly acquired statistical knowledge."

When we asked Mary about her mentoring style, she said: "Because I don't have children, I've probably always looked at students a little bit differently than a lot of people do. I've had a lot of PhD students, but I've also had a lot of undergraduate students who I'm still in touch with after many years...It's probably the trait of looking out for other people. You're worrying about: Why aren't they doing what they should be doing? Couldn't they make a little bit better use of their talents than they're making? Are they about to make a bad choice? Sometimes they are about to make a bad choice in choosing a life partner. I always say to women in math that the best decision you often have to make is choosing a partner...There's not a lot you can do in advising people except maybe leading them through some of the pitfalls that they may not have seen. So that's what I look in on. What can we try to do to say that they're making the right decisions, not only about their careers, but where it's leading them."

Mary has also supervised 34 dissertations and 11 master's theses or projects. In addition, she has served as a member of many PhD dissertation committees, most in mathematics education.

American Association of University Professors. Individuals have benefitted from Mary's mentoring, but so have organizations. Her volunteer work is extensive.

Mary has had a particularly strong affiliation with the American Association of University Professors (AAUP). This organization fits Mary's professional outlook. It is dedicated to "advancing academic freedom and shared governance; defining professional values and standards; promoting the economic security of those who teach and research in higher education; organizing to make our goals a reality; and ensuring higher education's contribution to the common good" [2].

For many years, Mary was chair of the AAUP's Committee on Women, known as Committee W. Today the committee is merged with another committee into the Committee on Gender and Sexuality in the Academic Profession. It works to "formulate policy statements, provide resources, and report on matters of interest ... addressing such issues as equity in pay, work/family balance, sexual

Figure 3. Left: Mary Gray, mathematician Jair Koiler, and friend Rosa Maria in Rio de Janeiro. Right: Mary Gray at the Teatro Colon Buenos Aires in summer 2018. Photograph taken by mathematician Jair Koiler from Rio. Koiler is one of the mathematicians with whom Mary went to Uruguay years ago to get dissident mathematician Jose Luis Massera out of prison.

harassment and discrimination, Title IX, and the role of gender and sexuality in rank and tenure."

While Mary was chair of Committee W, it partnered with AAUP's Committee on the Economic Status of the Profession (known as Committee Z). They recruited and oversaw the work of statistician Elizabeth L. Scott to develop the Higher Education Salary Evaluation Kit. The Kit was highly influential in raising salaries for women across the county so that they would be equitable with men who had the same background and experience. For details, please see [8]. Mary identifies Elizabeth Scott as her biggest professional hero.

In 1979, Mary received the Georgina M. Smith Award from the AAUP. The award is presented to an individual who has improved the status of academic women or advanced collective bargaining in the profession. This award is not given on a regular basis, but only when someone especially deserving is identified as exceptionally outstanding as to merit.

Leadership and diversity. In the 1970s, an academic department chairpersonship was not considered a desirable position in the mathematics department at American University where Mary was a professor. As a result, the role was rotated among the faculty members in the department, and Mary served as department chair six times (1977–1979, 1980–1981, 1983–1985, 2001–2003, 2009–2011, 2016–2018). Female mathematicians, especially those who held leadership positions, were not common at the time. Even today, fewer than 30% of department chairs in the mathematical sciences are women [9].

When asked if there was something different about women as leaders, Mary had this to say: "I think it's difficult and probably unfair to make generalizations, but I think women are brought up to be more considerate of how they're affecting other people. No matter how evolved a family they grew up in, they probably have been treated a little differently than sons in the family, or they've been expected to look out a little bit more for younger children, or when dad comes home from work, they're the ones who produce the drinks or whatever. I think the training tends to be more to look out for others, and that probably carries over into the way people behave as department chairs. You can of course come up with exceptions. But I think if there's one attribute, that would be it." Mary's thoughts are consistent with some of today's business leaders, who note that inclusivity, empathy, and emotional intelligence are among the many reasons women make great leaders.

Mary has noticed that often there is diversity in only certain strata of the workforce. Therefore, she advises people to review the diversity and benefits of the full workforce of an organization. Expanding benefits, especially offering free tuition, is one of the best ways to improve diversity overall.

Mary remembered that perks—like the kind American University offered to her as a faculty member and that she used to finance her law school studies—can be useful to diversify a workforce. Mary found this way: "The university is always talking about how we need more diversity. We need more diversity, they say, and then they don't want to do much of anything. To have more students, particularly from African American and Hispanic communities, at American University is going to be difficult. One thing that we did—and I spent weeks and hours and months on this—was get American University to give their employees free tuition for their kids. And that's good."

But what about contract workers, Mary wondered. Can't their kids get free tuition too, and at the same time help to diversify the student body? Mary continued: "Not everybody who works at the university is an employee of the university. The people who do the cleaning and the cooking are contractors. They're not actually employees, so their kids didn't get free tuition. Well, almost everybody on the cleaning staff is Latino and almost everybody on the cooking staff is African American. And so, you know, it's a perfect place for diversity…I finally managed to get the university to agree to provide free tuition for the children of all of the subcontracting workers, as well as some benefits for the workers themselves. This opens up opportunity for a vast variety of people."

Mary realized that diversity would result only if a diverse group of people actually used the employee benefit: "You need to get out there and make the kids aware of it, make the parents aware of it…" And we shouldn't make assumptions about the people who could use the benefit, because those assumptions might be racist. "Then the administration said the benefit was only for the elementary courses, like the writing courses and the elementary math courses. But think about the inherent racism of what they were saying. Well, I rounded up a worker who I knew, and the worker came to talk in the next meeting. We had the people in charge there and one of the workers said: I really want to take a graduate course in management because I've got five barber shops. And I would like to expand my knowledge and network in the management training. That shut up all the trustees immediately."

Mary continues to raise the question herself as she visits other institutions: "That's one thing: When I go to other universities and sometimes people say, 'What can we do?' The first thing I ask is: Who cleans, and who cooks? Are they real employees? And what are their benefits? And almost always it opens up opportunities for them. And there's gender involved too, because almost all of the cleaners—or at least more than two thirds of them—are women." Mary continues to be a tireless champion for increasing diversity.

Collaborative networks. We noted that Mary has built a huge network through which she gets asked to be involved in a variety of issues encompassing many states and countries. When asked about how she builds the network and partnerships, she had this to say: "It's usually somebody different for everything. For this most recent insurance article, I corralled a retired insurance lawyer who lives in my neighborhood to help. For women in the military, I tracked down a woman who was a Colonel in the military. I'm always looking for somebody who knows more about [a topic] than I do."

Also, Mary gets involved when there are calculations to be made. She noted: "The reason I'm involved in this labor dispute on campus now is because the people who are doing the union negotiation don't like to have to look at numbers. I think people are not as well informed as they might be, and it's not only with the advent of Facebook and other social media. The traditional media that you and I grew up on is not pursued by enough people." That's why Mary's department created an elementary statistics course that would be useful for improving the statistical literacy of the masses. She said: "We call it, 'Read the *New York Times*,' because that's about as deep as they get into the statistics. If people would keep track of the issues that come up, then when it impinges on something that they might know something about or might be interested in, there's a way they can seek more information. Reading the newspaper is an old-fashioned way to get connected, and it still works."

How does Mary get her information about current events? She always listens to the news while she is driving to work: "It's a drive that varies from just under 15 minutes to just under 30 minutes, depending upon the traffic and which day of the week it is and so on. That's just enough time to get the morning news. American University has a public radio station called WAMU, and the British Broadcasting Corporation (BBC) news comes on at nine o'clock in the morning. So, I usually get a few minutes of US news from WAMU, and then at nine o'clock BBC switches in, and then I arrive at work at nine fifteen. I get a little bit of all of it."

Networking is an essential career element, including in the mathematical sciences. If networking doesn't come naturally to people, they can always intentionally learn to do it by—among other things—concentrating on their higher purpose, as Mary does. Since statisticians like Mary like to look at numbers, they can be of special use in networks that are formed to achieve a higher purpose.

International justice work. Mary has worked on many human rights issues across the globe: in the Palestinian Occupied Territory, Gaza, Iraq, Rwanda, the Middle East, and elsewhere. Her entry to this work has been through the mathematical societies. Also, she has worked with Amnesty International, and she is a member, former chair of the board, and an executive committee member of Amideast, an American nonprofit organization engaged in international education, training, and development activities in the Middle East and North Africa.

How did Mary first get involved in international justice work? Mary said: "The first case I got involved with was the mathematician who we got out of prison in Uruguay…I was doing the work as a representative of the American Mathematical Society, but I met people who worked in Amnesty International. And then I had a good colleague in the sociology department here at American University who was Palestinian. He got me interested in working on Palestinian issues, which of course never get resolved. Every day there's another atrocity, but that certainly got me interested. And then from those organizations, I spread out to see what else I could work on." Mary has made many radio and television appearances, and given many speeches, to the above-mentioned and other groups about a myriad of human rights issues.

Women's rights. What is Mary most proud of, given her immense body of work? Mary said: "I think probably the work I've done on women's rights. As a special part of social justice starting the Association for Women in Mathematics (AWM) and getting it up and running and seeing what it's progressed to subsequently…Some of the legislation, particularly the pension legislation, which made a measurable effect upon people's lives. So, gender and the sports, also the work in gender equity in sports. That I see has had a definite impact that lasts. Some of it we've lost, some of it we have to do again, but basically, it's there. Some of the other human rights work, I've done a lot of death penalty work, which seems to be futile. Some of the human rights work and some parts of the world also seem to be rather futile. So, it's probably the gender stuff that's more easily measurable and has had a more lasting effect."

Mary provided a roadmap about legal options for those who believe they have been discriminated against [12], [10], [14]. Asked about the overall progress that's been made for women in the profession, Mary said: "There's been progress. It just hasn't been enough…You would not know we have a vice president," Mary said, noting that Vice President Kamala Harris has a visibility problem that women professionals often have. "Every time I read about people's candidates for succeeding Biden, if he's going to be succeeded, we don't see anything about his woman vice president. Why isn't it being considered? It's exceedingly annoying." Asked about what makes the US different from the UK, Germany, and the Scandinavian countries, all of which have had women in the highest political leadership position, Mary remarked: "I don't know. I'm on some sort of email list for a company called Classic Firearms. The classic firearms that they advertise turn out to be North Atlantic Treaty Organization (NATO) rifles that you can buy online for a couple of thousand dollars, or kits where you can convert your pistol to being an automatic weapon, et cetera. People at the university can't seem to help me get rid of this. It keeps coming back all the time. The point is that it has an audience. There is an audience for guns like this. That's the big difference I see between us and some of the other countries that are like us in other ways."

Future work. What would Mary like to work on next? She said:

> I'm still very interested in tax policy. It's one of the things that I wanted to go into a bit more. There was an article in yesterday's newspaper about carbon tax. The author was making a point that you need to solve environmental issues in other ways than tax policy. I don't think that's true. With tax policy, I don't think you can solve all of the issues, but I think you can solve a lot of the issues that people aren't thinking about. In some sense, tax policy is a backdoor in, but it's an important backdoor because it's where the money comes from, and it gets people's attention.
>
> One of my favorite issues with taxes is: Why is social security not collected on your entire salary; why does it stop after you get to a certain cutpoint? This benefits people who are making a lot of money, and it doesn't seem to me to be fair. It seems to me, if everybody who made a million dollars had to pay whatever it is, 6.9%, there'd be a lot more money in the social security coffers for people who are not so well off. That's not suggested by Bernie Sanders. It's not suggested by Elizabeth Warren. It's not suggested by anybody. The question is, why not?
>
> Everybody says that people won't buy it. They won't like it. Well, of course they don't like it anytime people say to increase the taxes. So, I don't think that's a reason. One of the things I want to look into is whether there's a limitation like that in any other country. The systems operate differently, but if you look at some of the Western European countries which have similar systems, I don't think there's any sort of similar cutoff issue. I need to get another research assistant, or I need to go get one of my friends in the economics department here to see what, if anything, he or she knows about it.

When asked how she deals with frustration, Mary said: "I feel that I need some distractions that are far removed from the problems of the world. I like to go to an opera. I like to read a book. I like to travel when it's possible." Sometimes Mary publishes reviews of plays (*Arcadia*; *Calculus*; *A Disappearing Number*), films (*The Tourist*; *Agora*; *The Imitation Game*; *La Figure de la Terre*; *The Man Who Knew Infinity*; *Gifted*) and opera (*Dr Dee*; *Emilie*), in addition to books. Many of her reviews were published in *The Mathematical Intelligencer.*

Mary continues to be involved in improving American University. The unionized staff went on strike at the beginning of the fall semester, protesting stagnant wages [15]. Mary was in her office advising the strikers when it was time for us to meet for the final time on Zoom. She came to our meeting late, and she left early, in order to advise the strikers. "We will not give up," one of the strikers was quoted as saying, apropos to describing Mary Gray's career approach.

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Amanda L. Golbeck

Madhu Mazumdar

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Combining Science and History: Lis Brack-Bernsen's Contributions to the History of Babylonian Astronomy

John Steele

Introduction

Lis Brack-Bernsen is a historian of ancient astronomy and mathematics who for many years conducted research and taught in the history of science program at Regensburg University in Germany. Now retired from the university, she remains very active in research into the history of Babylonian astronomy and astrology. She is the author of two books, author or coauthor of more than 35 research papers published in journals and edited collections, and editor of one coedited collection. For many years she served on the editorial board of the history of science journal *Centaurus* and was founder of the "Regensburg" series of workshops on the history of Babylonian astronomy. Her work, which is characterized by detailed mathematical and astronomical analysis alongside careful study of the original source material, has transformed the way that we understand the relationship between ancient Babylonian mathematical astronomy and its empirical basis and along the way revealed a whole new side of Babylonian astronomy that no one knew existed. In 2017, Brack-Bernsen's colleagues presented her with a Festschrift in her honor [21].

John Steele is professor of the history of the exact sciences in antiquity at Brown University. His email address is john_steele@brown.edu.

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Education and Training in the History of Science

Born and brought up in Denmark, Brack-Bernsen studied mathematics, physics, and astronomy at the University of Copenhagen. As a student, she also took courses in the history of ancient mathematics and astronomy taught by Olaf Schmidt, who profoundly influenced Brack-Bernsen's approach to the study of ancient science. Schmidt had himself graduated from the University of Copenhagen in 1938 with a degree in mathematics, and, just as Brack-Bernsen had been introduced to the history of ancient astronomy and mathematics by him, he had been introduced to the subject by Otto Neugebauer, unquestionably the most influential scholar of the history of ancient exact science during the twentieth century. In 1939, Neugebauer left Europe and took up a professorship in the mathematics department of Brown University. Schmidt followed Neugebauer to the USA and was appointed an instructor in mathematics and a research assistant to Neugebauer at Brown. Because of the German occupation of Denmark, Schmidt was forced to stay at Brown until the end of the war. Although personally difficult for Schmidt, his time at Brown was scientifically very fruitful. In 1943, he was awarded a PhD for his thesis "On the Relation between Ancient Mathematics and Spherical Astronomy," in which he demonstrated the importance of the observation of phenomena on the horizon and their role in the problem of determining the arc of the celestial equator which rises or sets in the same time as a given arc of the ecliptic [6].

In 1953, Schmidt was appointed a member of the Mathematical Institute of the University of Copenhagen, and it was here, a decade later, that his and Brack-Bernsen's paths crossed. As Brack-Bernsen recalled in her obituary notice for Schmidt:

> Olaf Schmidt was a careful, friendly, and very pedagogical teacher, and a brilliant and sharp thinker. He was also modest and unafraid of explaining things thoroughly. He admired and respected the ancient scientists, analysing their problems and results by means of modern science and, at the same time, trying to understand them on their own terms. [6, p. 132]

The last point is crucial: Schmidt saw modern science as a tool to help analyze ancient scientific material, but he was always careful to ensure that ancient science was not interpreted as the same as modern science, nor "translated" into its modern equivalent. Brack-Bernsen explains:

> Since our modern mathematical notation implies much knowledge which was unknown to the ancient mathematicians, he [Schmidt] never used modern notation in his lectures but invented a notation that came as close as possible to ancient mathematical thought. In his own words: "I want my representation to be such that an imaginative ancient Egyptian or Greek sitting in the back row of this room and listening to my lectures would nod his head as if saying: 'yes, that is right, that is fairly what I meant when I wrote it.'" [6, p. 132]

Schmidt's research and teaching covered the history of astronomy and mathematics in Greece, Egypt, and Babylonia. His classes led directly to some of Brack-Bernsen's own work, though, as she told me, Schmidt always wanted his pupils to publish alone, without his name. Only occasionally would he reluctantly consent to be named as a coauthor. As Brack-Bernsen recalls:

> I had to force him to publish the paper "On the Foundations of the Babylonian Column Phi" together with me. Section 7 was his part of the paper. It was what he had taught in his lectures on Babylonian astronomy—and based on this knowledge I had a deeper understanding of what was going on. But still he did not want to be second author. [1]

Schmidt's influence on Brack-Bernsen's work can be seen in other, less direct, ways too. Like Schmidt, Brack-Bernsen has emphasised the importance of horizon observations in ancient astronomy and has successfully and carefully used modern mathematical methods and synthetic astronomical data to investigate ancient mathematical astronomy, but has always taken care to ground these investigations in the original source texts and to explain these texts as much as possible from a Babylonian, not a modern, perspective.

Schmidt was influential in the development of Brack-Bernsen's career in other ways too. In 1969, while still a student at the University of Copenhagen, at Schmidt's invitation she published her first paper in volume 14 of the journal *Centaurus*, a special issue in honor of Otto Neugebauer's retirement. The paper "On the Construction Column B in System A of the Astronomical Cuneiform Texts" was written in response to a question posed to Brack-Bernsen by Schmidt about how the monthly change in the position of the moon at conjunction and opposition is influenced by solar and lunar velocity. Brack-Bernsen calculated the moon's position for more than 223 consecutive conjunctions and oppositions by linear interpolation from data found in the *Nautical Almanac*. She plotted the monthly change in lunar position and showed that the dominant effect was the solar anomaly: the graphs show an overall sinusoidal form with a period of the solar year. Working by hand, she then plotted yearly curves of the same data, with each year drawn with a different color pencil, and found that the variation of the curves up and down was due to lunar anomaly and had a period of 223 months, a period known as the Saros. As I have seen firsthand, this approach of drawing curves in different colors to identify the less-dominant contribution to a function's variation has been instrumental in several of Brack-Bernsen's later studies of the interplay between lunar and solar anomaly.

Following his retirement, Brack-Bernsen would visit Schmidt at home to discuss her ongoing research. Brack-Bernsen has also inherited her teacher's generous nature as a scholar, encouraging and supporting the work of others, including myself, and providing kind but crucial comments at critical stages in their research. As she wrote in Schmidt's obituary:

> I myself feel deeply indebted to my teacher whom I have the privilege to know for almost 30 years. Through his lectures and the interesting problems he posed to me, he introduced me to the exciting domain of the history of astronomy and transmitted his deep insight into the questions and working methods of ancient mathematics and astronomy. I gratefully remember countless occasions at his home, where I could also discuss my newest results with him. [6, p. 133]

Babylonian Astronomy

Roughly five thousand cuneiform tablets recovered from the site of Babylon and other ancient cities in what is now Iraq provide evidence for the practice of astronomy and astrology [20]. These texts mostly date to the middle and end of the first millennium BC but it is apparent that some form of astronomical activity already existed in the second and perhaps even the third millennium BC. Babylonian astronomy in the latter half of the first millennium BC encompassed a wide range of activities including the regular and systematic observation of selected astronomical phenomena, the development and use of methods to compute future astronomical phenomena, and the astrological interpretation of astronomical data.

The astronomical phenomena of interest to the Babylonian scholars in the first millennium BC were predominantly the regular or cyclical phenomena exhibited by the five planets visible to the naked eye (Mercury, Venus, Mars, Jupiter, and Saturn) and the moon. For the planets, these phenomena are the dates of the passages of a planet past a group of reference stars in the zodiacal band (called Normal Stars in modern scholarship) or past another planet together with the distances of the planet above and below the star or planet on that date, and the dates and positions of a planet when it exhibits one of its synodic phenomena (first and last appearance, station, or acronychal rising). For the moon, the regular phenomena are the dates of the passages of the moon past those same Normal Stars or planets with the distances of the moon from the other body, lunar and solar eclipses, and a group of six time intervals between the sun and moon crossing the horizon on specific occasions each month. These six intervals, collectively known as the "lunar six" in modern scholarship, have been a particular focus of Brack-Bernsen's scholarship and so it is worth explaining them in detail here.

In the Babylonian calendar, the month was tied to the cycle of the moon. The month began on the (observed or computed) evening of the first appearance of the new moon crescent shortly after sunset, an event that occurs a day or two after conjunction of the sun and moon. Due to the variability in the length of the time interval between one conjunction and the next (this interval is called the synodic month) caused by the varying speeds of the sun and moon in their apparent orbits as we see them around the Earth, the Babylonian month can have either twenty-nine or thirty days. Full moon can occur on the thirteenth, fourteenth, fifteenth, or sixteenth day. The lunar six time intervals were measured on the evening of the first day of the month, on the evening and mornings around full moon, and on the morning of the last visibility of the moon before conjunction. The intervals are listed below (confusingly, the Babylonians named two of these intervals "NA," and so I will use NA_{new} and NA_{full} to distinguish them):

NA_{new}: The time between sunset and moonset when the moon was visible for the first time after conjunction (i.e., on the first day of the month).

ŠÚ: The time between moonset and sunrise when the moon set for the last time before sunrise.

NA_{full}: The time between sunrise and moonset when the moon set for the first time after sunrise.

ME: The time between moonrise and sunset when the moon rose for the last time before sunset.

GE_6: The time between sunset and moonrise when the moon rose for the first time after sunset.

KUR: The time between moonrise and sunrise when the moon was visible for the last time before conjunction.

The size of these intervals, the days of the month on which they occur, and even the order in which ŠÚ, NA_{full}, ME, and GE_6 take place are very complicated to determine in advance because they depend upon several factors including the variable speed of the sun and moon (solar and lunar anomaly), the moon's latitude at the conjunction or opposition, the angle between the ecliptic and the horizon, and the time of conjunction or opposition relative to sunrise and sunset. The lunar six time intervals themselves were measured to a precision of 10 NINDA or 1/6th of an UŠ, where one UŠ is equivalent to four of our minutes and so 360 UŠ equals a whole day of 24 hours. The time intervals were written using the sexagesimal number system where each "digit" is sixty times greater than the "digit" to its right. It is conventional to transcribe sexagesimal numbers with commas separating each "digit" and a semicolon separating integers from fractions.

Regular, night-by-night, reports of astronomical observations were recorded in a group of texts called "regular watching" by their authors or "Astronomical Diaries" by modern scholars. These texts typically cover half a year and are divided into sections for each month. Most of each section is taken up with daily accounts of astronomical phenomena. At the end of each section we find summary statements of the positions of the planets during the month, the value of staple commodities in the market at Babylon, the river level as the Euphrates flows through the city, and selected historical events. Although the main aim of the Diaries was to record observed astronomical phenomena, bad weather often prevented observations being made. On these occasions, a prediction of the phenomenon was often recorded instead. Predicted phenomena could include the lunar six intervals, eclipses, and planetary synodic phenomena.

The Diaries provided the source material for various texts containing compilations of astronomical data. Of particular interest are the so-called "Goal-Year Texts" which gather together data which are to be used in making predictions of the astronomical phenomena which will occur in a given year. These texts exploit the fact that there are certain periods, different for each planet, after which phenomena will recur on more or less the same day in the Babylonian calendar. For example, phenomena of Venus recur nearly precisely after eight Babylonian years. Thus, predictions of the phenomena of Venus for a coming year can be produced by simply going back to the observations made eight years previously (in practice, a few small corrections were applied to make the predictions slightly better) [19]. The Goal-Year Texts also contain data for eclipses and the lunar six. However, the lunar six, in particular, do not repeat exactly after one period and scholars were for a long time baffled as to how they Babylonians predicted them.

By at latest the end of fourth century BC, the Babylonians had also developed a second type of predictive astronomy which calculated lunar and planetary phenomena using purely mathematical methods which did not rely upon the direct use of previous astronomical observations [17, 18]. Instead, in this mathematical astronomy the dates and celestial longitudes of the moon at new and full moon or of a planet when it exhibits one of its synodic phenomena were calculated using arithmetic procedures. These procedures were used to produce tables of computed lunar or planetary phenomena. The entries in each row of the table are computed either directly from those in the row above or else by adding values found in earlier columns of the current row. The planetary systems are relatively simple and apply one of two mathematical functions known as step functions and zigzag functions to the date and position of the planet when it exhibits a particular phenomenon to calculate the date and position when it next exhibits the same phenomenon. Step functions assign a particular value that is to be added to the previous date or position according to the position of the planet in the zodiac. Zigzag functions increase and decrease by a constant amount between fixed maximum and minimum values and determine the value which is to be added to the value in the previous row of a table. As I will discuss below, the lunar systems are both more ambitious in what they calculate and more complicated in how they do so than the planetary systems.

Astronomy had many roles in Babylonian society. Two of its main uses were in regulating the calendar and in providing data to be used in astrology. As mentioned earlier, the Babylonian calendar used lunar months which can least for either 29 or 30 days. The sequence of 29- and 30-day months, however, is extremely irregular: although, on average, more or less the same number of 29- and 30-day months will occur within a given year, these months are unevenly distributed. For example, while sometimes 29- and 30-day months may alternate for several months, on other occasions, three or even four months of one kind may occur in a row. This irregularity in the length of the month is caused by the large number of astronomical factors which influence the visibility of the new moon crescent including the variable speeds of the sun and moon (solar and lunar anomaly), the variation in the moon's latitude, and the variation of the inclination of the ecliptic to the horizon over the course of the year, not to mention the observing conditions (in particular due to weather). In addition, twelve 29- and 30-day months fall somewhat more than ten days short of the length of the solar year. In order to keep the months in line with the seasons, it is necessary to add an extra "intercalary" or "leap" month roughly once every three years. Determining in advance when a month begins and how many months there are in the year are therefore complicated astronomical questions. Babylonian astrology during the late first millennium BC also became increasingly mathematical, relying upon calculated data and mathematical schemes drawn from astronomy.

Lis Brack-Bernsen's work has spanned all aspects of Babylonian astronomy, including the mathematical schemes found in one of the earliest astronomical texts known as MUL.APIN and the mathematics and astronomy underlying some late first millennium BC astrological schemes [9, 14]. In the following, I will focus on two key areas in which her work has had a particularly large impact on our understanding of the history of Babylonian astronomy.

The Origin of Column Φ of Lunar System A

Lunar System A aims at calculating the time of conjunction and opposition of the sun and moon, the moon's longitude at that moment, the visibility of the moon around conjunction and opposition, in particular the visibility of the new moon crescent and therefore the length of the lunar month, and whether or not an eclipse of the sun or moon will happen at that conjunction or opposition and, if so, the magnitude of the eclipse. The moon's longitude at conjunction and opposition is computed by a simple two-zone step function. Calculation of the time of conjunction or opposition, the visibility of the moon, and whether or not there will be an eclipse is much more complicated, however. For example, the time of conjunction or opposition depends upon both solar and lunar anomaly (i.e., the variable speeds of the sun and the moon), and therefore cannot be modelled by a simple mathematical function. Instead, the Babylonians developed a method of separating the effects of the two anomalies, modelling them individually, and then combining their contributions to calculate the time of conjunction or opposition. Similarly, the determination of whether or not there will be an eclipse involved modelling the moon's motion in latitude by combining functions for the moon's longitude with one for the motion of the moon's node, and then using a further calculation to derive the magnitude for the eclipse. If the magnitude falls within certain limits then an eclipse is predicted. Otherwise, there will not be an eclipse.

The internal mathematical structure and the astronomical meaning of the various functions within the lunar systems (usually referred to as "columns" because they appear as such in the tables) was placed on a firm footing by Otto Neugebauer's detailed 1955 study *Astronomical Cuneiform Texts* [17]. However, Neugebauer did not address the question of the empirical basis of the functions or how they were constructed. It is this question that has been the focus of much of Brack-Bernsen's work.

At the heart of lunar System A lies column Φ. Column Φ is a linear zigzag function with minimum 1,56,47,57,46,40, maximum 2,17,4,48,53,20, and line difference 2,45,55,33,20. Column Φ has a number period of 6248, meaning that it takes 6248 lines before we reach the same number in the zigzag function. Over that period, values have gone up and down the zigzag 448 times. 6248 lines corresponds to 6248 lunar months or just over 505 years. In the lunar tables, column Φ is the first column calculated, appearing right after the date column. Within the tables, Φ serves purely to generate two other columns: column F, which represents the daily displacement of the moon on the day of syzygy (i.e., the lunar velocity), and column G, which is used in the determination of the length of the synodic month. The length of the synodic month is determined by adding together columns G and J to give the excess length of the month over 29 days. These two columns represent contributions due to lunar and solar anomaly respectively. Thus, column Φ is used to generate two functions, column F and column G, both of which are directly related to lunar anomaly.

Within a System A lunar table, column Φ is used purely as a generating function. However, some prose texts which set out calculations using column Φ and other functions, as well as some other, nonstandard, tabular material, show that column Φ itself represents the lunar contribution to the length of 223 synodic months, a period known today as the Saros after which there is a close return of both lunar anomaly and the latitudinal motion of the moon (as a consequence, the Saros is an eclipse period). Like the length of the month, the length of the Saros depends upon lunar and solar anomaly. Column Φ is the lunar contribution to the length of the Saros in excess of 6585 whole days; this contribution would need to be complimented by a solar contribution, although no such function has yet been found in the preserved texts. Column Φ, the lunar contribution to the length of the Saros, and column G, the lunar contribution to the length of the synodic month, are both normed for maximum solar anomaly. In other words, the solar correction is zero when the solar anomaly is greatest, and negative for smaller values of the solar anomaly.

Since column Φ is connected to the length of the Saros, it might be expected that it could simply be derived for measurements of the time between two eclipses separated by 223 months. Here is the rub, however: Column Φ represents only the contribution of lunar anomaly to the length of the Saros but by modelling the variation in the length of the Saros using modern data Brack-Bernsen showed that the variation in the length of the Saros is dominated by the effect of solar anomaly – lunar anomaly plays a much smaller role [3]. Thus, simple observations of the length of the Saros cannot have been used to derive column Φ. How was it derived, then?

Brack-Bernsen's answer to this question brings us back to the lunar six time intervals discussed in the previous section. Recall that the lunar six intervals are the time between the moon and the sun crossing the horizon. Individually, the lunar six intervals are highly complex functions, depending upon several factors including the lunar velocity, the inclination of the ecliptic to the horizon, the moon's latitude and the time of conjunction or opposition relative to sunrise and sunset. However, as Brack-Bernsen has shown, several of these factors cancel out when some of the lunar six are added together [14]. In particular, if we add the four intervals around full moon (ŠÚ, NA_{full}, ME, and GE_6), which together, following Brack-Bernsen, we call the "lunar four," all of the factors except for lunar velocity are eliminated (at least to a first approximation). Thus, the sum of the lunar four varies only due to lunar anomaly.

Through a systematic analysis of a large body of synthetic "observations" of the lunar four, Brack-Bernsen also discovered that the sum of the lunar four not only varies according to lunar anomaly, and thus is in phase with column Φ at full moon, but also that the amplitude of variation of the sum of the lunar four is very close to that of column Φ [4, 5, 7]. The values of column Φ are offset from the sum of the lunar four by about 100 UŠ; Brack-Bernsen has recently suggested a plausible explanation for how the Babylonians made this 100 UŠ adjustment to obtain column Φ drawing upon Babylonian methods of determining the time of eclipses [10].

Brack-Bernsen's groundbreaking discovery of the astronomical information embedded within the lunar six time intervals and how the individual intervals can be combined in order to provide information on lunar anomaly has opened up the doorway to understanding the empirical origin of column Φ. Whether or not it is correct in all of its details, which remains to be seen, it certainly takes us a good part of the way there. Indeed, important aspects of her proposal have been used by other scholars proposing their own (and, in my opinion, less likely) reconstructions of the development of column Φ [15, 16].

The Prediction of the Lunar Six Intervals and the Length of the Month

Brack-Bernsen's careful investigation into the astronomical meaning of the lunar four time intervals and their partial sums also led to the remarkable and totally unexpected discovery of a simple, elegant, yet very accurate method developed by the Babylonians to predict these intervals. This method draws on the fact that, although the individual lunar four are dependent upon many factors, most of these factors are eliminated or at least minimized if we take the sums ŠÚ+NA_{full} and ME+GE_6. Indeed, ŠÚ+NA_{full} corresponds to the setting time of the arc of the ecliptic between the moon's position when it sets on the two consecutive mornings around full moon, and ME+GE_6 is the rising time of the arc of the ecliptic between the moon's position when it rises on the two consecutive evenings around full moon. The setting time and the rising time correspond to the projection of a portion of the ecliptic onto the celestial equator. The two intervals therefore respectively correspond to the stretch of the celestial equator which rises or sets in the equivalent to one day's worth of the moon's motion.

Brack-Bernsen further demonstrated that many of the effects upon the size of the lunar four intervals repeat after 1 Saros of 223 synodic months. However, 1 Saros is equal to approximately 6585 + 1/3 of a day. Because of this 1/3rd of a day, the value of the lunar four time interval after 1 Saros will vary by the rising or setting time corresponding to 1/3rd of a day's worth of lunar motion. And since the equivalent value for one whole day is equal to ŠÚ+NA_{full} and ME+GE_6, then the values for 1/3rd of a day will be 1/3rd of ŠÚ+NA_{full} and 1/3rd of ME+GE_6, depending upon whether we are dealing with the lunar four which take place in the morning or the evening. This can be expressed by the following four equations, where *n* corresponds to the month we wish to compute for [8].

$$\begin{aligned} \text{ŠÚ}(n) &= \text{ŠÚ}(n-223) + 1/3(\text{ŠÚ}(n-223) \\ &\quad + \text{NA}_{\text{full}}(n-223)) \\ \text{NA}_{\text{full}}(n) &= \text{NA}_{\text{full}}(n-223) - 1/3(\text{ŠÚ}(n-223) \\ &\quad + \text{NA}_{\text{full}}(n-223)) \\ \text{ME}(n) &= \text{ME}(n-223) + 1/3(\text{ME}(n-223) \\ &\quad + \text{GE}_6(n-223)) \\ \text{GE}_6(n) &= \text{GE}_6(n-223) - 1/3(\text{ME}(n-223) \\ &\quad + \text{GE}_6(n-223)) \end{aligned}$$

Brack-Bernsen further showed that because the situation at the eastern horizon (i.e., at sunrise) is identical to that at the western horizon (i.e., at sunset) half a year later and vice versa, these rules can be expanded to the computation of the lunar six intervals at new moon:

$$\begin{aligned} \text{NA}_{\text{new}}(n) &= \text{NA}_{\text{new}}(n-223) - 1/3(\text{ŠÚ}(n-229) \\ &\quad + \text{NA}_{\text{full}}(n-229)) \\ \text{KUR}(n) &= \text{KUR}(n-223) + 1/3(\text{ME}(n-229) \\ &\quad + \text{GE}_6(n-229)) \end{aligned}$$

Having discovered these methods of predicting the lunar six, Brack-Bernsen then found direct textual evidence that the Babylonians knew them. The text TU 11 was identified as containing astronomical rules and a drawing of the tablet published already in 1922. However, despite attempts by several scholars, no one had succeeded in interpreting the text until Brack-Bernsen realized that part of it contained rules for computing the lunar six which were equivalent to the equations given above. Working with the Assyriologist and expert in translating Babylonian astronomical texts Hermann Hunger, Brack-Bernsen undertook a detailed investigation of this and related tablets [11, 12], recovering these rules for the lunar six and rules for determining whether or not a month had 29 or 30 days. I quote below one part of one of these texts describing the procedure for computing the value of NA_{new} and whether the preceding month had 29 or 30 days:

> In order for you to calculate (whether the month is) full or hollow, you hold in your hand your year, you return 18 years behind you, and in the 18th year[...] you return 6 months behind you, and you take ŠÚ and NA of Month VII and add (them) together, and [you take one third of it, and] you subtract it from the NA of the 1st day of Month I of the 18th year, and whatever[...] when it[...] if it is less than 10 UŠ, predict (the month) as full. [12]

The tablet on which this text is written is damaged (missing passages of text are indicated by [...]). But we can see that the text tells us to take the value of NA_{new} from 18 years (i.e., 1 Saros or 223 months) earlier, then take the values of ŠÚ and NA_{full} from a further 6 months earlier, add these latter two together and take one third of the result, and then subtract it from the value of NA_{new} from 18 years earlier. This procedure is exactly the same as the rule given in the form of an equation above. The following text is damaged but from related texts we know that it was stating that if the value of the NA_{new} that has been computed is smaller than 10 UŠ, the new moon crescent is too small to be seen and so it is necessary to add the whole of ŠÚ and NA_{full} to it to get the value of NA_{new} for the next day when the moon will be seen for certain.

Babylonian texts provide evidence that this procedure for computing the value of NA_{new} was regularly used in practice. Let us consider the following example. The text ADART VII 30 is a Normal Star Almanac containing predicted astronomical data for the year SE 96. The section for Month IX gives 19;50 UŠ for the predicted value of NA_{new} that month. In order to make the prediction, we need the value of the NA_{new} from 223 months earlier and the sum of ŠÚ+NA_{full} from 229 months earlier. These values are preserved in the Goal-Year Text ADART VI 10:

NA_{new} for SE 78 Month IX = 13;0 UŠ

ŠÚ for SE 78 Month III = 4;50 UŠ

NA_{full} for SE 78 Month III = 5;30 UŠ

Following the procedure as described in the text quoted above, we start by adding ŠÚ and NA_{full} for SE 78 Month III:

$$\text{ŠÚ} + NA_{full} = 4;50 \text{ UŠ} + 5;30 \text{ UŠ} = 10;20 \text{ UŠ}$$

Next, we take one-third of ŠÚ+NA_{full} and round to the nearest 0;10:

$$1/3(\text{ŠÚ}+NA_{full}) = 1/3 \times 10;20 \text{ UŠ} = 3;30 \text{ UŠ}$$

We subtract this 1/3(ŠÚ+NA_{full}) from the NA_{new} for SE 78 Month IX to give the value of NA_{new} for SE 96 Month IX:

$$NA_{new} = 13;0 \text{ UŠ} - 3;30 \text{ UŠ} = 9;30 \text{ UŠ}$$

However, 9;30 UŠ is below the visibility limit of 10;0 UŠ for NA_{new} and so we must add the whole of ŠÚ+NA_{full} to this result:

$$NA_{new,corrected} = 9;30 \text{ UŠ} + 10;20 \text{ UŠ} = 19;50 \text{ UŠ}$$

This result, 19;50 UŠ, is exactly the value given in the Normal Star Almanac.

The discovery of these methods for predicting the lunar six values has transformed our understanding of Babylonian astronomy.

Final Thoughts

Lis Brack-Bernsen's work on the history of ancient Babylonian astronomy serves as a model for how the detailed investigation of ancient astronomical theories and methods, with judicious and careful use of modern mathematics and astronomy as tools, can yield important insights into ancient scientific practice. Behind much of her work has been an interest in exploring the empirical basis for the Babylonian methods of astronomical computation. Through the surviving Astronomical Diaries and related texts we know what phenomena were regularly observed by the Babylonians and with what precision positions and time intervals were recorded. The preserved records are fragmentary, however, and as such not suited to large-scale analysis. In much of her work, therefore, Brack-Bernsen has turned instead to simulated observations, which can be analyzed both systematically and on large scales. In particular, she has made extensive use of the computer code "AA" developed by S. Moshier to simulate Babylonian lunar six data. Through her analysis of this data, Brack-Bernsen was able to find and confirm the correctness of empirical rules for calculating lunar phenomena. Sometimes these rules are hinted at in texts, but not fully explained. For example, the Goal-Year Texts indicated that the partial sums of the lunar four for eighteen and a half years before the target "goal" year were calculated, but offered no indication of how they were to be used. Through her analysis of a large body of synthetic data of these partial sums, Brack-Bernsen was able to identify the astronomical significance of the sums and ways that they could be used in making predictions of future phenomena, and was subsequently able to find these rules written in texts that had hitherto resisted interpretation. This approach also underlies her work in reconstructing column Φ of lunar System A.

Combining this scientific type of analysis with a commitment to paying close attention to the ancient evidence itself and to trying to understand ancient astronomy through the eyes and the methods of the ancient astronomers themselves, as Brack-Bernsen has done throughout her career, brings us as close as possible to understanding ancient astronomy. Her work demonstrates that in the history of science it is essential to consider both the history and the science. Without one or the other, we can only gain a partial, and often misleading picture of ancient scientific practice. But by carefully combining both historical and scientific approaches, we can gain a deeper insight into ancient science itself.

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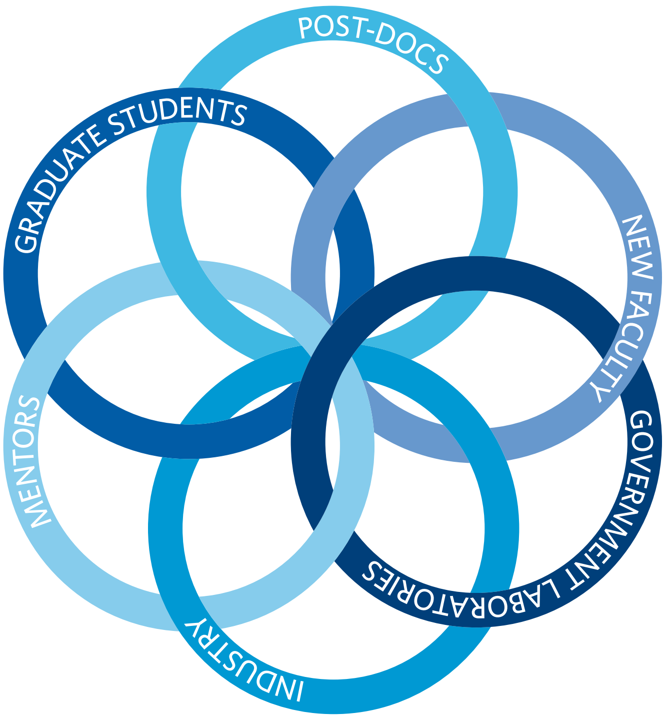
John Steele

Credits

Opening photo is courtesy of Lukas Pokorny.
Photo of John Steele is courtesy of Rebecca Barclay.

The Early Career Section offers information and suggestions for graduate students, job seekers, early career academics of all types, and those who mentor them. This month we feature an article by the students of Karen Uhlenbeck and two excerpts from the book *Aspiring and Inspiring: Tenure and Leadership in Academic Mathematics*. Next month's focus will be on careers at primarily undergraduate institutions.



In Celebration of Women's History Month

Advice from Our Advisor: Karen Uhlenbeck

Steven Bradlow and Magdalena Czubak

Introduction

When asked by the *Notices* to coordinate contributions from Karen Uhlenbeck's students for this "Advice from our Advisor" column we quickly agreed. The timing was perfect: perfect for the upcoming 2023 Women in History Month but also for a tribute to Karen on the occasion of her 80th birthday, which had just been celebrated in August 2022.

Karen has had nineteen PhD students. Below we present contributions from nine of them. Some clear common themes emerge which characterize Karen's role as an advisor: her infinite generosity with her time, her importance as a mathematical and personal anchor for her students, and of course the many ways she served as an inspiration for all.

Alexandre Casassola Gonçalves

"Go talk to other people. Meet other mathematicians, and their students; ask them about their problems, and show them your problem." This is one of Karen's most memorable pieces of advice. It was perfect for me at that time, for I used to be a person who resists asking for help, believing in the fallacy that "I should do it on my own." While most of the time, mathematicians try to work on their own, there are gaps that can only be filled by other people's contributions. Mathematics, after all, is a collaborative work, and lack of humbleness is not a virtue here.

Another piece of advice from Karen was to write down results, as soon as you get them. Don't just solve them in your mind, because you only know you understand something when you are able to tell other people about it, based on what you wrote. Karen used to ask for drafts of

Steven Bradlow is a professor of mathematics at the University of Illinois at Urbana-Champaign. His email address is `bradlow@illinois.edu`.

Magdalena Czubak is an associate professor of mathematics at the University of Colorado Boulder. Her email address is `czubak@colorado.edu`.

Alexandre Casassola Gonçalves is an assistant professor of mathematics at the University of São Paulo. His email address is `acasa@ffclrp.usp.br`.

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research articles we were writing, as well as for excerpts of our on-going dissertation. She would then give us valuable hints about mathematical academic writing, including "what not to write in detail, because people know it."

When she was not "on leave," Karen would meet her students once a week. She would spend at least one hour with each student, but this time could grow up to two hours when trying to figure out a particularly hard problem. These regular meetings with her PhD students seemed to stand on the same "priority level" as her other (important) professional commitments. I don't remember ever having our meeting canceled or even abbreviated because she had to receive some speaker for a conference, or some co-worker who just arrived.

One thing that stood out in our meetings was her patience at explaining a point I had not understood. She would talk, and then repeat, and then write on the board, and repeat again, over and over ... that could last for several meetings in a row, until I was able to achieve a breakthrough. Though, that does not mean she didn't leave a large bite for her students. Her famous advice after a long session—"Work out the details"—comprised the core of understanding we needed in order to arrive at where she was leading us.

This is not written down anywhere, but a large part of the work of an academic advisor surely is providing encouragement, and motivation to her students. How did Karen do that? By being available and open to talk about whatever your diculties were, academic or not. I remember that, as a student, most times when I left her office, I was feeling better than when I arrived simply because she had convinced me—again—that I would able to write my dissertation and earn my PhD degree. But it has to be pointed out that she did not consciously intend to convince me of this. It came out naturally from her attitude, during our conversations on the mathematical concepts and challenges I was facing. Metaphorically, she held my hand and carefully brought me out of the fog of my doubts.

Karen has been an outstanding prominent mathematician for many decades. I knew that when I first met her, at a conference. One year later I met her for the second time, in her office at UT Austin, after I had just arrived to start my PhD program. Obviously I was a bit nervous. For I didn't really known the person who would guide me and play a major role in my mathematical formation. All these (legitimate) concerns were quickly dismissed as soon as we started our scheduled weekly meetings, when I could observe and feel the way she talked to me, and to everybody else. I realized I had nothing to fear, for a very busy person with no time to spare, still can treat others with kindness.

In summary, we learn not only from what people tell us, but also from the example they set in their daily lives. I learned a lot from Karen (as well as from my former master's advisor, recently deceased), and I imagine them whenever I face the challenge of helping my current students: "work hard, be objective, write it down, talk to people, be patient, be kind, and don't ever think you are the best..."

Andrea Young

One of the great lessons that Karen taught me was not via advice but by modeling behavior. When I was a graduate student at the University of Texas at Austin, the geometry seminar was mostly attended by men. Indeed, more often than not, Karen and I would be the only women in the room. She would often sit quietly and listen to presentations by guest speakers, but almost without fail, she would ask a seemingly simple question that got to the very core of what was being discussed. And once she asked her question, she would continue to question the speaker until she was satised with the response. During these exchanges, I observed her male colleagues listening closely as they knew there was something interesting to learn if Karen was asking about it.

Seeing her do this every Thursday afternoon taught me two non-mathematical lessons. First, it taught me the value of asking questions and of not being satisfied with a simple answer to my questions. Secondly, and most importantly, she showed me how to use my voice unapologetically and how to take up space in a male-dominated environment. Karen could be assertive—aggressive, even—in her questions, but it always seemed to me to be from a place of genuine interest and love of mathematics and knowledge. I am as grateful for these lessons as I am for all the math she taught me. They have been transferable skills that have served me even as I have moved out of mathematics and into higher education administration.

Antonella Marini

I recall many moments with Karen, the first dating back to when she was at the University of Chicago, and I was a first-year graduate student, sweet and tough ones. I cherish both.

Tough love. Of my undergraduate paper on gauge theories she said "nice, but not an earth-shaking result." A severe comment, a compliment, and motivational, all bundled in one. Of a proof she said "I won't go over details, but I'll tell you it is wrong." And so I learned to not trust signs and factors of two and to try and develop a healthy mathematical intuition instead.

A novel feminine way of being a mathematician. Karen described mathematical realities making them appear concrete. She once said "the hardest mathematical concepts are not something one ever understands. One simply accepts them." Because of her teachings, I try to pass on to

Andrea Young is the vice president for finance and administration and an associate professor of mathematical sciences at DePauw University. Her email address is andreayoung@depauw.edu.

Antonella Marini is a professor of mathematical sciences and chair of the department of mathematical sciences at Yeshiva College and Stern College. Her email address is marini@yu.edu.

my students the notion of a mathematician as a humble artisan who learns intuition through sweat, and with that, the skill of uncovering the statue from inside the block of stone. The aspiration is to acquire familiarity with the mathematical landscape and move freely. Karen is a true feminist, a woman of incredible mathematical intuition, who invented her unique way of being a mathematician, as a person and as a female.

A sense of humor. Of her article, "Solutions to Yang–Mills equations that are not self-dual," co-authored with the Sibners, often referred to as a "landmark paper," she once said "I still don't understand the topology."

It took time to understand her. Thomas Otway, who was at UT as part of her team in the late eighties, once said to me "When you talk with her about our work, write down what she says, verbatim. It'll make sense LATER." In one of her visits, Leslie Sibner exclaimed, "You assigned the Dirichlet problem for Yang–Mills as a 'WARM-UP' problem!?" That ended up being my dissertation (who knows what the follow-up problem was supposed to be).

Personal advice. Once she said: "you've grown up to sound like a mathematician." And, "marry a mathematician!" (She thought it hard for a female mathematician to be understood by a non-mathematician.) When I claimed I was not ambitious, she called "you are, you climbed Mont Blanc."

A devoted advisor. When Karen moved to UT Austin, her Chicago students followed her. The world around was competing for Karen's attention at lunchtime, but she made sure that we had a "reserved spot" at her table. She was adamant that we should receive her undivided mathematical attention, while at our dedicated meetings, and be left independent afterward. She scheduled weekly meetings with each of us.

A "*PDE person*" and much more. She is an analyst at heart (one of her favorite results is "Regularity for a class of non-linear elliptic systems," *Acta Math.*, 1977), but truly also one of the founders of modern geometric analysis. I am filled with appreciation for having come in contact with multifaceted mathematical and human greatness. Watching her doing mathematics, like an apprentice learns at shop from the master, has been the greatest gift.

Brendan Guilfoyle

I studied for my doctorate under Karen's supervision at UT from 1991 to 1997. I can honestly say that, without her support at key moments, I would not have been able to graduate.

When I arrived, she had something of a fearsome reputation amongst the graduate students—at one time a computer game was going around featuring various characters with superpowers representing different members of the faculty and Karen's avatar had the ability to turn you to stone with its intense gaze. Anyone who has talked mathematics with Karen will know that gaze!

However, in person she was always friendly and helpful and I was delighted when she accepted me as a doctoral candidate. As a foreign student, I was excluded from the loan programs that my fellow US graduate students were tapping into to make ends meet. While I did have a teaching assistantship, this was a period of massive fee increases at UT—it went from hundreds of dollars a semester to thousands of dollars over the space of a few years.

As a consequence, I became involved in graduate student activism, helping to found the Graduate Student Assembly (which still exists at UT) and a graduate student newspaper. At the time we were campaigning for fee waivers for TAs, as it was manifestly unfair that you had to give back a month's wages every semester, mostly for nine "dissertation hours" a week which were in reality a one- to two-hour meeting with your supervisor.

In any event, my high-profile involvement was frowned upon by the powers that be in the mathematics department and I recall being threatened Mafioso-style in an RLM elevator one day by a disgruntled senior Faculty member. Luckily for me, Karen took a different view and liked the fact that I was involved in student politics. Karen's student days had been in Berkeley in the 60s, so she had witnessed tear gas going off in the classrooms etc. Her main concern was that I would give up mathematics and become a full-time activist.

As it was, I finished in a reasonable amount of time, made a career in mathematics, and have kept in touch with Karen down through the years. Without her support throughout this period, I very much doubt that it would have been possible for me to finish the programme.

Brendan Guilfoyle is a professor of mathematics at Munster Technological University. His email address is `brendan.guilfoyle@mtu.ie`.

Caio José Colletti Negreiros

Karen Uhlenbeck was my PhD's advisor at the University of Chicago. After attending a Seminar run by Karen I was moved by her contagious enthusiasm for mathematics. We met once a week in her office (as far as I recall it was Thursday about three o'clock in the afternoon). It was an unforgettable experience to have learned with her how to produce new results in mathematics.

The fundamental very first advice she gave me was her suggestion for my thesis problem. As she predicted, this problem opened a way to many related mathematical questions. In this period at Chicago she gave me a lot of important mathematical ideas and advice related to my thesis.

She taught me how to face a specic mathematical question from several viewpoints in order to possibly overcome some mathematical difficulty.

After five years in Brazil I spent one year with her at the University of Texas at Austin as a post-doc starting

Caio José Colletti Negreiros is a retired professor of mathematics at the University of Campinas. His email address is `caione@unicamp.br`.

September 1992. In this post-doc I learned many possibly new directions to move forward in mathematics. But months after my return to Brazil I became very ill and it took me almost three years to fully recover.

She has taught me that we must see our students in a broader sense. Karen was concerned with me as well with my family and acted effectively in a very human way to support us in some difficult situations that we faced in our lives.

Other fundamental advice from her was to make my professional decisions considering mathematics the first thing to be taken into account. This advice was very decisive in my professional life.

All my mathematical career is based on questions that have some relation to my thesis. After years a few of these related questions were solved mainly togethe with colleagues or PhD students at University of Campinas. By the way, my original thesis problem, as far as I know, remains unsolved.

In brief, it was a very memorable experience to have learned and worked in mathematics during those years in Chicago and Austin with a mathematician like Karen. From my own experience I can assure you that she was very demanding with respect to mathematics and very human concerning other issues.

Georgios Daskalopoulos

I am very happy to have the opportunity to express my deep gratitude and appreciation to Karen Uhlenbeck for her continuous mentorship and support throughout the years.

I first met Karen in 1985 when I joined the University of Chicago as a PhD student. At the time, Karen was giving lectures on the new exciting developments of gauge theory. After taking her class on variational problems (I still have the notes from this class), it became clear to me that I wanted to become her student. In 1986, after she returned from a leave in San Diego, I asked her for a thesis problem. She suggested that I work on making rigorous the approach of Atiyah–Bott on the Morse theory of the Yang–Mills functional on Riemann surfaces. I was very excited to work on this problem under her supervision and when she left Chicago I followed her to UT Texas where I completed my thesis. During this time, I also wrote a small paper with her on infinite-dimensional transversality and a few years later (jointly with R. Wentworth) another paper on stable extensions of holomorphic bundles. Recently, I have been extremely lucky to collaborate with Karen on a long-term project and experience again the unique way she does mathematics.

Throughout the years, Karen has been an incredible source of inspiration for me. From very early on I noticed that her mathematical thinking is based on simple geometric arguments, a great wealth of intuitive small steps fitting together perfectly to create a complex puzzle. One important piece of advice she gave me from the start was always to think about a special case of a problem before moving to the general case. "Never try to solve something nonlinear," she reiterated, "unless you fully understand the linear case." Evidently, most would agree that this is a common principle about any type of research. However, it was only after observing Karen work through different questions that I realized her ingenious capability for taking a complicated, maybe even intractable problem, and by slightly changing a few parameters, making it into one that is both tractable and interesting.

Karen has always been interested in physics and encouraged her students to study physics. The notion of symmetry plays a big role in her mathematical thinking. I recall that during my graduate studies in Chicago, she asked me to take a class by the theoretical physicist Prof. Nambu (who got the Nobel Prize a few years later for his discovery of the mechanism of spontaneous broken symmetry). The class was all about infinite-dimensional Lie algebras. I didn't get too much out of the class, and it was only years later that I understood Karen's prospective about the role of symmetry in mathematics and physics. More specically, it was when she explained to me the work of Hilbert and Noether on conservation laws in geometry and physics and applied these ideas in our joint work to study certain types of variational problems.

Another striking aspect about Karen's style of doing research is that, unlike most mathematicians of my generation who have learned geometric analysis from its tail end, Karen knows the origins and motivation of each problem very well. She rarely needs to go to references as she prefers to derive everything from first principles. This may not always mean giving a complete argument, but certainly enough to convince herself of the validity or invalidity of the claims at stake. I recall she always encouraged me to read less and think more.

Several times in my mathematical career, when stuck on a problem, I have gone back to her for advice. She has always been very eager to think about my problem and has often given me a response which at first has left me mystified. It took me a lot of effort, sometimes several months before I discovered what she meant to say, but at the end I always realized the incredible wealth of her answer. Like the oracle of Delphi with its famous $\chi\rho\eta\sigma\mu o\iota$ open to different interpretations, yet full of enigmatic wisdom, Karen's prophecies have enlightened my mathematical life on numerous occasions.

Georgios Daskalopoulos is a professor of mathematics at Brown University. His email address is georgios_daskalopoulos@brown.edu.

Magdalena Czubak

"People publish too much," is one of the most memorable comments from Karen during graduate school. "One good idea per year is very good," is another one.

Karen was generous with her time, and we always had a weekly meeting.

I wanted to do everything by myself, and Karen let me, even at a point when I do not think I would have been upset if she helped. Toward the end of writing up my thesis, I realized there was an issue with one lemma, and I was stuck on it for what felt like forever at the time: a whole month. I would go into our meeting and tell Karen what I tried but this is why it did not work, but now I had a new idea to try. Karen would say nothing, and when I finally figured it out (in a shower) and was so happy to tell her, and said that I was getting worried, she said, one month to be stuck is not that long. This is obvious now, and I am grateful that Karen let me find a solution on my own as this built the foundation for future math endeavors.

In retrospect, I wish I had asked more questions back then, so I could have benefited more from the vast knowledge and deep understanding Karen has, even if I probably would not have initially understood what Karen would have said. Karen does have a "nonlinear" way of thinking, and certainly in grad school I was very linear, and have been trying to embrace nonlinearity only in recent years. Karen was also famous for half-finished sentences, so this did not help me, but I was elated that by the time I was graduating, I knew what the end of the sentence was!

I also learned from Karen, not necessarily from any single comment, but from our interactions, to always put math first. What is best for math? That was how I chose where to go for my postdoc, and how I make other professional decisions to this day.

It took me several years after graduation to start addressing Karen as Karen instead of Prof. Uhlenbeck. Before it happened, I was thinking that I could always address Karen as Prof. Uhlenbeck; there are few people that I respect as much, both as a mathematician and a human being.

Michael Gagliardo

Karen Uhlenbeck had an incredibly important impact on the trajectory of my life even before she was my PhD advisor. After leaving UT Austin in 2000 with my master's degree mostly because I believed I was not graduate school material, I found myself working as an actuary at an insurance company. I hated it and while looking for teaching opportunities, I asked Karen to be a letter writer for some community college positions. Instead of just saying yes (or even no) and moving on, Karen told me that I should think of returning to graduate school and that she felt I was capable of producing good work. Due to the timing, it took me another year to return to UT Austin, but Karen looked out for me (and many other graduate students) while I was there, eventually becoming my thesis advisor. Every time I have a chance to think about how much I enjoy my position at California Lutheran University and the opportunities it provides me both personally and professionally, I am so thankful Karen thought to say, "you should come back." I get to be where I am today because of her mentorship.

Michael Gagliardo is a professor of mathematics at California Lutheran University. His email address is mgagliar@callutheran.edu.

Steven Bradlow

I became Karen's student in 1985 at the University of Chicago. At that time Karen already had three other PhD students and during my time she took on two more. One graduated soon after I arrived, one was working on harmonic maps, and the rest of us were all working on gauge theory projects. We were a reasonably close-knit group. I think we were all very aware of how extraordinary our advisor was and we realized how lucky we were to be working with her. Undoubtedly we also bonded over our common struggles with some of the special challenges posed by working with Karen, including:

1. Her inexhaustible energy. I (and I think all of my mathematical siblings) had weekly meetings with Karen. At these meetings I would present my progress of the week at the blackboard and we would discuss—sometimes in minute nitty-gritty detail—the latest obstacles and what should come next. I forget what the scheduled length was for these meetings, but invariably they would run well past two hours and I would always run out of energy long before Karen did. Karen understood well that she had more energy than most other human beings. I recall her saying that one thing she had noticed about most MacArthur fellows was that, like her, they were all extremely high-energy people. Also, she once described how as a young postdoc she was unable to settle down to work unless she first did some vigorous physical activity to burn off her excess energy.

2. Her cryptic communication style. I'm sure it comes as no surprise to anyone who knows Karen if I report that unfinished sentences and rapid jumps made it hard to follow her train of thought as she hop-scotched from one peak to the next in the mathematical terrain. We knew that whatever she said was profound and certain to be helpful, but often I left my meetings with Karen wondering what exactly it was that I had just been told. I recall many sessions with my fellow student Georgios Daskalopoulos puzzling together over messages we got from Karen, trying to crack the code. We didn't always succeed but when we did, we were always richly rewarded—and rather proud of our selves for having done it.

Among the many aspects I found remarkable about Karen was her clear-eyed view of her place in the mathematical firmament, and her acceptance of the obligations that followed from that. In the late 1980s (when I was her student) Karen was unquestionably a mathematical superstar. She clearly understood, without any hint of hubris, that as a result of her status wherever she went people would be clamoring for her attention. She seemed to accept the ensuing claims on her time in the best possible spirit of noblesse oblige (where in this context the term should not be burdened with any negative connotations!).

I would like to mention just one more of the many pieces of wisdom that I gleaned from Karen. I forget the precise circumstances—maybe it was in the aftermath of a seminar in which some piece of mathematics was not clearly explained—but Karen made an offhand remark about how, after years of constructing them herself, "I know what a proof is." I remember my sense of puzzlement—surely anyone with a basic grasp of propositional logic would know a proof when they saw it—but the remark struck a chord. Only later did I figure it out: logical correctness is a necessary but not a sufficient component of an argument that needs to be engaging, understandable, and persuasive, i.e. of a proof.

The glass ceiling for women persists in mathematics, perhaps less overtly than in times past. But academic mathematics retains cultural practices, norms, and conventions that act to hinder women's (and other groups') advancement. As long as these barriers remain, those women who wish to persist will need role models, mentors, and useful advice. *Aspiring and Inspiring: Tenure and Leadership in Academic Mathematics*, a collection of essays from successful female mathematicians, provides some of those role models and some of that advice. The chapter authors were asked to reflect on their own experiences and help point younger colleagues toward success. Several themes emerge strongly from the collection: Networks of support are important; feeling a sense of belonging is critical; individual mentorship matters; one should formulate one's own definition of success. Of course, the strongest theme is that academic mathematics has not achieved equality of opportunity. Until we do achieve it, we'll need books like this.

The essays were gathered and edited by Rebecca Garcia, Pamela Harris, Dandrielle Lewis, and Shanise Walker. Below we reprint excerpts from the collection. The full volume will be available from AMS Books in Spring 2023.

Excerpts from "Intersectionality as Impetus and Impediment"

Erica Graham

Community

To successfully navigate the mathematical world, I needed to surround myself by people who collectively provided me with connections, counsel, and confidence. At every stage of my career, beginning with choosing a graduate program, I have had to adjust who and what filled these roles for me.

The advice (or warning) I often given students looking to pursue graduate degrees is, "Grad school is going to suck, so you might as well surround yourself by people who don't suck." The same can be said of the road to tenure. Sure, some joy and passion makes it easier, but in order to deal with the externalities associated with pursuing tenure and establishing oneself in mathematics, it is helpful to have a community of people who make the process significantly less daunting. Here's how I built my community of connectors, counselors, and confidence-builders.

. . .As graduate student. The summer prior to beginning my graduate program, I participated in the Enhancing Diversity in Graduate Education (EDGE) summer program. Remember that calculus 2 professor? It turns out, Rhonda Hughes was also the co-founder, along with Sylvia Bozeman from Spelman College, of this program for women in mathematics,[1] which was designed to provide an extensive and sturdy support network to facilitate the success of women in graduate programs in the mathematical sciences. At the time, EDGE was a way for me to get back into the swing of doing mathematics—I'd been working in the real world for two years—before undertaking the momentous task of getting a PhD. That summer, I experienced mathematics in a completely different way. There was nothing about the program's coursework that was easy, but the network that EDGE gave me is something that has kept me going throughout all stages of my career. I'll come back to this.

In the process of deciding where to go to grad school, I was self-aware enough to know that the environment in which I planned to immerse myself for the next several years was considerably more important to me than anything else. Recruitment visits provided an invaluable window into what life might be like in my prospective program. As a Black woman originally from Queens, no one could have predicted that I'd end up attending graduate school in Utah. But, the feeling I got from visiting the math biology group, a subset of the graduate mathematics program, at the University of Utah told me what I needed to know. At the time, the community dynamics I witnessed among the faculty and students within the program instantly felt familiar. And, thankfully, my gut was right.

As expected, grad school was difficult, to say the least. But, the community I created for myself made it doable. My personality and working style were more or less compatible with my advisor, who provided many examples of 'how to be a research mentor' that I am constantly working to emulate. I haven't gotten there yet. I had a great collaborative group of fellow classmates, with whom I spent hours toiling on homework, studying for prelims, and later, practicing thesis defenses and job talks. We had older graduate

Erica Graham is an associate professor of mathematics at Bryn Mawr College. Her email address is ejgraham@brynmawr.edu.

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[1] https://www.edgeforwomen.org/

students who served as ‘us, a year or four from now.’ The older students willingly showed us how to open Matlab on a Linux system (true story), shared their own experiences, and gave us samples of teaching statements and job interview questions. From beginning to end, the stress, frustration, commiseration, and tears were intermingled with the weekly *Lost* watch nights (when tv was still watched in real time), cabin parties in the mountains (where snowshoeing was the only way in or out), and football games (go Utes![2]). Having heard grad school horror stories from BIPOC and white colleagues alike, I am grateful for the unicorn-esque experience that I had.

During grad school, I found that getting a Ph.D. was about much more than being a good test taker, which is how I got through my preliminary exams. It was about pressing onward when I didn’t think I could. It was about believing that some part of me had what it took to succeed without having a concrete definition for success. It was about knowing that I could not compare myself to my peers because my story was different from the next person’s. It was about resolutely deciding that I would continue putting one foot in front of the other until the graduate powers that be kicked me out (it never happened). It was about being just foolish enough to not have made alternative plans. It was about faking it until I made it.

It’s clear that my blackness played a large role in this clear case of *imposter syndrome* that I had/have. But given the sense of camaraderie I felt from being able to have similar conversations with fellow women graduate students, I suspect that intersectionality was at play. The imposter syndrome impedes my ability to consider that anything that I do is different from what anyone else would do if given the opportunity. For a long time, I wondered whether the only reason I was accepted to grad school was because I was Black. About half of the students in the program identified as women, so that wasn’t it, and I was clearly there for some other reason that had nothing to do with my academic ability. Clearly.

Even once I passed prelims, it didn’t really sink in. Even when I started doing research, it could never be as good as the others’ around me. ‘It’ was never enough. Well, if that wasn’t imposter syndrome, then I don’t know what is. Layla Saad has described this so aptly: “I always felt like I was invisible, I felt like people didn’t really see me, and yet I was always in these leadership positions. I felt like I didn’t really earn it when I got it; especially if it came easy, I would question it” [9]. The reality of what I was experiencing is even more sobering. Saad also describes this as a feeling of internalized unworthiness, one that stems from the culture of white supremacy that governs many of our lives from the moment we are born. “You have been receiving messages that Black people are nothing, and that when we are given anything, it’s either an exception, or it’s luck somehow, or you killed yourself to get there and you have to keep going at that same speed. You have to keep working yourself into the ground to maintain that same level” [9].

The telltale sign of imposter syndrome lies at the intersection of making something look easy and discounting its value. What’s more is the types of people who suffer from severe cases of imposter syndrome are often at the intersection of marginalized identities. I view my ability to navigate structural barriers to my success by being in the right place at the right time, in the vein that it may not be something that I necessarily deserve, because these are the messages that have been subliminally fed to me from the day I entered the world. What are those messages? That I do not deserve the success that I have, and that the hard work that I put into doing well in school was in an effort to maintain an invisibility that I did not know I sought. I constantly discount the amount of work that I do and the degree to which I excel because it feels the most ‘natural.’ And yet, I am all-too-aware, as emerged from a recent conversation I had, that mediocrity is a privilege that I as a Black woman simply do not have. If I write all the things I do as a mathematician and tell myself it’s the bare minimum, is it true? This is where the reality check is needed. My reality is so deeply ingrained in a culture of white supremacy and anti-Black racism, that I needed a community that could replace my distorted perception of reality with something slightly closer to the truth. I needed people who could both validate my experience and fact-check my insecurities. Here’s where being part of the EDGE network made the difference for me.

. . .As EDGEr. After participating in the EDGE summer program, I returned as a graduate student mentor one summer, made new connections and renewed old ones. This was my second priming against the infectious environment of academic mathematics. I was given guidance I hadn’t realized I needed from the faculty teaching the courses, from local program coordinators, and from the program directors alike. In fact, my decision to remain in academia upon completing my degree stemmed from the connections, counsel, and confidence I received that summer.

As a result of my relationship to EDGE and its ever-expanding professional network, I received financial support to attend national conferences and presented at an international conference as a graduate student. As a postdoc, I’d been invited to speak and/or co-organize conference sessions, received job references and colloquium invitations, and gained research collaborators. Some of the professional service that I’ve done was also the result of EDGE connections. The multi-dimensional support from EDGE has always been available to me at all stages of my career, not the least of which still includes being able to text, call, or video chat, with my favorite people for advice,

[2]*The Ute Indian Tribe has had a formal agreement with the University of Utah since 1972, which includes partnership in the areas of name and logo use, education and educational access, advisory representation, and outreach. The most recent Memorandum of Understanding can be found at* https://admin.utah.edu/ute-mou/.

support, some laughs, or even a much needed meme. Through EDGE, I gained the type of community where a mentor drives from Washington state to Salt Lake City for my dissertation defense. Because, unlike the other cues that I receive on a daily basis, I am worth it.

In the EDGE summer program lineage, I've gone from student to graduate mentor to faculty. In doing so, I've learned to appreciate my mathematical journey through someone else's lens. My own insecurities aside, I've been able to reflect on the ways in which my positioning provides opportunity to the next generation of mathematicians. When I can't find a reason to push through for myself, I look for ways to support others because I know what lies ahead. When I connect with EDGErs at conferences, I am often reinvigorated, if not for myself, then for them.

. . .As postdoc. I began my postdoc at North Carolina State University not knowing whether I would remain in academia or go into industry. It was a three-year postdoc, which was the ideal length for me. Here's why. In year one, I spent most of my time adjusting to a new living and working environment. I had to confront the reality of life as a queer Black woman in an interracial relationship in a new place: Moving from Utah to North Carolina was a big shift, and I had to adapt to how the new forms in which racism and bias played themselves out beyond my graduate school bubble. With the massive change, it was very difficult for me to hit the ground running with research. Whether it was pushing past the invisability (invis-ability = invisible disability), trying to publish work from my doctoral thesis, or doing background reading for work with my postdoc mentor, I struggled in year one. I was forced to create my own gauge of productivity. By year two, I managed to focus entirely on my new project (even making some headway!), without the headache and stress of having to apply for a new position. By year three, I didn't have a choice but to apply for a job, even if every other part of me didn't feel as though I was ready. Sometimes it's best simply to rip the bandaid off.

It's a challenge to balance a description of what I did during my postdoc versus how I felt. My invisability made everything a thousand times harder. But prioritizing self-care[3] helped a bit. At the end of the day, no one could know me better than I knew myself, and I was adamant about using my flexible position to take care of myself before I even attempted to do good research. This doesn't work for everyone, but I personally had no other option.

I did take advantage of the low teaching load and travel funding that came with my postdoc, and . . .well, I traveled. A lot. I attended conferences, workshops, and organized sessions. I met people. A lot of people. Then I was invited to give colloquia and organized more sessions. It was exhausting, but it was also a way for me to expand my professional network. I knew that I'd have to apply for jobs down the line and was very intentional about talking to people about their experiences and various career paths. This included folks who were in industry, academia, or government. It was such a busy and also meaningful time in trying to figure out who I wanted to be as a mathematician.

[3]*I fully participated in this revolutionary act, as described by Audre Lorde.*

Excerpts from "I Am Not Your Typical Role Model (or Do Not Follow My Steps)"

Ivelisse M. Rubio

The road to tenure

Norms, expectations and goals. Even if you find the place where you can fulfill your goals, the department might have other expectations for you. When a department recruits for a position, there are some areas they need to strengthen and are looking for people with certain qualifications and interests. The norms and criteria for obtaining tenure and promotion vary from place to place, and they might not be clear. It is important to know them.

When I started at the UPR-H, the rules and expectations for obtaining tenure were made clear. Among other things, the mathematics department was looking to strengthen their research. Hence, in addition to teaching, I was expected to have a research agenda, publish in peer-reviewed journals, and obtain external funding. Some of the colleagues in the department thought that starting an REU during my first year was a mistake, that this was something that should be done by people with more seniority and who are already established researchers. They were probably right but the timing and conditions were perfect to start the summer program and this could not wait five or six years. For this reason, I ignored the advice and decided to take advantage of the good conditions to start the summer program.[4]

During my first year at the UPR-H, I worked on a paper related to my thesis research, submitted the paper to one of the best journals in the area, and it was rejected. I did not know that this was part of the process: that one learns from the reviews, revises the paper, and submits it to some other journal. Between the work dealing with the bureaucratic processes to run SIMU, mentoring undergraduate research students during the academic year, writing and administrating grants, serving on committees, and teaching

Ivelisse M. Rubio is a professor at the University of Puerto Rico, Río Piedras. Her email address is `ivelisse.rubio@upr.edu`.

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[4]*I was one of the two Co-Directors for the Summer Institute of Mathematics for Undergraduates (SIMU), where the two research components were led by two other Research Advisors. The research topics differed each year.*

my classes the responsibilities on my plate were enough, and I just quit research. Part of the expectations for tenure were related to research but, luckily for me, the evaluation system at the UPR-H was based on points. I had plenty of points, given my results in the other areas, and had some publications related to the summer program [8]. In three years, I was promoted to Associate Professor, and tenured two years later.

Isolation and mentoring. *Imposter syndrome* [1] can manifest in different ways. For example, not revising and resubmitting a paper after a rejection because you think that the work is not good enough or not looking for collaborators because you think that you do not have the skills needed to contribute to a group. Looking back, I now realize the importance of having a research network, a group of peers with common interests, with whom you can collaborate, share experiences and ideas. A group of colleagues that value you, and with whom you can consult with no worries of being underestimated [3]. For my outreach projects I had a great network and received the encouragement and support from Barbara Joudry and James Schatz (NSA). Also, I had a wonderful group of friends and colleagues at the UPR-H with whom I could discuss issues related to teaching, gender, and politics, but not about my research area. My research collaborators were my former PhD advisors who were no longer working on research or eager to publish. *In order to keep an active research agenda when starting in academia, it is good to have collaborators who are active researchers and might have more experience in the publication process.*

Time management. Two skills that I have always lacked are good time management and maintaining focus. In academia, you are expected to be successful in different areas: teaching, research, obtaining external funding, and service. In addition, you have a personal life and other interests. I have always enjoyed the academic environment: being at the university, attending talks and activities, spending time with the students and chatting with my colleagues. But I also love going to the beach, playing with my pets, gardening, making handcrafts, and spending time with friends and family. I am also involved in political and social activism. How could I find time to do it all?

In Hispanic culture, women are expected to be caretakers. The cultural and social expectations are that we take care of the children, the house, and the parents. Many think that being a professor gives you the advantage to manage your schedule and, hence, be able to do all this. But first, it is not true that your work is done once you are done teaching. Second, family and home should not be the sole responsibility of women. I get very upset when I see panels about "How to manage career and family" where all the panelists are women. This limited gender composition of the panel is a reinforcement of gender roles. A spouse should not be a person who "helps" with family and home responsibilities. Instead, a spouse should be a person who shares those responsibilities. Having the encouragement and support of my former spouse in all my activities during my tenure period was instrumental to many of my accomplishments. But, as I said before, I did not know how to focus in order to have the motivation and time management to include research in my academic activities.

Workplace bias (Negotiating duties). Some tenure-track professors are assigned to several committees and service duties. They might feel an obligation to these duties because they demonstrate that the professor is a "good citizen." Also, the type and amount of duties or teaching responsibilities assigned to female professors sometimes are different from the duties assigned to male professors [11]. In general, women also tend to work harder and receive less pay than their male counter parts for the same work [10]. Service activities can be time consuming and, hence, diminish the time and focus available for other responsibilities for which he/she will also be evaluated. I know of a case in which a woman professor was assigned a lot of service work. She did not complain because she was on probation but, as a consequence, she was unable to carry out her research agenda, which was needed to obtain tenure. In the end, she did not get the research publications, nor the tenure.

Assigned duties should be negotiated with the personnel committee.[5] All tenure track professors should be treated equally. They should be given the time and resources necessary to fulfill the research expectations, regardless of gender, gender identity, sexual orientation, race, ethnicity and other backgrounds.

Student evaluations. Another element of the evaluation for tenure is student evaluations. I love to teach. I prepare well for my courses, give homework and spend a lot of time grading it. During my first semester at the UPR-H, I enjoyed teaching so much that I organized study sessions and stayed late helping students with their Abstract Algebra homework. This reflected positively on my teaching evaluations. At the bachelor's level, the mathematics courses were usually gender-balanced, and I felt that both female and male students respected me. But, in graduate school, where there is no gender balance, the story might be different. As a graduate student, I witnessed some students not missing the opportunity to be vocal about any mistake made by a female professor who almost always came well prepared for class, while the same students did not say a word about a male professor who almost always came unprepared and made mistakes in class. If this happened now, one of those students could be accused of harassment. At that time we were all silent.

Gender stereotypes might affect how we perform and can be reflected in student evaluations, so they should be dealt with. If the situation that I described above had happened to me when I was younger, I probably would have shown

[5] *At the UPR, personnel committees advise department chairs and deans in personnel-related issues such as hiring and promotions.*

my anger to the student by saying something back in the classroom. This might have made the situation worse. It might be better to discuss the situation with the student privately and also talk to the chair of the department. But the situation should not be overlooked.

The road to Full Professor

Once you have your tenure, the next step is being promoted to Full Professor. Of course, the expectations will be higher than those to obtain tenure. Even though the evaluation system was still based on points, I needed research publications in order to become Full Professor. After five years of successfully running SIMU at the UPR-H [4, 5], the NSF-REU grant ended and we decided not to submit a new proposal. It was time to do other things. Since outreach was the focus of my work during those years, I felt the need to keep up the work with underrepresented minorities in mathematics. I partnered with Ricardo Cortez from Tulane University to develop an enhanced mathematics component for the SACNAS conference and secure funding for it [2]. I kept doing what I felt was important and did not dwell on the promotion to Full Professor.

In 2002, when SIMU was about to finish, the ADVANCE Institutional Transformation (ADVANCE-IT) program was starting at the UPR-H under the leadership of Idalia Ramos, a professor at the Physics Department passionate about women's issues [7]. The aim of the program was to increase the participation and advances of women in science at the UPR-H by changing the organizational climate and culture at the campus. Participating in this program increased my awareness of women's issues in the sciences and made me realize that I spent five years of my life promoting research among undergraduate minority students, but I was not doing research myself. I was not setting the example. As my mother would say, I was "predicando la moral en calzoncillos" (a Spanish saying that translates to "preaching morality in underwear"). To motivate Hispanic female students to engage in mathematics research I should serve as an example and be a role model. After five years of being at the UPR-H, I decided to do what most faculty do when they just graduate: develop a research agenda.

Developing a research agenda. At the same time that I got involved with the ADVANCE-IT program, a young computer scientist from the UPR-RP, Carlos Corrada, had the idea of studying applications of the monomial permutations that I studied for my master's thesis to the construction of interleavers for Turbo codes. Again, all the conditions were perfect for a new project. This time it was to develop a research agenda and take research more seriously. The project favored our collaboration as we complemented each other well: I had the mathematics background and Carlos had the engineering background and understood the application well. Our different backgrounds also solved the issue of the order of the authors in the publications easily. We decided that the focus of the journal would decide who goes first. If it were a more mathematical journal, I would be first author; for an engineering journal, Carlos would go first [8].

A good opportunity to present our work in an international conference on finite fields that was immediately followed by another in related topics appeared and I took advantage of both. During the first conference I consciously suppressed my friendly behavior, attended each one of the talks and kept working quietly during all the breaks. This could be interpreted *as assimilating and playing into the stereotypes* [6]. I did not want to reinforce any stereotype; what I wanted was to be known *first* as a researcher. For the second conference I relaxed, still went to all the talks, but I also enjoyed (a lot!) all the evening hangouts. Attending these two conferences gave me the opportunity to be exposed to the finite fields community and exchange ideas with other mathematicians who would become good collaborators, mentors and friends. For example, the work on monomial permutations was extended to Dickson permutation polynomials in a first collaboration with Gary Mullen from Penn State University [8].

An interesting research project to generalize an elementary approach to the divisibility of exponential sums from characteristic 2 to characteristic p was presented by Harold Mattson, Jr., an emeritus professor at Syracuse University, to Francis Castro from the UPR-RP, and to myself. We were later joined by Hugues Randriam from Télécom ParisTech. Francis and I were very good friends and were collaborating already on the Dickson polynomials project. He was an expert in exponential sums and had several publications on the topic with Oscar and Carlos Moreno. Mattson was a renowned researcher in topics related to coding and number theory. Working with them on a project for which I had no background could have been very intimidating. However, I felt comfortable because I knew that I could contribute and also because I had no trouble asking them questions. The imposter syndrome did not attack this time.

In 2006, I was promoted to Full Professor at the UPR-H. The students and working environment there were excellent, I had many friends, and I was respected and very happy. But, since I was already collaborating with two professors at the UPR-RP, it made sense to move to that campus. The Computer Science (CS) Department at the UPR-RP was a small department that offered an undergraduate degree with a strong algebraic component. The department was revising its program, looking to strengthen their undergraduate research, and had positions open. The conditions were perfect for another change and I moved to the UPR-RP in 2007 as I described earlier.

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MEMORIAL TRIBUTE

Remembering Georgia Benkart

Alejandro Adem, Tom Halverson, Arun Ram, and Efim Zelmanov

Georgia Benkart passed away unexpectedly in Madison, Wisconsin, on April 29, 2022. Georgia earned her BA from Ohio State University and her PhD in 1974 from Yale University under Nathan Jacobson. She was a profoundly influential scholar and leader in the fields of Lie theory, representation theory, combinatorics, and noncommutative algebra. She spent her career at UW–Madison, where she was the second woman to join the department and the second to earn tenure. At the time of Georgia's retirement in 2006, she was the E. B. Van Vleck Professor of Mathematics. Georgia is survived by her sister, Paula Benkart, who also attended Ohio State University and earned a PhD in History from Johns Hopkins University in 1975.

Georgia was an inspiring teacher, the advisor to 22 PhD students, and a mentor to scores of mathematicians around the world. She published more than 130 articles and research monographs and gave more than 350 invited talks, including plenary lectures at AMS meetings, the AWM Noether Lecture at the Joint Mathematics Meetings, and the Emmy Noether Lecture at the International Congress of Mathematicians. Her lectures were works of art. Without fail they were accessible to nonexperts, told a compelling and creative story, delighted her audiences with literary allusions and puns, and invited everyone into the fun.

Alejandro Adem is a professor of mathematics at the University of British Columbia. His email address is adem@math.ubc.ca.

Tom Halverson is a professor of mathematics at Macalester College. His email address is halverson@macalester.edu.

Arun Ram is a professor of mathematics at the University of Melbourne. His email address is aram@unimelb.edu.au.

Efim Zelmanov is a professor of mathematics at the University of California, San Diego. His email address is ezelmanov@ucsd.edu.

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Figure 1. Georgia Benkart at the graduation of her PhD student, Dongho Moon, on May 15, 1998.

Georgia's service to the mathematical profession is legendary. After retiring from teaching in 2006, she continued an active research program in which she published nearly 40 papers. At the same time, she focused her attention on service to her professional societies. She was president of the Association for Women in Mathematics and an associate secretary for the American Mathematical Society. In fact, she was the lead organizer of the 2020, 2021 and 2022 Joint Math Meetings, the last of which took place online just three weeks before her death.

However, Georgia's most meaningful and enduring impact on our community lies outside of what is found in her impressive CV. It is about the people whose lives she changed. She was a pioneer in building research collaborations, starting when she worked with teams of graduate students and continuing with the many research networks that she built among early-career women mathematicians.

A recent article in March 2022 *Notices*, "Gems from the Work of Georgia Benkart," profiles her mathematical work. We are enormously pleased that she was able to see this before her death. In this memoriam, we have asked several of her colleagues and collaborators to describe their experiences working with her. In our opinion, this is the best way to know our friend, Georgia Benkart.

Sheila Sundaram

I met Georgia in May of 1988, while I was a postdoc at the University of Michigan. Georgia had invited me to speak at a conference on Lie Algebras in Madison. As a recent PhD, I was overwhelmed by the distinguished list of other speakers, but Georgia's warm welcome and kindness helped to put me at ease.

Perhaps this marked the start of Georgia's interest in the work of the algebraic combinatorics community, from papers of Phil Hanlon, Bob Proctor, and John Stembridge. Indeed she initiated a long and fruitful research programme with her subsequent students and postdocs, investigating, for example, analogues of Cauchy identities and other classical character formulas due to Littlewood for the orthosymplectic Lie superalgebras, and developing the use of Robinson–Schensted analogues and jeu de taquin techniques in the representation theory of Lie algebras. A short list of my personal favorites among her papers that might serve as an introduction to Georgia's work for algebraic combinatorialists is: [BCH+94, BSR98, BR98, DG13].

Our paths crossed on a few occasions in the ensuing years. A particular highlight is a lunch we had at Yale with another legendary figure, Walter Feit, whom I would never have met were it not for Georgia. I had recently moved to Connecticut, beginning what was to be a long absence from academia. Some time later she took the trouble to reach out to me about a paper of mine. Personal circumstances had prevented me from submitting the final version; having just taken over as editor, she had found it in the files of the journal. Eventually I returned to doing mathematics, and wrote to her again. Almost two decades

Figure 2. Alberto Elduque and Georgia Benkart, in Asturias (Spain) in 1994.

had passed, and I was not sure she would remember me. I was astonished when she immediately wrote back, saying, among other things, that she had found the whole thread of the correspondence regarding my paper. She was solicitous and encouraging about my new mathematical endeavors, making it a point to attend a talk I gave at an AMS Special Session in Bloomington.

Georgia's mathematical legacy is of course legend. It is a source of comfort to me that she lived to read the beautiful tribute to her work by Tom Halverson and Arun Ram, which appeared in the March 2022 issue of the *Notices*. Most recently I saw her in November of 2021, during the virtual Northeast Women in Algebra and Combinatorics Conference, on the occasion of the 50th anniversary of the AWM. There, technical difficulties with Zoom notwithstanding, she gave an absolutely captivating talk on MacKay quivers, Schur–Weyl duality, and the rich interplay of algebra and combinatorics. (The slides are available online at `https://www.albany.edu/~sdc/AWM21/Georgia.awm50.pdf`, see Figure 8.) She had clearly taken great pains to make her "walking talk," as she called it, broadly accessible, and the delivery is interspersed with sly puns and subtle humor, both in words and graphics.

Georgia helped launch and establish the careers of countless young mathematicians. Her many collaborations are a testament to her dedication to sharing her passion and knowledge of mathematics. The loss to our community is indescribable. Those who knew her personally are left with deep sorrow, but also lasting memories of her extraordinary kindness, and her desire to help others advance in the profession.

Arturo Pianzola

Grace, Generosity, Kindness. We all know what these words mean, but just like in mathematics where there is a difference between understanding a proof and seeing the result behind it, knowing the meaning of these words is not the same as feeling them. If you were fortunate enough to have Georgia Benkart be part of your life, you certainly felt these words.

Georgia was the external examiner for my PhD. For some reason that I cannot explain, we remained close ever since. It is not that we saw each other on a regular basis, but whenever we did meet, we always found time for each other. You could vividly sense that she needed to know that you were doing fine. Whatever struggles or challenges you were facing, just talking to her had a soothing effect.

Her generosity and desire to help and serve was exemplary. A few years ago, even though she was overwhelmed with requests, she agreed to act as the main external reviewer of our mathematics department. She took the time to talk to students and support staff and left a lasting impression on them. The final recommendations of the report provided insightful guidance. It was not a "generic" document, but one that was written with care and a desire to force us to reflect. This was her style. To convince people, at times without them even noticing it, that they have the ability to be better.

We had an inordinate number of meaningful coincidences, like unknowingly working on the same mathematical problems for completely different reasons. Sometimes these coincides were simply hilarious. Georgia had invited me to Madison and had booked a special reservation at a Japanese restaurant (my favorite food). Upon looking at the menu, the following exchange took place:

- (me) Right on! Yellowtail and take me to jail, baby!
- (Georgia, with a mixture of perplexity and her unmistakable laughter) What?
- I love yellowtail, Georgia. Anyways. It is a long story. The expression comes from a live performance by Ben Sidran, an American jazz piano player. As you know, I think that Jazz is the greatest contribution that America has made to the arts.
- Ben Sidran?
- Yes. Have you heard of him?
- He is in my yoga class!

The next time I saw her, more than one year later. We went for coffee, and she said, "I got you a little something." It was an autographed copy of a book that Sidran had written. Great book. I am reading it again and when I hold it in my hands, I know that it was also in hers at one point, and I let my mind wander through the times we spent together.

In a play where any part of the script can be rewritten except for the ending, fond memories are a valuable currency. You've blissfully touched so many lives, Georgia, and will always be remembered. You were never meant to be gone, but rather just not to be around any longer. You will be missed. Until we meet again.

Arturo Pianzola is a professor in the department of mathematical and statistical sciences at the University of Alberta. His email address is a.pianzola@ualberta.ca.

Figure 3. Zongzhu Lin, Georgia Benkart, Brian Parshall, Jens Carsten Jantzen, Daniel Nakano, James Humphreys; Representations of Algebraic Groups, Quantum Groups, and Lie Algebras: AMS-IMS-SIAM Joint Summer Research Conference, July 11–15, 2004, Snowbird, Utah.

Ruth Charney

Reading through tributes to Georgia from her colleagues, former students, and friends, one cannot help but be impressed by the impact she had through her research, her mentorship, and her collegiality. In this note, I would like to focus on another aspect of her life's work, namely her contributions to the profession through her involvement in professional societies.

Georgia retired from her academic job at the University of Wisconsin–Madison, in 2006. Far from gearing down her involvement in mathematics, she continued full speed ahead on her research (publishing nearly 40 papers post-retirement) and turned her attention to advancing the profession through service with professional societies. In 2009, she became president of the Association for Women in Mathematics (AWM). This is an organization with a small staff and limited budget that is highly dependent on volunteers. As described in a recent issue of the *AWM Newsletter*, her accomplishments as president

Ruth Charney is Theodore and Evelyn Berenson Professor of Mathematics at Brandeis University. Her email address is charney@brandeis.edu.

over the next two years were extensive. They culminated in a spectacular conference celebrating AWM's 40th anniversary, "40 Years and Counting: AWM's Celebration of Women in Mathematics." This was the first in what subsequently became a biannual series of AWM Research Symposia. After her term as president ended, she continued her involvement in AWM. In 2015, AWM initiated an NSF-funded program of Research Collaboration Conferences for Women, designed to promote collaborative research projects and create research-based networks. Georgia was one of the first to organize such a workshop, the WINART1 (Women in Noncommutative Algebra and Representation Theory) workshop. With Georgia's help, this network has continued to grow and to thrive. The most recent workshop, WINART3, took place in April 2022.

In 2010, Georgia agreed to serve as an associate secretary for the American Mathematical Society (AMS). The AMS has four associate secretaries, one for each geographic section, whose primary responsibility is to oversee the scientific meetings of the Society. In the case of sectional meetings, that includes approving proposals for special sessions, screening abstracts, and inviting speakers for named lectureships. In addition to organizing two sectional meetings per year, the responsibility for organizing the annual Joint Mathematics Meetings (JMM), as well as yearly international meetings, rotates among the four associate secretaries. Associate secretaries work largely behind the scenes, so most people are not aware of the important role they play in planning and implementing successful meetings. Georgia was the associate secretary for the Central Section which spans a large region of the US and Canada, north to south from Manitoba to Texas, east to west from Ohio to Nebraska.

As a highly respected member of the community, Georgia was able to recruit top-notch speakers and session organizers for these meetings. In addition, she was instrumental in engaging the Portuguese and European Math Societies to hold a joint international meeting with the AMS in 2015. Most importantly, she served as the primary organizer for four JMMs, including the most recent one, JMM2022. In the past, the AMS has shared responsibility for these meetings with the Mathematical Association of America (MAA). The 2022 JMM, scheduled to take place in Seattle in January, was the first "JMM reimagined" in which the AMS took over as primary organizer. This required considerably more work for the associate secretary in charge. Then, at the last minute, the meeting in Seattle was cancelled due to concerns about the pandemic, and it was rescheduled as a remote meeting to be held in April. This resulted in a whole new set of challenges for the organizers. Georgia, as usual, took the reins with confidence and guided the transition smoothly. Sadly, though none of us knew it, by the time the JMM took place in April, she was suffering the effects of her cancer and she died just three weeks later.

In addition to her work with AWM and AMS, Georgia served on multiple other committees including the National Academy's US National Committee for Mathematics (2013–2020) and the Board of Trustees of the Mathematical Sciences Research Institute (2011–2022), as well as the editorial boards of several journals. Viewed from the outside, Georgia's service contributions are indeed impressive. She was extremely good at her job, highly organized and methodical, and made sure everything got done as planned. Seen from the inside, by those who worked directly with her, she was humble, kind, and treated everyone with the greatest respect. She also had a delightful sense of humor and brought a sense of joy to her work. So many remembrances comment on what a pleasure it was to work with her.

For me, Georgia will always be a role model—for her devotion to research, to service, and to mentorship, and for her unfailing ability to welcome everyone with an outstretched hand and a smile. I know that many of you feel the same.

Figure 4. WINART2 (Women In Noncommutative Algebra and Representation Theory 2), Leeds 2019.

Ellen Kirkman

Georgia Benkart was an extraordinary model for excellence in research, exposition, communication, service to the community, and expanding participation in the mathematical community that she loved.

An Extraordinary Communicator

Georgia and I worked in different research areas of algebra, and I first knew of Georgia from her beautifully

Ellen Kirkman is a professor of mathematics at Wake Forest University. Her email address is `kirkman@wfu.edu`.

constructed invited lectures that were understandable to wide audiences. She was an MAA Pólya Lecturer (2000–2002), an AWM-AMS Noether Lecturer (2014), and an ICM Noether Lecturer (2014). She visited Wake Forest University in 1994–1995 as our tenth Gentry Lecturer, giving a lecture aimed at a general audience: "Life in the Aftermath—The World Beyond the Symmetric Group" and a more specialized talk: "Commuting Actions—A Tale of Two Groups" (on one of her favorite topics, Schur–Weyl duality!). There was usually a pun or a literary reference; "A Tale of Two Groups" began: "It was the best of times, it was the worst of times." Her papers were carefully constructed so that a novice could gain admission to some lovely mathematics. Both her lectures and her papers appeared so polished because she worked intently on every detail.

A Prolific Researcher

Georgia wrote over 130 research publications, with over a hundred collaborators, spanning a wide variety of research areas. Her paper [BR98] on down-up algebras (with almost 100 citations) was constructed from her point of view of generalizing the representation theory of $\mathsf{U}(\mathfrak{sl}_2)$, but the graded down-up algebras provide examples unexpected by their original context: to those studying noncommutative algebra and algebraic geometry they are a convenient family of examples of Artin–Schelter regular algebras of global dimension three, generated by two elements, with two cubic relations, in contrast to the other family of Artin–Schelter regular algebras of global dimension three (including commutative polynomial rings in three indeterminates) that are generated by three elements with three quadratic relations.

Georgia was always exploring new areas of research. Our work with the WINART groups arose because Georgia wanted to learn more about Hopf algebra actions. Georgia told me that one reason that she retired early was so that she would have younger colleagues who could expand her mathematics.

A Facilitator of Research Collaborations

Georgia was one of the organizers of the "Connections for Women Workshop," held during the MSRI semester "Noncommutative Algebraic Geometry and Representation Theory" in 2013. When Chelsea Walton started WINART (Women In Noncommutative Algebra and Representation Theory), Georgia was one of the co-organizers as she had experience with similar collaboration conferences for women including Algebraic Combinatorixx (May 2011) and Algebraic Combinatorixx2 (May 2017).

The first WINART conference was held at BIRS in May 2017. Forty-eight women were divided into eight research groups and spent the week working on research projects that generally continued following the conference. Georgia and Rosa Orellana led a WINART1 group "Diagram algebras, tensor invariants, representation theory." Georgia was a co-organizer of WINART2, and she and I led a WINART2 group "Tensor invariants under Hopf algebra actions," at the University of Leeds in May 2019. Before most virtual meetings Georgia would send TeXed notes that outlined clearly where we were in the project. This group produced two papers [BBK+22a, BBK+22b]. With Georgia's encouragement to apply for further opportunities to work together, we received offers to continue our research at both MSRI and IAS in summer 2020. Unfortunately COVID interrupted these plans, but we continued meeting virtually every week. Three members of the WINART2 group met at IAS for two weeks in June 2022 under their Summer Collaborators program. Georgia was to be part of this group and she was very much missed.

Preparations for WINART3 began in 2021, and Georgia and I again led a collaboration group that met weekly in 2022. As Georgia was the AMS associate secretary in charge of the Joint Mathematics Meetings, she was able to meet with us only virtually. This time our topic was "Verlinde-type formulae for fusion coefficients." Our work on this exciting project continues.

Georgia was an encouraging mentor to me, personally, and to all the women in our groups. Her enthusiasm for mathematics was infectious. She provided expert mathematical and professional advice, was incredibly patient as we made mistakes, and wrote countless letters of recommendation. Even when intensely busy in her role as an AMS associate secretary, Georgia would somehow magically appear at the AMS session when one of our group members spoke. Georgia was a kind, gentle, warm, brilliant, humble, and generous person, with a wonderful sense of humor and much common sense. Her commitment to equity, diversity, and inclusion was clear.

So Much Service to the Profession!

Georgia provided service to the profession in many different ways. She was a longtime member of MSRI's Board of Trustees and Committee on Women in Mathematics, the Scientific Advisory Board of the American Institute of Mathematics, an AMS associate secretary, a president of the Association for Women in Mathematics, and she served on a number of editorial boards. From 2015–2016 Georgia agreed to be a member of a three-person external evaluating committee for my department's program review, a task I knew she had performed for several other universities. Typical of Georgia's many service activities, the carefully written review was helpful to the department as it advocated for new tenure track positions, and it was supportive of the department's efforts in teaching and research.

Georgia Benkart, extraordinary model.

Figure 5. Eleanor and Frank Lemire, Dan and Susan Britten, Paula and Georgia Benkart; dinner at the Benkart's during the 2002 AMS meeting in Madison.

Rolf Farnsteiner

During the academic year 1987–1988 Georgia Benkart and Marshall Osborn organized a Special Year on algebraic Lie theory in Madison. They had obtained NSF funding to invite long-term visitors and to host several meetings. This was a good time for such an undertaking: Richard Block and Robert Wilson had just completed the classification of the restricted simple Lie algebras, there were interesting developments concerning certain infinite-dimensional Lie algebras, the so-called Kac–Moody algebras, and, by work of Friedlander-Parshall and Jantzen, the geometric approach to the representation theory of restricted Lie algebras, first set forth by Jon Carlson in the context of finite groups, had emerged as a powerful tool.

Having the privilege of being a long-term visitor, I got a closer look at Georgia as a researcher and teacher. I had met her before at conferences and heard of her even earlier, but I had not witnessed her interaction with her students and colleagues to such an extent. Her warm personality greatly facilitated the collaboration of her various guests. She valued informal discussions and organized a number of social gatherings that included her local colleagues working in group theory and ring theory. I have fond memories of our weekly tennis matches and, like many others, I enjoyed her great sense of humor. All of this was part of her leadership and guidance that proved to be very effective and in my opinion was key to the success of the Special Year.

During the Special Year the classification of the finite-dimensional simple Lie algebras over algebraically closed fields of characteristic $p>0$ was the main focal point. This was a long-standing problem, whose inception is often dated back to Witt's 1937 discovery of a nonclassical simple Lie algebra. The challenges involving the classification were perhaps best described by George Seligman, one of Georgia's mentors at Yale, in his 1967 monograph: "It is not simply the case that new methods must be found to establish analogues of the theorems for characteristic zero, but rather that almost the only analogues which remain true (with the same degree of generality) are those whose traditional proofs turn out to have been independent of the characteristic anyway."

In their above mentioned work, Block and Wilson had overcome these obstacles in the context of those simple Lie algebras $\mathfrak{g}$ over an algebraically closed field of characteristic $p>7$ that afford a p-th power operator $x \mapsto x^{[p]}$ that enjoys the formal properties of taking p-th powers of square matrices. This self map turned out to be of major importance and even though at the time many simple nonrestricted Lie algebras were known and a conjecture was formulated, it appeared that the relevance of George Seligman's comments had resurfaced on a different level.

There were, however, initial results indicating the validity of the generalized Kostrikin–Shafarevich conjecture, which stated that simple Lie algebras over algebraically closed fields of sufficiently large characteristic $p > 0$ are either classical or of Cartan type. Building on earlier work by Kaplansky, Ermolaev, and Block, Georgia Benkart and Marshall Osborn had shown in their 1984 *Annals* article that the conjecture was true for simple Lie algebras having a one-dimensional Cartan subalgebra in the cases $p>7$. In their technically sophisticated paper, Benkart and Osborn showed that, aside from $\mathfrak{sl}(2)$, only the so-called Albert–Zassenhaus Lie algebras can occur. During the Special Year, Georgia and Marshall, partly in collaboration with Helmut Strade, made further significant contributions which paved the way for the announcement of the proof of the conjecture by Strade and Wilson a few years later.

One important milestone in the classification was Kac's Recognition Theorem from 1970. This result, which gives technical conditions ensuring that a Lie algebra is classical or of Cartan type, was widely used even though its proof contained some inaccuracies. Together with Tom Gregory, Georgia started in 1987 a long-term project which aimed at providing a complete proof of Kac's Theorem. First results, which improved the theorem by showing that one of its conditions was redundant, were published by them in 1989. It took another 20 years until the entire story was told in a monograph by Benkart–Gregory–Premet. The publication of this work in my opinion represents a great service to the community and exemplifies Georgia's attention to details, clarity, and accuracy that was a hallmark of her work in general.

The foregoing remarks do by no means adequately reflect the breadth of Georgia's interests and contributions. Aside from her list of publications, a quick glance at the topics of the 22 PhD theses written under her guidance provides a good impression of Georgia's sharing her broad expertise with the mathematicians of the generations following her. Moreover, not only her students have benefitted from her captivating lectures and the clarity of her exposition.

Figure 6. Left to right: Alberto Elduque, Consuelo Martínez, Georgia Benkart, and Santos González, in Zurich, on the occasion of the ICM in 1994.

Alberto Elduque

I had the good fortune to meet Georgia Benkart during my first mathematical workshop, near Boston, in 1983. I was a first-year graduate student, it was the first time I had crossed the Atlantic, and I was excited and nervous about the possibility of meeting mathematicians from different places around the world. I vividly remember how impressed I was after Georgia's beautiful talk, so carefully prepared, with the right pace, some puns here and there, deep mathematics,... I was absolutely fascinated. There I had the model to follow. I never told her how much she had impressed me and had influenced and strengthened my desire to become a mathematician.

After that initial encounter, we continued to be in touch, and met either at workshops, or in short visits either of mine to Madison, or hers to Spain. She once wrote to me "Zaragoza is one of my favorite places." And certainly she enjoyed visiting Spain, and not just mathematically, as can be seen in Figure 2. Our mathematical collaboration started during the academic year 2000–2001. I was on sabbatical leave and asked her about the possibility of spending it at the University of Wisconsin. She was very supportive from the beginning and, together with Marshall Osborn, helped me and my wife and daughter, who was ten years old at that time, to settle smoothly in Madison. When we arrived, she had even put flowers in our apartment and bought us food. This care for the details, both in mathematics and otherwise, was one of her wonderful characteristics.

I cannot explain in a few words how much I enjoyed doing mathematics with Georgia throughout that year, benefiting from her insight, her advice, and her brilliant questions, which forced me to think deeper and deeper. She had been working with Efim Zelmanov [BZ96] on the description of Lie algebras graded by non-simply-laced root systems, following the work for simply-laced root systems initiated by Berman and Moody [BM92]. The clue to understand these Lie algebras is a construction, due to Tits [Tit62] that relates Lie and Jordan algebras. During that year we tackled the analogous problem for Lie superalgebras.

I would like to share with the readers one "aha" moment that Georgia and I enjoyed together, motivated by her ability to ask the right questions. It was during the spring of 2001. Georgia had many students and visitors, so we had fixed a weekly one-hour meeting. In one of these meetings we had just finished successfully some computations that we needed and we were relaxed talking about different things. The conversation turned to the exceptional basic classical simple Lie superalgebras $D(2,1;\alpha)$ of dimension 17, $G(3)$ of dimension 31, and $F(4)$ of dimension 40. The classification by Victor Kac [Kac77] of the finite-dimensional simple Jordan superalgebras over algebraically closed fields of characteristic 0 was based on the famous Tits–Kantor–Koecher (TKK) construction of a Lie algebra starting from a Jordan algebra.

Kac used his former classification of Lie superalgebras to check which of them are 3-graded as above, and to conclude from there the classification of the simple finite-dimensional Jordan superalgebras. Among these there is a family of 4-dimensional Jordan superalgebras related to the Lie superalgebras $D(2,1;\alpha)$ and a mysterious 10-dimensional Jordan superalgebra, nowadays known as Kac superalgebra K_{10}, related to $F(4)$. Kac gave a multiplication table for this exceptional object. No direct proof that this was indeed a Jordan superalgebra, other than through its connection with $F(4)$, was known. Well, as usual, Georgia started to ask natural and interesting questions: *what does the structure Lie superalgebra of* K_{10} (that is, the homogeneous component $\mathcal{L}_0$ above) *look like?, and the Lie algebra of derivations?* A few easy mental computations with the dimension of the subspaces that appear inside $F(4)$ convinced us that this structure Lie superalgebra had to be the orthosymplectic Lie superalgebra $\mathfrak{osp}(2,4)$, and the Lie algebra of derivations should be the direct sum of two copies of $\mathfrak{osp}(1,2)$. The next question Georgia asked was: *and* *then, what is the structure of K_{10} as a module for its derivation algebra?* And at that point we looked at each other realizing at once that this was the clue! The Jordan superalgebra K_{10} had to be the direct sum of a trivial 1-dimensional module spanned by the unit element, plus the tensor product of the 3-dimensional irreducible module for each of the two copies of $\mathfrak{osp}(1,2)$. But this module is the so called "tiny Kaplansky superalgebra" K_3, related to $\mathfrak{psl}(2,2)$ by TKK. Therefore, we must have $K_{10} = \mathbb{F}1 \oplus (K_3 \otimes K_3)$ [BE02]. Of course, there was some extra technical work to do, but we both knew that the multiplication in K_{10} was essentially determined from this decomposition, something we completed in the next few days.

Some years later, in 2014, my daughter returned to Madison, but this time as a new math graduate student at the University of Wisconsin. The last time I met Georgia in person was on the occasion of my daughter's doctoral thesis defense in 2019. Georgia had retired from teaching some years ago, but she continued to be very active in research and service, and we had the opportunity to talk about some mathematical problems. She was planning to attend, as an invited speaker, a conference on Nonassociative Algebras organized in Coimbra (Portugal) in 2020, to celebrate my 60th birthday. The pandemic forced the conference to be postponed to July 2022. At the meeting we all remembered her, we all missed her.

Konstantina Christodoulopoulou

Georgia was a great mentor and an inspiration not just to me but to many mathematicians. I became aware of her work as an undergraduate student at the University of Athens in Greece and during my first semester as a graduate student at UW–Madison I took her representation theory course. That course was an unforgettable experience; not only did it provide me with a strong background in Lie theory but it also revealed its beauty and complexity, and inspired me to work in that area.

I was truly lucky to have had Georgia as my PhD advisor. We met almost every week for three years and I am grateful for her patience and gentle guidance! I am also grateful for the semesters that Georgia provided me with a research assistantship that made it possible for me to concentrate on my research.

Georgia's wisdom, generosity, kindness, and humor will always stand out to me as I think back to graduate school. After I graduated, Georgia continued to support me and advise me as I navigated life after graduate school. I will always be indebted to Georgia for her encouragement, guidance, and inspiration.

Konstantina Christodoulopoulou is an associate instructional professor and undergraduate coordinator at the University of Florida. Her email address is kchristod@ufl.edu.

Sarah Witherspoon

Georgia was my postdoctoral mentor at the University of Wisconsin twenty-some years ago. I had first met her when I was a PhD student working in group theory under Jon Alperin. At that time, Georgia was writing a paper in combinatorics with my husband Frank Sottile, also a student. I could count on one hand then the senior women mathematicians I knew of in my research area or a closely related field. Georgia was one of them, and an important role model. She was an inspiring speaker, presenting mathematics in beautiful diagrams and pictures, keeping it as simple as possible, with many jokes and puns. She was an exceptional mentor to many young mathematicians, with a resulting impact on mathematics that goes far beyond the excellent research she personally contributed.

I first benefited from Georgia's enthusiastic welcome of all young mathematicians shortly after I had graduated and was at an AMS sectional meeting for a special session. The topic was Hopf algebras, which arose in my PhD thesis but was only tangential to the group theory community I had gotten to know as a student. I felt a stranger to this new community and I was desperate to be a part of it. Georgia remembered me and reached out to make sure I was invited to join others for meals. A few years later, when my tenure track job search was not going well and I was pregnant with my first child, Georgia pushed for Van Vleck Assistant Professorships for both Frank and me at the University of Wisconsin. This was a fantastic opportunity for both of us to join the vibrant culture there in algebra and combinatorics, with many students and postdocs and seminars and teas. Much activity centered on Georgia and her research group.

Georgia greatly influenced my career over the course of many years. My time in Madison was a very uncertain one for me due to new parenting responsibilities. Georgia expressed confidence in me as a mathematician and pushed me gently to begin and to keep working on a joint project she suggested, on down-up algebras. These are algebras that are defined combinatorially and are connected to Hopf algebras and quantum groups—rings that not only have a multiplication but also a dual structure called a comultiplication. Quantum groups arose in mathematical physics and representation theory, and down-up algebras can be viewed as subalgebras of some quantum groups in a natural way. In our regular meetings on

Sarah Witherspoon is a professor and head of the department of mathematics at Texas A&M University. Her email address is sjw@math.tamu.edu.

down-up algebras and beyond, Georgia was always full of ideas and questions, probing for what I could best learn and contribute, facilitating mathematical and professional growth.

Georgia and I continued collaborating long after I moved on to jobs elsewhere, providing important encouragement during my second pregnancy as well. She mentored me through my first PhD student, Mariana Pereira, whose thesis grew out of ideas from my ongoing research collaboration with Georgia. In fact the three of us have a joint publication on representations of quantum groups, research that was primarily carried out during a semester that we all spent at the Mathematical Sciences Research Institute.

Over the many years since, I continued to see Georgia at conferences, and would sometimes visit her in Madison. She was always warmly welcoming, often opening the doors of her and her sister Paula's house to current and former students and postdocs, of whom there were many, and we would enjoy the excellent food and company and conversation with Paula and Georgia.

In recent years, I often saw her at AMS meetings where she was in charge as associate secretary for the AMS Central Section. There she would calmly deal with any number of issues that came up, and pop in and out of special sessions as time permitted. I am honored and proud to have known Georgia as a mentor, as a collaborator, and as a friend. Her impact on mathematics and on many mathematicians is immeasurable and enduring.

Figure 7. Pandemic working smiles: Shraddha Srivastava, Rosa Orellana, Elizabeth Manosalva, Georgia Benkart on August 6, 2021.

Efim Zelmanov

On February 19, 1992 I entered the USA with my family and a green card and proceeded to Madison, WI, to join the Department of Mathematics. Before that I stayed for two semesters at Yale and the University of Virginia as a visitor.

In Madison, Georgia and Marshall Osborn became our guardian angels. I cannot think of a word that would fit Georgia better than angel, the way she behaved, spoke, even looked.... Georgia was a very special (rare) angel, the one with down-to-earth common sense and a fantastic (also down-to-earth) sense of humor. Now when I am writing these words I imagine Georgia looking over my shoulder and making comments. I know that it won't happen and that is painful.

Rosa Orellana

An inspiration, a role model, a mentor, and a friend, these are the words that come to mind when I think of Georgia Benkart. I didn't realize until many years later how important it was to have such a strong female role model before I knew that I needed one. I first met Georgia when I was a second-year graduate student at a Connections for Women Workshop at MSRI. I remember that she came to talk to me, she knew my advisor, and asked me about my research. At the time I didn't know who she was, but I will always remember that she made the effort to come and talk to me. I continued to meet Georgia at conferences and our friendship developed into a mentor-mentee relationship.

In the years that followed, she always demonstrated interest in my work by inviting me to speak if she were organizing a conference. She visited me at Dartmouth in 2009, when she became the first female mathematician to deliver our department's distinguished Kemeny Lecture series. At this time, I had the opportunity to ask her why she retired so young, if she continued working as hard, if not harder. She answered my question with a smile and said that sometimes to do more meaningful work, you must quit your job.

In 2011, Georgia co-organized Algebraic Combinatorixx with Monica Vazirani and Steph van Willigenburg. The workshop was organized at Banff and the objective was to create small research groups of female mathematicians. Part of the activities included mentoring meetings where everyone shared personal experiences and tips about what worked for them to keep research going. To me, personally, this was Georgia's most meaningful contribution to our community. At the time I was the mother of a young child, had recently experienced the loss of my second child, and was finding it difficult to get research going again. I was one of the lucky people to get an invitation to the first

Rosa Orellana is a professor of mathematics at Dartmouth College. Her email address is Rosa.C.Orellana@dartmouth.edu.

Algebraic Combinatorixx at Banff. This workshop transformed my career. I was able to make professional connections that led to collaborations; it focused my research plan that became my next research proposal which was funded by NSF, and it allowed me to feel confident to apply for full professorship two years later.

In 2015, Georgia invited me to co-lead a research group for the newly funded WINART group. The first workshop was going to be held at Banff and it was going to follow a similar format as the Algebraic Combinatorixx. Although, she didn't say it directly, she was training me to take a more leading role. It was with her encouragement that Julie Beier, Tricia Brown, Steph van Willigenburg, and I co-organized the second Algebraic Combinatorixx at Banff in 2017. An admirable quality of Georgia was that she knew that she didn't have to be a co-organizer to contribute. She was equally happy being a participant, a group leader, or a co-organizer.

Toward the end of 2020, it was becoming clear that the pandemic was negatively affecting the research output of many female mathematicians. In fact, a letter by the European Women in Math, among others, discussing this issue was one of the factors that led Susanna Fishel, Pamela Harris, Steph van Willigenburg, and me to organize a Research Community in Algebraic Combinatorics workshop to support women research groups. The workshop was organized with the support of ICERM and all the research activity was done via zoom meetings. Georgia, although she was already leading another group for WINART, decided to also lead a group for the ICERM workshop.

Georgia was an inspiring speaker, and her talks were full of beautiful mathematics. In 2014, she delivered the AWM-AMS Noether Lecture on the topic of the McKay correspondence. A big portion of her later work was related to the McKay correspondence. This correspondence says that the subgroups of SU(2), the group of 2 x 2 unitary matrices with determinant 1, are in bijection with some graphs known as the simply laced affine Dynkin diagrams. The McKay matrix arises as the adjacency matrix of these graphs. In 1985, R. Steinberg generalized McKay matrices to any group using tensor products. Two of Georgia's last publications were with her WINART group. In this work she generalized McKay matrices to finite-dimensional Hopf algebras. Our collaboration, which was ongoing at the time of her death, was on generalizing McKay matrices to finite-dimensional monoids.

At the time of her death, Georgia was leading three all-female research groups, had advised 22 PhD students, and had 134 publications with over 90 collaborators, of which I count myself lucky to be one. She had organized many conferences, workshops, and many AMS meetings as associate secretary of the AMS for the central section; her conference organization work led to the inclusion of more women and minorities as speakers. In numbers her contributions are immensely impressive; however, her most impressive and more difficult to quantify achievement is the impact that she had on many people like me. By meeting her at the beginning of my career at the Connections for Women in Math, it made it feel normal that a woman could achieve such a level of success as a mathematician.

Alejandro Adem

In 1988 I made a trip to Madison to deliver a colloquium talk in the Department of Mathematics at the University of Wisconsin. Being a topologist, I didn't know much about the algebraists there. That changed very quickly after meeting wonderful people like Lou Solomon, Marty Isaacs, and (of course) Georgia Benkart. She was extremely kind to me, and struck me as exactly the kind of person I would like to have as a colleague. One year later I received a tenure track offer from UW–Madison and I remember thinking that it would be a great place to be for a **very algebraic** topologist, and so I became a faculty member there for the next 15 years.

One of the highlights of my time in Madison was being a colleague and friend of Georgia Benkart. She was a world-class algebraist with many celebrated research accomplishments who commanded the respect of her colleagues. At the same time she was kind, modest, and highly supportive of other mathematicians, especially young researchers and students. She was a remarkable lecturer, I remember vividly how amazed I was by the meticulously prepared, entertaining lectures she delivered in her undergraduate courses. Her work on departmental committees was extraordinarily thorough, thoughtful, and impeccably presented. She was a role model for me and I will always be grateful for her advice and support when I was beginning my career. Attaining Georgia's high standards in research, teaching, and service was of course impossible for the rest of us! But she showed us that it could be done, and how to do it.

Georgia was also a dear friend of my family, and her participation in social events was an important ingredient in the wonderful experience we enjoyed while living in Madison. A visiting seminar speaker was typically treated to dinner and then a dessert party, and of course Georgia hosted some of the nicest events (the cakes were also amazing). During the bleak winter months the social glow of colleagues like Georgia helped keep us feeling warm and fuzzy. I also fondly remember a trip we made in January 1995 together with Efim Zelmanov to attend an algebra workshop in Morelia, Mexico, where we escaped from the snow to enjoy excellent mathematics and great sightseeing.

I discovered that she was also a delightful travel companion.

Georgia and I overlapped on the AMS Council for many years, and it was always a pleasure to meet her at the meetings and catch up on news from Madison. Her selfless work on behalf of the AMS was yet another example of her love for mathematics and mathematicians, but equally important was the fact that she was extremely effective, and our entire community benefited enormously from her work.

Georgia Benkart was one of a kind and she will be sorely missed by all who knew her. She was an extraordinary human being and a remarkable mathematician who leaves behind outstanding scientific and human legacies for us to celebrate and cherish.

Hélène Barcelo

It is a great honor to have known Georgia Benkart for more than 25 years. We first met in the early 1990s at various conferences in algebraic combinatorics. During the 1996 year-long program in combinatorics at MSRI, I had the good fortune of beginning a long-lasting friendship with her. Georgia was the epitome of a scholar-academician and showed boundless generosity with her knowledge, time, and friendship. Discussing mathematics with her was a privilege; she never once indicated surprise at my ignorance but rather shared her knowledge with patience and dedication. Finding someone with whom I could be myself without fear of negative judgment was an exceptional experience for me in those days. Despite Georgia's erudition, she had a humble approach to mathematics even in areas where she was a leading figure. Our mathematical conversations remained of this type through to the end.

In 2011, Georgia joined MSRI's Board of Trustees, which allowed us to work alongside one another for more than ten years. Georgia's commitment and diplomatic skills shined throughout her tenure and I came to rely on her invaluable counsel. Even while she was occupied as associate secretary of the American Mathematical Society, Georgia distinguished herself by never missing an MSRI board meeting in all her years of service. Georgia also played an instrumental role on MSRI's Committee on Trustees and Committee on Women in Mathematics, serving as chair of the latter from 2015 to 2020. In particular, she shaped MSRI's support of a collection of Celebratio Mathematica volumes in honor of women mathematicians.

One exciting event Georgia, Estelle Basor, Jill Pipher, Frank Ferris, and I organized was the inaugural American Women in Mathematics (AWM) Research Symposium in March 2013, marking the 40th anniversary of the AWM. With nearly 300 women in attendance, this launched a new series of biennial symposia showcasing the work of women in mathematics. It was a memorable event and a testament to Georgia's passionate commitment to gender equity.

In 2017, Georgia and I participated in the BIRS workshop Algebraic Combinatorixx II with the goal of bringing together women from a wide variety of backgrounds and experiences to strengthen their presence in the mathematical community by catalyzing research collaborations. While the intensity of the program was palpable and energizing, a week is inevitably a very short time to make progress on new research. Toward the end of the workshop, Georgia and I worried that many of the women would return home to heavy professional and personal obligations and that maintaining research momentum would be difficult. We discussed how to enable these research groups to further their collaborations and bring their projects to fruition after leaving BIRS. This marked the inception of MSRI's Summer Research in Mathematics (SRiM) program, which provides the opportunity for two weeks of in-person collaboration to small groups of mathematicians, especially women and gender-expansive individuals who are disproportionately affected by various obstacles.

From its modest beginnings, SRiM has gained remarkable popularity. During the pilot program in summer 2017, 13 women from the BIRS workshop Algebraic Combinatorixx II met at MSRI. The program was formalized in summer 2018 and the number of applications exceeded expectations with 81 women applying from among 22 groups. Since then, SRiM has become increasingly competitive with the number of applications nearly doubling year over year; 291 women applied from among 80 groups in 2020. Georgia's support and advocacy on the Board of Trustees were key to the development of SRiM.

Deeply committed to service and as generous a friend as she was a colleague, Georgia leaves an extraordinary legacy in mathematics which I know will continue to inspire us all. She was a steady presence, and I will forever miss her mathematical insights, friendship, and kindness.

Hélène Barcelo is the deputy director at the Mathematical Sciences Research Institute, Berkeley. She is an emerita professor of mathematics of the school of mathematical and statistical sciences at Arizona State University. Her email address is hbarcelo@msri.org.

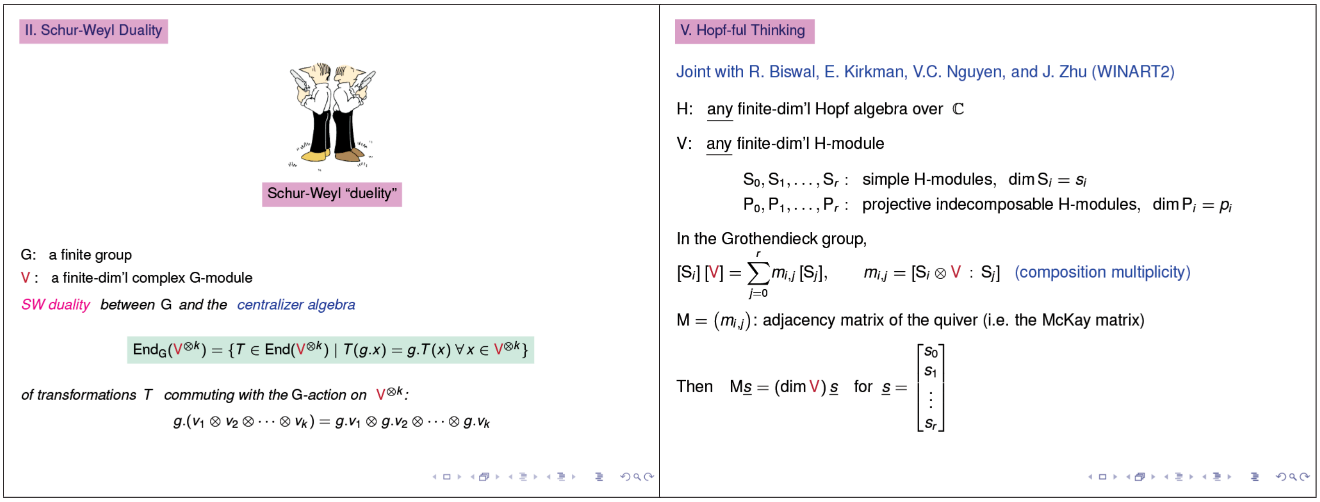

Figure 8. Schur–Weyl duality and Hopf-ful thinking: Slides from Georgia Benkart's talk at AWMs 50th anniversary conference, November 20, 2021.

Daniel Nakano

On February 4th 1992, I met Georgia Benkart when she was a Professor at the University of Wisconsin–Madison and I was an NSF Postdoc at Northwestern University. I invited Georgia to give a talk in our algebra seminar. Little did I know that this invitation would lead to a strong mentoring relationship and profound friendship that lasted over 30 years until her untimely passing.

Georgia was a unifying force in bringing people together and had a wonderful talent for community building. During my postdoctoral years (1991–1995), I had many significant interactions during my visits to Madison. Georgia introduced me to Arun Ram, Dan Britten, Frank Lemire, Tom Gregory, and her PhD student Karl Peters. Karl and I wrote four papers together (with Jon Carlson and Steve Doty). One early wintery Saturday morning, Georgia called us up on the phone. She said that Efim Zelmanov, who had just joined the faculty at Madison, "wants to go out to eat blueberry pancakes." Later that morning, my wife RuthElizabeth and I met them halfway between Chicago and Madison at a quaint pancake joint in southern Wisconsin.

Even with her busy schedule, Georgia was able to find time to visit us twice during my six years at Utah St. University (1995–2001). On one visit to give a research colloquium, my adventurous colleague, LeRoy Beasley, led us on a challenging hike in Logan Canyon to the top of the Crimson Trail (a 4.7 mile hike with a 1345 ft elevation gain). We all know that Georgia gave beautiful talks. She negotiated the hike as smoothly as she presented her colloquium talk.

Daniel Nakano is a distinguished research professor in the mathematics department at the University of Georgia. His email address is nakano@uga.edu.

Georgia had a very close relationship with her older sister, Paula, throughout her life. They were born in the same calendar year, both attended Ohio St. University as undergraduates, and both earned PhDs. Paula moved to Madison to help take care of their mother in the 1980s, and Georgia and Paula lived together since that time. Our daughters, Susanna and Aneesa, were born in Logan, Utah. They are three years apart in age, and had the opportunity to interact with Georgia many times during her visits and at mathematics conferences throughout their childhood. RuthElizabeth and I are convinced that our daughters were divinely inspired by Georgia and Paula's sisterhood. Our daughters shared a bedroom, and participated in the same activities (golf, orchestra, and journalism) in high school. Georgia was very excited when she heard that Aneesa would be attending U.C. Berkeley and joining her sister Susanna who was currently there as an undergraduate.

I moved to the University of Georgia (UGA) in 2001 and was awarded a Distinguished Research Professorship in 2010. On both occasions, Georgia wrote recommendation letters on my behalf. Back in 1977, Georgia received a tenure-track offer from the University of Georgia, and often toyed with the confusion, honor, and puns that could have been generated if she had taken the job.

In 2004, Georgia and I served together on the organizing committee with Brian Parshall, Jens Carsten Jantzen and Zongzhu Lin for the AMS Summer Research Conference in Snowbird, Utah (see Figure 3). This was a high-pressure gig. The AMS provided a large amount of funding for the conference with a clear expectation that the conference should attract the best mathematicians in the field. A conference volume, with the organizing committee serving as editors, was published. In retrospect, Georgia's

Figure 9. A Georgia pun: The state of Georgia signed by Georgia (recreated by Daniel Nakano from a conference memory).

leadership on the organizing committee played a major role in making Snowbird 2004 the landmark international conference in Lie and representation theory in the early 2000s.

Georgia visited UGA five times since 2001 to speak at research conferences, give colloquia and seminar talks, and interact with our algebra group. On one of Georgia's visits to UGA, I was serving as VIGRE director and also co-led our UGA VIGRE Algebra Research Group (which included faculty, postdocs, graduate students, and undergraduates). Our research group published seven papers under the aforementioned name and was described in her President's Report in the May–June 2009 issue of the Association for Women in Mathematics (AWM) newsletter. At this point, I would like to make an addendum to the AWM article. Georgia was one of first pioneers in the modern era who was able to successfully vertically integrate research and education. I drew insight and ideas for designing our research group from Georgia's successful six author collaboration with her five PhD students on the walled Brauer algebra that was published in the *Journal of Algebra* in 1994.

Georgia was a frequent research visitor to Korea and I was on her first flight to and her last flight from Korea. In 1995, we were invited to speak at one of the first representation theory conferences in Korea organized by Seok-Jin Kang (a Yale classmate, and Georgia's future collaborator). In those days, in order to get decent airticket prices to Korea one had to go to a wholesale vendor. RuthElizabeth and Liz Jurisich, who was a postdoc at the University of Chicago at that time, also travelled with us. So we bought four tickets in economy class on an old rickety 747. The experience was surreal: as the plane was descending into Seoul, we were surrounded by a bevy of eager travelers who stood up on their seats to snag their overhead luggage while the airline flight attendants were grabbing them and scolding them to get back into their seats.

Fast forward 20 years later in 2014: I see Georgia at the Incheon International Airport after the Seoul ICM. Georgia had presented the prestigious Noether address at the ICM. She was flying back to the states in first class. I was fortunate on that flight to be bumped up into first class, and was assigned to the seat across from her during the flight. At one point, while we were enjoying our gourmet meal, we looked at each other and almost on cue started to say the same thing: "Remember the first time when we flew to Korea, ... ".

Georgia was a woman with a strong faith conviction. After her magnificent journey of earthly life that has made a tremendous impact on all of us, I truly hope that she is seated in "first class" in her final resting place.

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Credits

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Lynne Heather Walling (1958–2021)

Jay Jorgenson, Thomas Shemanske, and Lejla Smajlović

Introduction

Lynne Heather Walling lived with an enduring passion for community and a profound belief in the importance and value of everyone and everything. Throughout her life she thought deeply about the world of mathematics and mathematicians. When someone encountered unfairness, Lynne felt their struggle, and she had a lifelong quest to address the wrongs she saw.

Lynne Walling passed away on May 28, 2021 at her home in Bristol, England. About five weeks earlier, she had a seemingly innocent health event from which the subsequent medical examinations uncovered that she had aggressive and terminal cancer. In Lynne's final days, she surrounded herself with friends not because of fear or sadness, but rather to show those near her how to accept her circumstance. It is said that a wise person learns from the experience of others, and Lynne was allowing others to understand what she was facing.

This article consists of remembrances from some of Lynne's colleagues with the purpose of describing her contributions to mathematics and the mathematics community. The editors of this article wish to thank Erica Flapan, the Editor-in-Chief of the *Notices of the American Mathematical Society*, for initiating this project. By accepting Erica's request to edit this tribute to Lynne, it is our aim to recognize, and celebrate, the important and unique place in the mathematical world which Lynne occupied.

Lynne was born on October 9, 1958. As we learn from comments below by Lynne's father Stuart, Lynne's early life in Northern California was filled with a sense of community and equity for all. After finishing high school, Lynne attended the University of California at San Diego but left after one year. Two years later, she enrolled in Sonoma State University, at first to study accounting, but then she returned to mathematics. After graduating from Sonoma State University, Lynne began her PhD studies at Dartmouth College in 1982.

While at Dartmouth Lynne's mathematical work focused on Hilbert modular forms, and she completed her dissertation in 1987 under the direction of the second-named editor (T.S.). The contribution by Rainer Schulze-Pillot gives an excellent overview of Lynne's research contributions. As Schulze-Pillot states, Lynne often tackled "the difficult technical questions that arise when one tries to make general results as explicit as possible." This style of "hands-on" work was manifest in all other aspects of Lynne's life. She would sew her own dresses, make the food for her animals, take on home construction projects such as roofing and plumbing, and build sculptures and artwork for herself and her friends. Lynne sought to participate in all aspects of life, and she did so with the traditional "heart, mind, and soul," as well as with her hands.

After Dartmouth, Lynne briefly held faculty positions at St. Olaf College (Northfield, Minnesota) and Bates College (Lewiston, Maine) before moving to the University of Colorado (Boulder, Colorado) in 1990. She took a

Jay Jorgenson is a professor of mathematics at CUNY. His email address is `jjorgenson@ccny.cuny.edu`.

Thomas Shemanske is a professor emeritus of mathematics at Dartmouth College. His email address is `thomas.r.shemanske@dartmouth.edu`.

Lejla Smajlović is a professor of mathematics at the University of Sarajevo. Her email address is `lejla.smajlovic@efsa.unsa.ba`.

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two-year leave from Colorado and worked at the National Science Foundation (Alexandria, Virginia) from 2000 to 2002. In 2007, she accepted a position at the University of Bristol (Bristol, UK) and stayed until her passing in 2021. Various contributors describe their interactions with Lynne at each of these times and places, so we will not duplicate their comments. Rather, we now describe some of Lynne's highly influential efforts regarding issues of inclusion and diversity.

In 1987 Audrey Terras and Dorothy Wallace organized the first *Automorphic Forms Workshop* (AFW). The subsequent series has become an important annual event, and it is most often held in the Western United States. Soon after Lynne moved to Colorado, she emerged as a driving force in AFW. With Lynne's influence, the workshop grew in the manner originally set forth by Audrey and Dorothy, which is to be, primarily, a friendly environment for mathematicians with all different *types* of experience; Lynne disliked the use of the word "level." There were no invited speakers nor featured talks. There was only one allowed distinction between attendees and that was based on a single point: For any specific mathematical question, did you know the answer or not. Egos and all preconceived notions of hierarchy were not tolerated, and Lynne was unwavering when ensuring that the welcoming atmosphere was maintained.

In 1994 Lynne participated in a program in Berkeley, California whose aim was to encourage undergraduate women to attend graduate school in the mathematical sciences. The proceedings of the meeting is published in the book titled *Women in Mathematics: Scaling the Heights*, edited by Deborah Nolan. This wonderful volume is just as significant and relevant today as it was in 1994. Lynne's contribution, titled "Quadratic Reciprocity and Continued Fractions," consists of lectures followed with material intended for small group work. This style of teaching, which seems to stem from Lynne's upbringing, became part of many of the classes she taught in the following years.

In 2002, Lynne attended a conference in honor of Audrey Terras's 60th birthday. It is worth repeating Audrey's statement below, which is that Lynne gave "a talk on an uncomfortable subject: 'Women in Mathematics: Participating, Surviving, and Succeeding.' " For Lynne, the important issue was not that of comfort, hers or anyone else's. Rather, only by openly and honestly discussing the problems of the mathematical community can one attain any meaningful improvement. Without question, Lynne retained and believed in the sense of community and equity she experienced in her childhood.

After Lynne moved to Bristol in 2007, she often said that she missed the environment of the AFW. As a result, she created the Building Bridges (BB) workshop and summer school. To date, there have been four BB conferences, and some of the contributions below describe these truly wonderful events. As with AFW, Lynne continued to promote the notion of community and inclusion for all, and again it was not always comfortable. As a panelist at one meeting, Lynne and an audience member engaged in a somewhat heated disagreement regarding the progress a particular institution was making toward equitable treatment of its members. The exchange was quite surprising, and uncomfortable, for everyone. After the panel discussion, Lynne, the audience member with whom she disagreed, and several others met for dinner, then drinks, and more discussion which went on for hours. For Lynne, the disagreement did not become disagreeable. Lynne was keen to continue the dialogue, because she felt that with such exchanges perhaps we could achieve further and, in her opinion, necessary change.

The fifth Building Bridges conference, BB5, was scheduled to be held in Sarajevo in July 2020, but it was postponed because of COVID-19. In March 2021, the organizers began in earnest to plan for BB5 to be held in Sarajevo in August 2022, and Lynne was leading the way. The summer school teachers were in place, and several grant applications were submitted. Soon it would be time to arrange local accommodations. Many of the participants from past BB conferences had contacted Lynne and asked about BB5, in anticipation of renewed friendship and mathematics. Anyone who spoke with Lynne at that time could sense her happiness and elation as she prepared for her next community event.

But then, all too suddenly, Lynne was gone.

It is often felt upon the passing of a friend or loved one that you did not have a chance to say "Good-bye." Not only is that true with Lynne, but it seems as if we did not have enough time to say "Thank you," so let's do so now. Thank you, Lynne, for sharing your mathematical insight. Thank you for leading the way in seeking a more fair and equitable community and for showing us how to do our part. And to paraphrase what Misha Rudnev wrote in his contribution, thank you for demonstrating that one can face one's own mortality by holding your head high in a manner which can only be described as steadfast and determined.

Thank you Lynne. May the memory of you and the lessons you taught us carry on.

Early Years

Stuart Walling

I believe one of the things that developed Lynne's character was the great variety of her experiences during her first twenty-two years. From age nine through high school, we lived in a large house in Santa Ana, California. This is the house Lynne always felt she "grew up" in—and she did! It was inhabited by me, her father; Anne Daniel, her stepmom; Lynne; her two-years-older brother, Alan; and David, Annie's two-year-old son.

It was the time of Vietnam War protests, peace marches, love-ins in the park, a steady flow of Unitarians in and out of the house. Diverse ideas and opinions flowed freely, with Lynne absorbing it all like a sponge. We did a lot of hiking and camping, including a two-month cross-country trip with a station wagon, tent trailer, five kids, and two alleged adults. Lynne was eleven.

Figure 1. Lynne at age 12 or 13.

We gave her a lot of freedom to try things, but I believe we also instilled a strong sense of personal responsibility. As a 10th grader, her Spanish teacher took a small group to Durango, Mexico, where they lived with local families—a great experience. She went to a girlfriend's fundamentalist church, just to see what it was like. She took a music appreciation class in high school, "just because," and questioned the teacher's opinions on the music's meaning. He gave her a "B"—the only non-A she ever got!

She had a disillusioning freshman year at UCSD; she found it hard to find enough people with whom to talk Math. She drifted for a couple of years—to Washington D.C. where she acquired her first dog, Chuck. She then gravitated to Sonoma County, California, where she eventually enrolled at Sonoma State College, and she loved it!

I truly believe all her experiences, both positive and negative, were a big factor in producing the unique person she was. She will forever be in my heart and mind, and I hope remembered with fondness by all the students she taught over the years.

Stuart Walling is Lynne Walling's father; Donna Janesky is Lynne's stepmother. They live in California. Their email address is janesky40@comcast.net.

Thomas Shemanske

Lynne entered graduate school in 1982, my second year as an assistant professor at Dartmouth. Her first two years consisted mostly of classes and the required oral exams. I still remember bits of her algebra exam. She walked into the classroom and the other examiner and I asked her where she would like to start. She quipped about perhaps classifying all the finite groups. My colleague and I smiled and said that would be an excellent start. We did eventually segue to other topics.

As Lynne was my first PhD student, the mentoring process was evolutionary. We met regularly; she presented results; I asked questions; we explored options to circumvent obstructions and new paths to follow. Finally, perhaps near the end of her fourth year, she had amassed a nice set of results on integral weight Hilbert modular forms arising from theta series and explicit actions of Hecke operators on them. Already, those results were probably more than enough for a thesis, but it was clear that Lynne had a good deal of talent, momentum, and a well-developed tool set, so after telling her those results were very nice, I naturally asked, "What about the half-integral weight case?" I believe there was an emotional response on several levels to that question, and it remained a favorite story for her to tell to students who considered working with me.

Figure 2. Lynne working on her thesis while on a road trip with her parents.

Since we are remembering a number theorist, it is no surprise that the notion of reciprocity played an important role in Lynne's life. Now this is not about quadratic or Artin reciprocity, rather something else. At the end of Lynne's fourth year of graduate school, my wife and I bought a house. This was the 1980s with interest rates in the teens, and we ended up purchasing a definite fixer-upper spending virtually every dime we had. We had seen the house and had it inspected in the winter. After closing in May, we peeked in the attic and saw bright sunshine filtering through a myriad of holes in the roof. Since we were then paupers and roof integrity tends to be a high-priority item, it fell to me to expand my own tool set to include roofing skills. So my comment to Lynne was that if she wanted to talk math with me, it would have to be on my roof. Well, at least that is how Lynne remembered what I said.

Now Lynne had always been a fan of the wood and metal shops at Dartmouth, so there was no question that she would show up. She would grab a bundle of shingles, hoist them over her shoulder, climb up the ladder, and help me roof while talking math. Admittedly a number of my neighbors did inquire "Who is that woman? She works harder than most guys I know."

Years later, now working at Boulder, Lynne bought a house in Longmont; ah yes, you probably see where this is going. She and I were in the midst of a joint paper, so it hardly came as a surprise when she invited me out to work on it during a break. We worked hard that week, but I was not flying out until that Monday, which left the weekend. In unseasonable 90-degree heat we ripped off cedar shingles, redecked the roof with plywood, and covered everything with roofing felt to keep the rain out. That Monday when I left, it was 30 degrees and several inches of snow had fallen overnight. But the roof remained water tight, and Lynne (relying on her multi-faceted training as a graduate student) managed to finish the job that week. Whether she was mentoring any of her own graduate students at that time is unknown.

To return to Lynne's graduate student years, she entered graduate school intent on procuring a job at a good liberal arts institution. Given her strong thesis, I encouraged her to apply for postdocs as well as to some liberal arts institutions, but advisors are only that. She did land a very nice job at St. Olaf. After two years, she was again on the market, this time landing at Bates, another wonderful institution. The next year she was on the market yet again, but in requesting a letter of recommendation, said she was now absolutely sure that mathematical research was her driving force. She loved doing math and wanted to be somewhere where that focus would be paramount. And so she began at Boulder.

Dorothy Wallace

Once Lynne and I got into a friendly wrestling match. I outweighed her by quite a bit, yet she rather easily flipped me upside down. I will never forget this. In fact, everything about Lynne is hard to forget.

Lynne made substantial contributions to the field of modular forms, writing papers with a large number of coauthors. I was not one of these coauthors, and so I will leave mathematical details to others. For me, Lynne was first and foremost a friend, from the moment I met her. She was generous and loyal as well as fiercely protective, especially of her female colleagues.

When she was still a graduate student, an older colleague told Lynne she should stop wasting time and just finish her thesis. Lynne asked him, "Do you make your own dinner? Do you do your own laundry? Do you pack your own lunch? Do you clean your house?" The answer to all of these was no—his wife did all of those things. Lynne then explained that she did all of these chores for herself, and they took time. Lynne was never silent when she perceived an injustice, no matter how slight. And she was usually forgiving, taking personal affronts with grace and an unmatched sense of humor. Years later at a math related party, Lynne was first on the dance floor with this same colleague.

The year Lynne moved to Boulder she gave me several things that would not fit in her car. Foremost was a six-foot-tall stuffed velvet bird built on a sturdy iron base she welded herself. She called it the "lesser known greater emu" because it had the wrong number of toes for an ostrich. It sat over my daughter's crib, then over her bed, then was disassembled (with Lynne's permission) to become a plant trellis. It is now being refurbished as an outdoor sculpture, and will rise again as "Anhinga borealis splendens var. Wallingbird," and be placed in a visible location for anyone to see. As a tribute to Lynne it will be inadequate. But it always has, and always will, remind me of her every time I look at it.

Dorothy Wallace is a professor of mathematics at Dartmouth College. Her email address is Dorothy.I.Wallace@dartmouth.edu.

John Cremona

Lynne Walling was my good friend and colleague for nearly 40 years. We first met at a Halloween party in the Mathematics Department common room at Dartmouth in 1982. I had just arrived from the UK as an instructor, and was not sure how to approach such a distinctively American event, but I need not have worried: in walked Lynne, dressed as a witch, putting me at my ease and making me feel at home.

I have never met a graduate student quite like Lynne. She overshadowed her peers with her enthusiasm, hard work, and achievements. When she asked Tom Shemanske (who later became her advisor) to put on a course in class field theory, he responded by giving her some book titles and telling her to give the course herself. So that year's Dartmouth students, and I, learned CFT out of Janusz's book from lectures by Lynne. She was still, even to this year, regularly ribbing me about the qualifying exam which Tom and I gave her in number theory, which lasted a couple of hours though we knew from the first minutes that she would pass.

As well as working hard at mathematics, Lynne was the social focus of the Dartmouth graduate students. She instituted a weekly breakfast for faculty, cooked by the students in the common room, which ran from 10 a.m. to noon on a weekday. She also regularly turned her office into a barber's shop and cut the hair of many of the other students—and some staff, though in my case she came to my apartment so that the proceedings could be overseen by my baby daughter. Lynne was one of her first babysitters.

Another of Lynne's extra-curricular activities was making models of unusual objects out of satin. I have a cactus made by her, and she once managed to fly to the West Coast to visit her father carrying a life-sized parking meter (made of satin) as a gift. And at a party at her house in rural New Hampshire, she had to warn guests visiting the bathroom that one of the two toilets in there should not be used—it, too, was white satin.

The conferences and workshops which Lynne organized were distinctively different. The "Building Bridges" series which she ran every couple of years after her move to Europe, following a pattern she had set with a similar series in the US, were a bridge between the US and Europe and also a bridge between theoretical work on automorphic forms and more explicit or computational methods. As I saw for myself in the two of these I lectured at (Aachen 2012 and Bristol 2014), these meetings were characterized by a refreshing and stimulating atmosphere, which were truly inclusive and diverse, with participation including inexperienced people as well as big names, those from small colleges and top research institutions, and above all, well-balanced between female and male, among speakers and students. This is just one example of the service which Lynne gave to the community. Another is the quality of her reference letters: I remember one such, written by her about a younger colleague's teaching, as being by far the best teaching reference I have ever read, not in how positive it was (though it was positive), but in the way that it told the reader everything they needed to know about the candidate as a teacher, rather than repeating generalized platitudes.

Lynne and I did not meet in person between my leaving Dartmouth in late 1984 (I last saw her driving away in the snow with my washing machine in the back of a van) and her arrival in Bristol in 2007, where by chance I was visiting for a year. That year gave us the chance to renew our friendship, which continued right up until this year. Whenever I visited Bristol we always met up for dinner, usually at the Lido (she always had scallops and Pedro Ximenes ice cream) so that she could go straight on to the pub quiz next door at the Victoria; but when I was there last week, the place seemed empty and quiet without Lynne. I will miss her.

Years at Boulder and the NSF

Eric Stade

Lynne Walling was captivated by mathematics that's beautiful, and by beauty that's mathematical. Here at the University of Colorado Boulder, we came to expect and to appreciate her exclamations of "Ganz schön!," which she would frequently declare in response to a bit of mathematics that she found particularly beautiful or elegant.

Lynne was equally enthusiastic about sharing this captivation with others. She was driven to impart her passion for mathematics on learners at all levels—students in service courses, math majors, mathematics graduate students, fellow mathematicians.

When Lynne arrived at Boulder, in 1990, the department was in the beginning stages of a large-scale, still-ongoing effort in course and curricular reform, especially at the lower level. Lynne became an active force behind these initiatives, both as a Mathematics Department "private citizen" and in her roles as undergraduate chair and, later, department chair. Culture change requires commitment and energy, and Lynne brought both of these, in substantial amounts, to our reform endeavors.

Through these and a variety of other enterprises—creation of a professional development series for math graduate students, efforts to encourage and support women in mathematics, and so on—Lynne showed an unstinting dedication to helping others learn and appreciate mathematics, and thrive in the mathematical community.

Lynne left our department for greener pastures in 2007, but the impact she has had on us, here at Boulder—particularly on our educational mission—persists. I know she's had similar impacts throughout her journey in this world, and I hope her pastures are even greener in the next one.

Kathy Merrill

Lynne Walling is one of my heroes. She was smart, creative, brave, generous, fun, and unapologetic. Her strong passion for mathematics was matched by an equal passion for living life boldly and on her own terms. No task was

Eric Stade is a professor of mathematics at the University of Colorado, Boulder. His email address is stade@colorado.edu.

Kathy Merrill is a professor emerita of mathematics at Colorado College. Her email address is kmerrill@coloradocollege.edu.

too daunting for her to take on, whether in mathematics, administration, or house reconstruction.

I met Lynne when I spent a sabbatical year at the University of Colorado from 1991 to 1992. I am a harmonic analyst, and was at the time working on questions in ergodic theory. In spite of my lack of proper background, Lynne generously invited me to collaborate with her on a project studying sums of squares over function fields.

Jacobi and Hardy solved the classical problem of counting the number of ways an integer can be represented as a sum of k squares for small k by examining Fourier coefficients of powers of the classical theta function. Lynne proposed using analogous techniques to count the number of ways a polynomial with coefficients in a finite field can be represented as a sum of k squares with restricted degrees. The classical theta function would be replaced by a function field analogue. We carried out Lynne's proposal by first finding an inversion formula, and then transformation formulas under the full modular group for this theta function. Using the inversion formula to move to a fundamental domain, we were able to find recursive formulas for the Fourier coefficients. This led us to the representation numbers in closed form for polynomials of small degree. The other representation numbers that came out of the recursive relationships involved sums of Kloosterman sums, and for these we could only provide asymptotics [MW93]. In joint work with Jeffrey Hoffstein [HMW99], we were later able to use Poincaré series to find elementary formulas for all of the representation numbers.

This is just a glimpse of a small bit of Lynne Walling's mathematics. Other contributors will discuss more of her impressive body of work. But even beyond her many papers and talks, Lynne substantially reshaped the mathematics community by enabling so many nontraditional mathematicians to succeed. She did this through friendship and teaching, as well as formal outreach efforts from her leadership positions.

Lynne's own mathematical career had a nontraditional start: she dropped out of college, worked some low-paying jobs, then returned to get an accounting degree, before graduate school at Dartmouth. In addition, she faced the onslaught of small slights and unnecessary obstacles that are all too often thrown at women mathematicians. Lynne survived because she was talented, worked hard, and loved mathematics. She knew the importance of showing toughness and confidence, something I needed to learn from her.

Teaching was central to Lynne's work, and her approach reflected her own experiences. She knew that potential mathematicians benefit from the chance to develop mathematics on their own early on. Nontraditional students, in particular, thrive under conditions of challenge and support rather than either coddling or cutthroat competition. Based on this, she developed discovery activities for all levels of her classes, which the students could work on either individually or in groups, with restrained help from the instructor. Lynne shared these activities widely with colleagues, and constantly revised them based on their success in the classroom.

Lynne built community with both senior women mathematicians like Audrey Terras, and women just starting their careers. She offered the latter advice based on her own experiences, as well as concrete opportunities to give talks, write papers, and apply for grants. When she held leadership positions, such as department chair and NSF Program Officer, Lynne used her power to make sure women received fair evaluations, and also to establish programs that supported young mathematicians in general.

What I have written, while true, is much too serious to capture Lynne Walling. When I think of her, the image that comes first is playful and fun. I still have earrings she made for friends out of matchbox cars. When we were together and not doing math, we drank too much scotch and red wine. She helped my son learn to drive, by telling him to make a revving noise like **vrrroooommm!** when letting out the clutch so that he wouldn't stall. This is the Lynne I will never forget.

Andrew Pollington

I first met Lynne Walling in 1991. BYU had organized a special year of research in number theory for the academic year 1990–1991, and Lynne was invited to give a talk in our seminar. The University of Colorado was, at the time, the strongest number theory department in the Mountain West and Lynne was one of its young stars. Lynne arrived in Provo and rapidly became an integral part of the special year, both giving talks to the number theory group and taking an interest in the ambitions and research of the BYU students. Her enthusiasm for Mathematics and joy for life were infectious. That year we were doing some remodeling of our house and we soon discovered another side to her talents. She volunteered to assist, and was soon in our basement, sledgehammer in hand, removing sheetrock and later helping to restore studs and put up the new walls.

One aspect of the special year was a week-long conference attended by experts in Diophantine approximation and analytic number theory. At the end of the meeting,

Andrew Pollington is a program director at NSF (Algebra and Number Theory). His email address is adpollin@nsf.gov.

These opinions are my own and do not in any way reflect the opinion of the National Science Foundation.

those who had not yet had to leave attended a dinner at a bistro in Salt Lake City. Lynne came to the dinner equipped with a bag of toys, including a radio-controlled car which was raced up-and-down the table.

This was an eventful year for me and was certainly enhanced by my now firm friendship with Lynne. Over the next few years, we would see each other a few times at conferences or when I traveled to Colorado to give talks. In 1999, I was invited to become an NSF rotator, a program officer with main portfolio in number theory. At the end of that year, I needed to find a replacement, and thought of Lynne. With the agreement of Philippe Tondeur, the division director at the time, we invited her to come to NSF for an interview for the position. The day before the interview I received a call from Lynne telling me, "I have fallen off a ladder and have broken my arm ... but I am still coming." She duly showed up, arm in plaster, on time for the interview. In September 2000 Lynne replaced me as a program officer at NSF. During her tenure we met a few times at conferences, such as a meeting NSF funded and organized at IAS, on the Langlands program. We also had many long discussions by phone concerning various aspects of the job of a program officer. Lynne spent two years as a program officer at NSF, and brought her encouraging persona to the position and did a lot to encourage underrepresented groups in mathematics.

I took a year-long leave from BYU to spend 2006 in Bristol working at the Heilbronn Institute. I had already had a number of connections to researchers in Bristol and continued to visit the University and the Heilbronn Institute for the next few years. This meant I was in Bristol when Lynne started her position at the University of Bristol. We would often meet in the department during subsequent summer visits.

Lynne had always been very active in conference and workshop organization and she and several colleagues in Europe and the US started a series of workshops, "Building Bridges," that brought together new mathematicians and mathematics students to learn more in depth about a chosen research topic. Lynne (whose idea this was) provided leadership and active participation. These were supported by several funding organizations including NSF. On the US side the NSF PIs have been: Olav Richter, Jennifer Beineke, Jay Jorgenson, and Jim Brown. These are just a few of those involved in these workshops. A meeting was scheduled for summer 2020, but was curtailed by COVID and has been postponed.

I last saw Lynne the summer of 2019. We worked clearing weeds in her garden, and talking about mathematics. Her death came as a great shock to me and I will certainly miss her. She was a real star, mathematically and through her encouragement of others to take up and participate in the joy of mathematics.

Years at the University of Bristol

Jonathan Keating

Lynne Walling moved to the School of Mathematics at the University of Bristol in 2007, establishing a new avenue of research in automorphic forms, theta series, and Hecke operators. She was highly active in the international community in these areas, being closely involved in raising funding for, and organizing, several major workshops and postgraduate summer schools. Lynne had previously been a program officer at the NSF and she wrote the best end-of-grant reports I have encountered.

Lynne was a truly outstanding teacher, introducing several new courses and being a key contributor to redesigning the curriculum. She was extremely popular with the students, who could sense her commitment and dedication to their education.

Lynne was also willing to take on leadership roles. She was for several years the head of the Pure Mathematics Group in Bristol. Her fierce protectiveness of, and loyalty to, members of the group were legendary. She also went out of her way to befriend and mentor new faculty members. Lynne felt passionately about issues relating to women in mathematics. She was a catalyst for attracting excellent women mathematicians to Bristol and she influenced many people's thinking on this, including my own.

Finally, Lynne was a distinctive character in the School. She was passionate in debate and unflinching in challenging policies she disagreed with (especially those coming from outside mathematics). She was also kind, generous, funny, and sociable (she rarely missed a post-seminar dinner), nurturing many deep friendships. It says a great deal that, when the end came, she was cared for, and finally laid to rest, by her mathematical friends.

Jonathan Robbins

When Lynne arrived in Bristol in 2007, it was like a breath of fresh air, or rather, a gale. She was fearless in speaking up for what she believed in, and was especially supportive of junior colleagues, delighting in their successes.

Jonathan Keating, FRS, is a professor of mathematics at the University of Oxford. His email address is `Jon.Keating@maths.ox.ac.uk`.

Jonathan Robbins is a professor of mathematics at the University of Bristol. His email address is `J.Robbins@bristol.ac.uk`.

Lynne and I had many occasions to work together when I became head of school and she became director of the Institute of Pure Mathematics. For her, it was a job she had done before (and she had been chair at Boulder, of course). I was the newbie. As a caricature of our relationship, I would cast myself as a newly licensed driver, nervously gripping the steering wheel of a vehicle of unaccustomed size. Lynne, sitting in the seat next to me, is, by turns, a wonderfully irreverent and distracting raconteur, a would-be joyrider urging "faster, faster," and a somewhat cross driving instructor informing me that over the last mile I would have failed my driving test. In reality, she gave me much wise counsel and encouragement, and I will always be grateful.

Mathematical Contribution

Rainer Schulze-Pillot

I met Lynne first when she was about to finish her doctoral thesis and was impressed by her energy, temperament, and enthusiasm. Over the years we met many times at various places in the world and this impression never changed. A central theme in Lynne's research activity was the theory of integral quadratic forms and their theta series, in particular the difficult technical questions that arise when one tries to make general results as explicit as possible. To try and make the readers feel some of the fascination that this classical but indeed quite technical subject had for Lynne (and also for me) let me give a sketch of the problems and tools.

For a positive definite symmetric $m\times m$ matrix A with integral entries consider the quadratic form $Q_A(x_1,\dots,x_m) = \sum_{i,j} a_{ij}x_ix_j$ in the integral variables $x_1,\dots,x_m$. Call two such forms Q_A, Q_B integrally equivalent if $B = {}^tUAU$ holds for an integral matrix U of determinant ± 1, and say they are in the same genus if $B \equiv {}^tU_{p,r}AU_{p,r} \bmod p^r$ can be solved for all primes p and all positive integers r with matrices $U_{p,r}$ as above. It is known that a genus consists of finitely many integral equivalence classes. If $r_A(n)$ denotes the number of integral vectors $\mathbf{x} = (x_1,\dots,x_m)$ satisfying $Q_A(\mathbf{x}) = n$, the theta series $\vartheta_A(z) = \sum_{n=0}^\infty r_A(n)\exp(2\pi i n z)$ defines a holomorphic function of the complex variable z in the upper half plane. It turns out to be a modular form of weight $k = m/2$ for a subgroup of the modular group $SL_2(\mathbf{Z})$ defined by congruences modulo an integer N (the level of the group) dividing $4\det(A)$, i.e., for all matrices $\left(\begin{smallmatrix} a & b\\ c & d\end{smallmatrix}\right)$ in that group one has $\vartheta_A((az+b)/(cz+d)) = (cz+d)^k\vartheta_A(z)$. Modular forms have deep connections to many problems of number theory and arithmetic geometry; in our case their analytic properties can be used to reveal information about the representation numbers $r_A(n)$. Consider for simplicity the space $M_k = M_k(SL_2(\mathbf{Z}))$ of modular forms of weight $k > 2$ for the full modular group $SL_2(\mathbf{Z})$; the theta series ϑ_A belongs to this space if the matrix A has even diagonal entries and determinant 1. One has a splitting $M_k = \mathbf{C}E_k \oplus S_k$, where $E_k(z) = \sum_{(c,d)\in\mathbf{Z}^2\setminus\{(0,0)\}}(cz+d)^{-k}$ is called the Eisenstein series of weight k and where $f(z) = \sum_{n=0}^\infty a_n\exp(2\pi i n z) \in M_k$ belongs to the space S_k of cusp forms if $a_0 = 0$ holds. By a celebrated theorem proven by C. L. Siegel in 1935 a suitably weighted average over a set of representatives $Q_{A'}$ of the integral equivalence classes in the genus of Q_A of the theta series $\vartheta_{A'}$ (called the genus theta series $\vartheta_{\mathrm{gen}A}$) equals E_k. In particular it is easy to calculate explicitly its Fourier coefficients, i.e., the weighted averages $r_{\mathrm{gen}A}(n)$ of the representation numbers $r_{A'}(n)$. Moreover, $r_{\mathrm{gen}A}(n)$ factors into a product over all primes p of p-adic densities $\alpha_p(A,n)$ which measure numbers of congruence solutions modulo powers of p. On the other hand, the differences $r_A(n) - r_{\mathrm{gen}A}(n)$ are the Fourier coefficients of a cusp form and hence by a theorem of Deligne, previously called the Ramanujan–Petersson conjecture, of slow growth for $n \to \infty$, which gives an asymptotic formula for the representation numbers $r_A(n)$. Analogous results with more complicated formulation hold for general A.

A further generalization due to Siegel, again with analogous results, involves the number $r_A(T)$ of solutions in integral $m\times g$ matrices X of the matrix equation ${}^tXAX = T$ for a $g\times g$ matrix T. The $r_A(T)$ give rise to a holomorphic function of several complex variables, called the Siegel theta series of degree g of A; it is a modular form for a group of integral symplectic $2g\times 2g$ matrices. Finally, one may allow the vectors and matrices above to have coefficients in the ring of integers of a totally real extension field of finite degree of the rationals and obtain Hilbert resp. Hilbert–Siegel modular forms. One of the arising complications in all these more general situations is that one has to deal in them with more than one Eisenstein series of given weight, and that it is difficult to express the genus theta series explicitly as a linear combination of basic Eisenstein series.

On all these spaces of modular forms one has for each prime p not dividing the level of the group a commutative algebra of linear operators, the Hecke algebra $\mathcal{H}_p$. On the space of cusp forms this algebra can easily be diagonalized with the help of a natural scalar product on the space; on spaces of Eisenstein series the diagonalization is a difficult technical problem. The deep connections

Rainer Schulze-Pillot is a retired professor of mathematics at the Universitat des Saarlandes. His email address is `schulzep@math.uni-sb.de`.

between the eigenvalues of such Hecke operators on modular forms and their generalizations on one hand and eigenvalues occurring in arithmetic geometry on the other hand are the subject of the famous Langlands program. In our context, an approach originating in work of Eichler, Andrianov, and Freitag is the study of the action of Hecke operators on theta series. In many cases, the space generated by theta series is invariant under the Hecke operators. Expressing $T(\vartheta_A)$ for an operator T explicitly as a linear combination of theta series one can then prove multiplicativity properties of the average representation numbers $r_{\mathrm{gen}A}$ as well as Siegel's theorem in a purely algebraic and very elegant way [Wal97]. Many of Lynne's papers belong to this circle of ideas, starting with her thesis [Wal87, Wal90] where she studied the explicit action of Hecke operators in the case of Hilbert modular theta series. In later papers, some of them with coauthors, she derived explicit formulas for this action in the cases of Siegel [Wal06a, Wal08, Wal19] and of Hilbert–Siegel theta series [FW21], considered applications to representation numbers of quadratic forms [MW93, HW99, HMW99], computed eigenvectors and eigenvalues of the Hecke algebra in spaces of Eisenstein series [Wal17a, Wal17b], and proved various auxiliary results which are also of independent interest for the various types of modular forms involved; see, e.g., [SW93b, SW93a, SW95, HW02, Wal06b, CW07, Wal13, Wal17b]. In one of her last papers she solved the abovementioned problem to find an explicit expression for the genus Siegel theta series as a linear combination of basic Eisenstein series [Wal18]. The problem had been around unsolved ever since Siegel's groundbreaking papers from the 1930s, and she was particularly proud of this achievement. With all these results, Lynne left a mark in this favorite subject of hers.

Solomon Friedberg

I first met Lynne when we were undergraduates at UC San Diego, and in later years when we crossed paths we always had that wonderful sense of seeing someone you've known and been friends with for a long time. We really got to know each other after she received her PhD from Dartmouth with Tom Shemanske. Our connection was a shared interest in theta series. These series provide a systematic way to construct automorphic forms out of certain geometric or algebraic information. Lynne's work on theta series began in her 1987 thesis "Theta Series Attached to Lattices of Arbitrary Rank" [Wal87] and her interest in them extended throughout her career—the latest paper [FW21], with Dan Fretwell, is "Hecke operators on Hilbert–Siegel theta series." Since these same functions appeared in my thesis, Lynne and I had a natural mathematical connection.

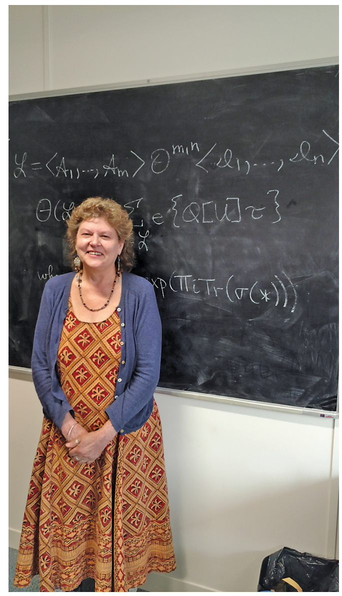

Figure 3. Dr. Lynne Walling at Inspiring women at Bristol University | International Women's Day 2020.

Theta series can be studied either as functions (following Eichler and Siegel, among others) or representation-theoretically (following Weil and Howe, among others). Lynne's approach was function-theoretic. Since I had a foot in both areas, as our careers progressed I was pleased to talk with Lynne, occasionally as a sort of an interpreter and more often simply as a scholar who respected and found value in both points of view. We also both cared about students and teaching, and valued a certain openness to mathematicians who contributed in different ways. With these commonalities and Lynne's energy, our meetings often led to intense discussions, about both math and about the profession.

Lynne was not only an engaged researcher who cared a great deal about mathematics, she was also a tremendous force in organizing events. Dorothy Wallace had begun an automorphic forms workshop in the 1980s that was specifically welcoming to scholars studying automorphic forms from all points of view, and Lynne soon became involved in the organization of the series. In the 1990s, I attended and spoke at three of these conferences that Lynne organized or co-organized, and she was also kind enough to invite me to Boulder to give lectures on several occasions. It was fun to catch up with her; she would always have a great deal on her plate but find time to introduce me to a graduate student or postdoc. After she moved to England, Lynne helped create and organize a new series, the EU/US Summer School on Automorphic Forms and Related Topics. At the second of these summer schools, in 2014, I had the opportunity to co-teach a mini-course with Jim Cogdell, and with Ameya Pitale as assistant, on the Langlands Program. Lynne and her co-organizer Jennifer Beineke were very particular about the mini-course. They wanted to be sure that it was at the right level for the participants. I appreciated the guidance a great deal.

Solomon Friedberg is a professor (McIntyre Endowed Chair) of mathematics at Boston College. His email address is `friedber@bc.edu`.

The theta functions that Lynne studied come in many flavors that are different; for example, there can be different underlying discrete objects that one sums over, and geometric configurations of different dimensions. Lynne showed that one may systematically understand spaces of theta functions attached to lattices by Hecke theory, and this understanding has ramifications.

For example, in her paper [Wal97] and in her paper [HW99] with Jim Hafner (of blessed memory), she gave an elegant new proof of Siegel's famous mass formula for an integral quadratic form, computing the representation densities explicitly, by combining Hecke theory for Siegel theta functions with computations of Eisenstein series. In many other works, she showed that theta functions behaved in specific, understandable ways, from extending the Eichler commutation relation to higher degree Siegel theta series to using them (in joint work [HMW99] with Hoffstein and Merrill) to count sums of squares over function fields. Her research, spanning more than three decades, constitutes a lasting contribution to the field.

Lynne's love of mathematics and her dedication to scholarship were an inspiration for a generation of students, and she was a role model for a generation of female scholars, leaders, and teachers. I was privileged to be one of her many math friends, one with a particularly long connection. Lynne was a one-of-a-kind person, deeply caring, a committed teacher, full of energy and good cheer, an excellent scholar, and profoundly dedicated to mathematics. Her passing is a loss for our profession and, for all of us who knew her, a personal loss as well.

Jeffrey Hoffstein

I first met Lynne Walling during a trip to Boulder in the early 90s and liked her immediately. We didn't engage mathematically until the special year on automorphic forms at MSRI in 1994–1995. I knew that Lynne liked thinking about sums of squares, and I had recently been spending some time working over the rational function field, and knew that there were no cusp forms when the level was 1. We ended up computing the spectral expansion of powers of the function field theta function, and using this to find exact formulas for the number of representations of an element as sums of a given number of squares. We could get exact formulas because in this context there are no cusp forms, so the expansion is very simple. The trouble was, I was supposed to be in charge of making sure the MSRI program ran smoothly, so I had a lot of trouble focusing. Luckily, this was no problem for Lynne! She just drove me like a merciless tyrant until the project was done. And it was fun being driven by this particular merciless tyrant. She loved what she was doing and was an irresistible driving force. I wish I had seen her more over the years and miss her tremendously.

Jeffrey Hoffstein is a professor of mathematics at Brown University. His email address is JHoff@brown.edu.

Larry Gerstein

I first met Lynne in the fall of 1985, when I was a visitor at Dartmouth College and had the opportunity to teach a graduate course on the theory of quadratic forms. Lynne was in the class, and her talent and intensity were immediately evident. At the time she was hard at work on her dissertation research under Tom Shemanske, under whom she had already been introduced to the basics of quadratic forms.

The focus of her research was on quadratic forms over algebraic number fields, particularly on their theta series: functions that encode the representation data of those forms. This was all new to me, and so I was astonished and delighted to discover that from time to time I could tell her something about the mechanics of lattices over rings of algebraic integers that she found useful. Lynne had a particular interest in connections between lattices and their duals, and as I look back over her career I see that this interest continued to bubble up.

Over the years she showed herself to be a valued collaborator with many others on a broad range of topics, for instance the action of Hecke operators on Siegel modular forms, but she retained her interest in quadratic forms. I am particularly struck by her applying her analytic techniques to the context of quadratic forms over global function fields. Just to give one example, I cite her demonstration (with her collaborators) that every polynomial over a finite field can be represented as a sum of four squares, and going on to count the number of ways this can be done by summands of bounded degree.

Though her move to England limited our interaction, I did see her at occasional conferences, for instance at Oberwolfach, and we continued to exchange emails on mathematical mysteries of interest to both of us. She never lost her sparkle, and I'm saddened by her death.

Larry Gerstein is a professor emeritus of mathematics at the University of California, Santa Barbara. His email address is gerstein@math.ucsb.edu.

Passionate and Supportive Outreach

Audrey Terras

Probably I first met Lynne when she was a graduate student at Dartmouth and subsequently I attended many of the meetings she organized. In fact, many of them were continuations of a meeting that my student Dorothy Wallace organized at MSRI in April, 1987. The name of the conference ultimately became Workshop on Automorphic Forms and Related Topics. It still exists. See the website for the 34th meeting. Lynne kept the spirit of these meetings which was friendly and open to all. The meetings were distinguished by having a large number of women speakers, something that was not true of meetings on the subject when I was a young mathematician.

She came to my 60th birthday party conference and gave a talk on an uncomfortable subject "Women in Mathematics: Participating, Surviving, and Succeeding." It is on her website. I recommend it. The talk includes 11 ways harassment and discrimination are manifested. The most all-encompassing is "perpetual condescension." Then it lists 23 survival strategies. My favorite is "prove theorems." Finally it lists 11 suggestions for the math community to improve matters. The talk should be applied not just to women but to everyone on the receiving end of discrimination, particularly minority mathematicians.

It would be an example of abuse not to mention that Lynne's mathematical work was indeed impressive. She was not afraid to work on higher-rank automorphic forms, a subject that is not for the faint of heart. Her explicit Siegel theory papers give a new and completely explicit version of Siegel's important formula for the weighted average of the representation numbers over isometry classes of lattices in the genus of L. The joint work with Jeff Hoffstein and Kathy Merrill on function field analogs of automorphic forms showed that, as in the classical case, the main term of the number of representations of a number as a sum of k squares of degrees $< m$, is given by the Fourier coefficient of an Eisenstein series. This implies that the representation number is nonzero for m sufficiently large. Earlier formulas were not easily seen to be nonzero. Serre had only proved the non-vanishing in the case $k = 3$.

Audrey Terras is a professor emerita of mathematics at the University of California, San Diego. Her email address is aterras@ucsd.edu.

Jennifer Beineke

Lynne was a dear friend—funny, caring, and a good listener. Whenever I visited the UK in recent years, we would have fun reconnecting, usually meeting up at a train station. I'd find her waiting for me in a corner, reading a mystery (both of us being lovers of detective fiction). After our day together, she would send me off with a bag of books she had finished, for the rest of my trip.

I first met Lynne in 1997 at the 11th Annual Workshop on Automorphic Forms and Related Topics (AFW), which she organized at the University of Colorado Boulder. The conference was advertised as "small and friendly." As a nervous graduate student, I thought it would be a good opportunity for presenting my research, and the workshop lived up to its promise. Lynne was a force to behold as organizer, not only running everything adeptly, but taking the junior researchers under her wing, making sure they interacted meaningfully with the senior mathematicians. I was amazed when she took the time to speak with me and introduce me to others to discuss research. She also invited me to call her in the future if I had questions or concerns. That was the beginning of our friendship.

Before she left for England, Lynne organized five more of these annual workshops, continuing to mentor the junior members of the number theory research community. In 2003, she introduced panel discussions into the meeting format, on topics such as research collaborations, applying for grants, publishing, and encouraging and retaining women and other underrepresented groups. When Lynne moved to Bristol in 2007, it was hard for her to say good-bye to the AFW. Then, at the 2010 conference "Durham Days on Modular Forms," organized by Jens Funke, I remember her moment of inspiration. We were sitting over pizza and wine, bemoaning the lack of an AFW in Europe, when it clicked—Why not start one? In her inimitable Lynne way, she made it happen, and the "Building Bridges: EU/US Summer School + Workshop on Automorphic Forms and Related Topics" was born.

Collaborating with Lynne on Building Bridges, I saw that she didn't just want to recreate the AFW in Europe—she wanted to foster new connections. She created partnerships across continents, finding funding from agencies in both the United States and Europe to support international graduate students and junior faculty. She created new connections between advanced researchers by stipulating that each mini-course be led by two specialists who had never collaborated. This arrangement also provided the students with different perspectives on a topic. Lynne

Jennifer Beineke is a professor of mathematics at Western New England University. Her email address is jbeineke@wne.edu.

added panels, poster sessions, and "speed talks" to the meetings. And, of course, the social events! Lynne was the life of the party.

At the 2014 Building Bridges workshop in Bristol, ahead of current trends, Lynne wanted to bring the lack of diversity in mathematics to everyone's attention. For the panel that year, we therefore chose the topic "Encouraging and supporting diversity in the mathematics community." We had hoped for a mix of panelists from the United States and Europe, but could only find one European participant—Nils Skoruppa—willing to serve. The other five were all American. Though the resulting discussion was tentative, it was a start, and thanks to Lynne's perseverance, the conversation has continued.

Always nearest to Lynne's heart, however, were the opportunities she created for junior mathematicians and women. At the workshops, many women (including myself) found we had been given more time to speak than we had requested. Lynne pointed out that women never asked for the longer slots—but men did. I also learned, behind the scenes, that the friendly, supportive atmosphere was due in no small part to Lynne keeping a close eye on the senior mathematicians. If one of them said something at the meeting that could have been viewed as disparaging or critical, she immediately took that person out to the hall to "have a word." With a fierce passion, Lynne made sure everyone was treated with kindness and respect.

Like many others, I owe an enormous debt of gratitude to Lynne and miss her deeply. When reflecting on our time together, I have found some solace in watching her online talks: "Women in Mathematics: Ambition in an ambivalent society" (2015, https://www.youtube.com/watch?v=-FQWpfZIPT8) and her University of Bristol's Best of Bristol Lecture, "The art and beauty of pure mathematics" (2016, https://www.youtube.com/watch?v=XoKIfCgiA_w). Lynne found inspiration in music, and enjoyed quoting one of her favorite singers, Ani de Franco, in some talks. To honor Lynne's memory, I hope each of us will continue to mentor and advocate for others the way she did, and, in the words of de Franco, be "willing to fight."

Samuele Anni and Jim Brown

Most mathematicians in the United States that work in the area of automorphic forms are aware of the annual Automorphic Forms Workshop (AFW). This is a long running and well-established workshop; the 34th AFW will be meeting in Spring 2022. Lynne was an active organizer and participant in the AFW while she was a faculty member in the United States and was known for promoting early career researchers and student involvement in the AFW. Lynne was particularly focused on increasing female participation both in attendance and giving talks. In 2017 Lynne relocated from the University of Colorado at Boulder to the University of Bristol in the United Kingdom. This relocation was the starting point for Building Bridges: EU/US Summer School and Workshop on Automorphic Forms and Related Topics (BB). Lynne saw the benefit of the AFW to researchers in the US, particularly those early in their careers, and felt the model could be beneficial to researchers in the EU as well. Moreover, rather than just having a conference to bring together researchers, a summer school was included. Automorphic forms can be a difficult research area to break into for new researchers; the school was designed to allow a smoother transition for students with a desire to work in this area. "Building Bridges" refers to building stable connections, bridges, between automorphic forms research communities in Europe and the US, but it stands for more. It also refers to Lynne's idea to "bridge" two instructors that had never worked together before for each summer school session and foster further collaborations among more senior researchers. It is also an occasion for seeding the future: PhD students have the chance of building connections, bridges for their tomorrow, and Lynne was always a strong support toward these achievements.

Figure 4. Building Bridges 2. Conference photo

In addition to stellar mathematics, Lynne was instrumental in ensuring that BB included panel discussions and presentations on important topics such as increasing diversity in mathematics. Recently "speed talks" have been introduced to promote a fun and positive atmosphere for both first-time speakers and more seasoned ones.

Various prizes, selected personally by Lynne, were given out after each session of speed talks to keep it light and fun. BB is a testament to Lynne's impact on the mathematics

Samuele Anni is the maître de conférences at Aix-Marseille Université. His email address is samuele.anni@gmail.com.

Jim Brown is professor of mathematics at Occidental College. His email address is jimlb@oxy.edu.

community, indeed during and after these events she was promoting and coaching early career researchers.

The first BB took place in 2012 at RWTH Aachen University in Germany and was co-organized by Lynne, Aloys Krieg, Martin Raum, and Olav Richter. The second BB was held in 2014 at the University of Bristol and was organized by Lynne, Jennifer Beineke, and Jonathan Bober; the third at the University of Sarajevo organized by Lynne, Jay Jorgenson, and Lejla Smajlović; the fourth at the Alfred Renyi Institute of Mathematics in Budapest organized by Lynne, Jim Brown, Gergely Harcos, Jay Jorgenson, and Árpád Tóth. Plans for the fifth BB were in place to be held at the University of Sarajevo in 2020 before the school and workshop were forced to be postponed because of the COVID-19 epidemic. While Lynne was always the driving force behind BB, the current organizers (Samuele Anni, Jim Brown, Jay Jorgenson, Almasa Odžak, and Lejla Smajlović) are determined to continue to realize Lynne's vision for an inclusive and welcoming school and workshop in automorphic forms for years to come. We currently plan to hold BB5 at the University of Sarajevo in the summer of 2022. While still in the process of finalizing plans for BB5, we will certainly include a tribute to Lynne's contributions to the mathematical community. We hope to see a large turnout of the automorphic forms community.

Personal Remembrances

Jeffrey Stopple

I first met Lynne at one of the very earliest Workshop on Automorphic Forms conferences, maybe 1988 or 1989. She called me afterward and informed me we were going to be friends, such a very "Lynne thing to do." Before we knew it, we were organizing the workshops ourselves in Santa Barbara and in Boulder. In those days invitations went out by snail mail and we had to look up addresses one at a time in the printed AMS Membership Directory. Lynne was from the beginning adamant that the workshop be friendly and inclusive, and the language about that on the earliest Workshop website is still copied and pasted from year to year. We even discussed the idea that maybe the name was too restrictive—maybe it should just be the "Workshop on ... and Related Topics." I still have in front of me the very first ceramic mug, from the 6th Annual in Boulder in 1992—Lynne discovered ceramic mugs could be custom printed in China cheaply, but the minimum order was a case, vastly more than we needed. I've been "gifting" excess workshop coffee mugs ever since.

I was always impressed by Lynne's independence and self-reliance. While teaching at St. Olaf in Minnesota, Lynne lived on a rural property with no indoor plumbing. When the outhouse was "full," she described to me the process of digging a new pit with a backhoe, and dragging the old outhouse over to sit above it.

Lynne was my best friend for over two decades, but we drifted apart when she moved to Bristol and I got married. I can't believe she's gone. She will be missed.

Donna Janesky

Figure 5. Lynne painting pottery from `https://www.lynnewalling.org/`.

My stepdaughter, Lynne Walling, was a truly unique individual. She had a true love of math, perhaps influenced by her father, Stuart, who had a lengthy career teaching math. But in addition to her math prowess, she was an expert in many other areas, and this expertise often came to her through self-learning. They say necessity is the mother of invention, and this was certainly true for Lynne. Need a new roof? A new dress, fence, garden, plumbing, iron bed? Lynne could do it. She always availed herself of the opportunities to learn new things, and quickly put her knowledge to good use. Those of you who knew Lynne are aware of her unique sense of style, fashioning her own clothes and surrounding herself with unique furnishings.

Lynne surrounded herself with plants and animals wherever she lived, planting lovely flower and vegetable gardens where her dog and cats could roam. And yes, even some chickens. "Au Naturale" comes to mind when remembering Lynne. There was no pretense or "artificial ingredient" injected into her joyous lifestyle. She truly loved what she did and spoke glowingly of her own mentors. She herself became a mentor and an example, especially to women entering this difficult field for women. Lynne gave 100% to her students, colleagues, family, and friends. She was truly one of a kind, and we were blessed to have her in our family.

Jeffrey Stopple is a professor of mathematics at the University of California, Santa Barbara. His email address is `stopple@math.uscb.edu`.

Donna Janesky is Lynne Walling's stepmother. Her email address is `janesky40@comcast.net`.

Suzanne Caulk

Lynne Walling has said "As mathematicians we look at things differently, which means that we can produce things that other people aren't going to, and that's exciting".[1] Well, I would say, that Lynne looked at things differently than anyone I have ever met and that meant she did things that other people wouldn't do and she was definitely exciting and fun to be around.

Lynne's mathematics was focused on the beauty of the objects she studied, and she effectively conveyed that to her students. She was a creative person who found beauty in many things, and expressed herself in a variety of ways. When she took me on as her PhD student, I didn't realize that together we would learn to tile her new bathroom, but we did!

One of the things Lynne was passionate about was diversity and justice within the mathematical and academic communities. This meant she took on leadership positions that required a lot of hard work because she believed this was how she could best help others and improve her community. Lynne's efforts will live on in all of us who she mentored over the years.

YoungJu Choie

Lynne left a lasting impression on me as a colleague and a friend with her strong but charming charisma since I met her for the first time at the special year program on Automorphic forms at MSRI, Berkeley, USA in 1994. In our generation, there were not many female mathematicians working on Automorphic forms, so that we naturally shared a looming anxiety, an uncertain, sometimes fearful future as inexperienced mathematicians at the early-career stage at that time.

Light drinking parties led by her with like-minded friends were always lively and enjoyable, full of life with her loud chatter. No one could stop her presence and charming charisma at all occasions. I am particularly grateful for her serving as the keynote speaker at the Memorial Workshop of the Foundation of Korean Women in Mathematical Sciences in 2005, visiting me and Korea several times afterward. I believe that even now Lynne may be somewhere talking about mathematics and other matters with her charismatic voice.

Suzanne Caulk was a professor of mathematics at Regis University; her current affiliation is unknown. Her email address is `scaulk@regis.edu`.

[1]*from Dr. Lynne Walling's web page at the University of Bristol* `http://www.bris.ac.uk/science/research/lynne-walling.html`

YoungJu Choie is a professor of mathematics at Pohang University of Science and Technology. Her email address is `yjc@postech.ac.kr`.

Misha Rudnev

On the evening of Monday 10 May 2021, Lynne Walling phoned me, after having had her first, and only appointment with an oncology consultant earlier on that day. "I'm gonna die," she said. "It sucks. I don't have much time. I want a party on Friday."

On Friday I arrived from my London flat at her Bristol home. There, I met some 30 friends, colleagues, and students, all in a similar state of disbelief. Lynne, too weak to engage in the mingling and beers, mustered her strength to have people follow her around the house, telling them to put their names on things they'd like to have after she was gone. She was matter-of-fact forcing on these arrangements as if she was making plans to cook jam out of the blackcurrants in the garden when they ripen.

Unflagging courage, directness, honesty, and generosity have always been among the defining characteristics of the person Lynne was. They largely accounted for the kind of mathematics that attracted her as a number theorist—explicit and constructive versions of classical questions in the theory of theta series and automorphic forms. They also laid foundational principles for her interactions with students, colleagues, and institutions. In response, students worshiped her; colleagues respected and trusted her; and institutions reckoned with, delegated powers to, and feared her.

Roughly half the time throughout her Bristol tenure that started in 2007 she headed the Pure Mathematics Division in the School. "I like being in charge of people," she would tell me. In positions of power, she would invariably look down, only in a good way. She would always do everything she could to help people in positions of lesser rank or privilege. Quoting one of her colleagues, she wanted to lift people up, not stamp people down.

As a feminist mathematician and true artist in all aspects of her life, she has become a role model to a whole generation of scholars, both men and women. Since 1998, she gave some 20 invited presentations on Women, Diversity, and Education. Her commitment to matters of equal opportunity, justice, and equality was so contagious that it reached even the most hardened cynics.

One of the exclusive luxuries in today's world rank-and-file is being in position to say no. Being uniquely fearless, Lynne could say no to God himself and face the consequences. This goes at least as far back as an incident of her giving an adult malefactor a stern lecture as to why one should not feed the bears in the Yellowstone National Park when she was seven. (Was this also an early example of her later universally renowned excellence in teaching?)

Misha Rudnev is an associate professor of mathematics at the University of Bristol. His email address is `m.rudnev@bristol.ac.uk`.

When she had to face death, she stood bold, undefeated, and slightly sardonic. Of all the many things and experiences she taught and shared with me, this may be the most precious and memorable one.

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Jay Jorgenson

Thomas Shemanske

Lejla Smajlović

Credits

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American Women Mathematics PhDs of the 1950s

Margaret A. M. Murray

Introduction

In the summer of 2022, Ukrainian mathematician Maryna Viazovska was awarded the Fields Medal. The Fields Medals, among the most prestigious awards in mathematics, are awarded every four years to mathematicians age 40 or younger to recognize both past achievement and future promise. Since the first Fields Medals were awarded in 1936, the prize has been awarded to 62 men—and only two women. Maryam Mirzakhani became the first woman to win a Fields Medal in 2014, three years before her premature death ([13]). Viazovska is now the second, and the only living, female Fields Medalist.

Both inside and outside the mathematical community, Viazovska's award has enlivened the ongoing cultural discussion about women in mathematics—a discussion that has, in the past, tended to emphasize the rarity of female mathematicians, particularly at the highest levels of achievement. Indeed, though women are still underrepresented when it comes to awards such as the Fields Medal ([5]), we live in a time when women are more visible participants in the mathematical community than ever before. Moreover, women have long been present in the mathematical community and, in various times and places, have made substantial progress in gaining recognition for their achievements. Even so, this progress has been by no means linear, and has been repeatedly punctuated by setbacks and reversals.

For many years I have made a careful study of historical patterns in the awarding of mathematics PhDs to women by institutions of higher education in the United States and Canada, with a primary focus on those women who earned PhDs during the years 1940–1959 ([15]). In an earlier article, I considered the American[1] women mathematics PhDs of the 1940s ([16]). In this article, the focus is on the American women mathematics PhDs of the 1950s—the decade in which women's share of US mathematics PhDs descended to its lowest level since the 1890s.

In the United States, we are living through a time when women's hard-won rights seem increasingly precarious. With this in mind, it behooves us to consider a time in history when women had fewer rights and were far less welcome in the mathematical community than they are now, and to understand how the women of that era were nevertheless able to persevere and to thrive as mathematicians.

Historical Context

Yale University has the distinction of being the first university in the United States to award the PhD in mathematics—and, indeed, the first PhD in any subject—in 1862 ([23]). In the decades that followed, other American universities followed suit. In 1882, Christine Ladd-Franklin completed all requirements for a PhD in mathematics at Johns Hopkins University—but, famously, Hopkins refused to actually *award* the degree she'd earned until 1926 ([8, p. 5]). It was not until 1886 that Columbia became the first US university to award a mathematics PhD to a woman, Winifred Edgerton (later Merrill) ([8], [9]). In the decade of the 1890s, women earned nearly 10 percent of US mathematics PhDs. In each of the next five decades, from 1900–1909 to 1940–1949, the proportion of mathematics PhDs awarded to women by US institutions remained consistently between 10 and 15 percent ([8], [14]). But in the 1950s, women's share of US mathematics PhDs

Margaret A. M. Murray is an associate professor of instruction in the departments of rhetoric and mathematics at the University of Iowa. Her email address is margaret-a-murray@uiowa.edu.

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[1]*For the purposes of this article,* American *refers primarily to the US, though my research also includes women who earned PhDs in Canada.*

Years	Total PhDs awarded	Women	
		Number	Percent of Total
1920–1929	352	49	13.9
1930–1939	780	113	14.5
1940–1949	835	87	10.4
1950–1959	2,325	109	4.7
1960–1969	6,407	363	5.7
1970–1979	10,877	1,086	10.0
1980–1989	7,356	1,159	15.8

Source: [14, Table 1.2], compiled from data in [11] and [22]

Table 1. Mathematics PhDs in the United States: Total and Share to Women, by Decade, 1920–1989.

dropped precipitously—just as the total number of PhDs awarded began a dramatic upward surge. Table 1 provides data, by decade, from the 1920s through the 1980s.[2] As the table makes clear, the raw numbers of women earning PhDs in mathematics actually increased slightly from the 1940s to the 1950s—but at the same time, the corresponding numbers of men nearly tripled.

As has been well documented, many social and political factors combined to shape the decline in women's share of US math PhDs in the 1940s and 1950s—as well as the gradual recovery during the 1960s, 1970s, and 1980s (for example, [8], [14], [24]). Arguably, World War II was the single most decisive event in shaping the American mathematical community in the 20th century. Prior to the war, US government investment in mathematics research and education was essentially nonexistent. But during and after World War II, mathematics (both pure and applied) came to be seen as essential to national security. In the war's immediate aftermath, federal funding of science and mathematics increased dramatically, with the creation of agencies such as the Office of Naval Research in 1946 and (especially) the National Science Foundation in 1950. At the same time, the Servicemen's Readjustment Act of 1944, commonly known as the GI Bill, helped to fuel a massive postwar expansion of the size and scope of higher education in the United States.[3]

One of the most striking effects of this postwar expansion was the spectacular dilution of women's representation in male-dominated fields—a dilution all the more striking given how robustly women had been recruited to work in such fields during wartime. During the war years, when men all but vanished from US college and university campuses, women were welcomed as both graduate students and instructional faculty in order to keep those institutions afloat. But at war's end the tables were turned: as campuses filled with surging numbers of returning GIs—nearly all of them male—fields like mathematics became much more competitive and increasingly attractive to young men. And in the US cultural milieu of the late 1940s and 1950s, the message to women was that their proper place was neither on campus nor in the workforce, but in the domestic sphere at home.

The factors that suppressed women's share of US mathematics PhDs persisted, to some degree, into the 1960s and 1970s. But at the same time, there were countervailing forces at play that led to a gradual and steady increase in women's representation—while at the same time facilitating professional advancement, albeit belatedly, for some of the women who'd earned PhDs in the 1950s. The first and most dramatic of these countervailing forces was the Cold War between the US and the Soviet Union. Events including the Soviet nuclear bomb tests of the 1940s and 1950s; the 1957 Soviet Sputnik launch, initiating the space race; and the Cuban Missile Crisis of 1962 all contributed to an increased sense of urgency about the need to develop research talent in the mathematical sciences. In specific response to the launch of Sputnik, Congress passed, and President Dwight D. Eisenhower signed, the National Defense Education Act (NDEA) and the National Aeronautics and Space Act, creating NASA, in the late summer of 1958 ([21], [20]). The NDEA helped to fuel graduate study for a new generation of students, while the establishment of NASA further stimulated an efflorescence of research in applied mathematics.

A second countervailing force was the movement for racial and gender equality of the 1960s and 1970s. Major gains for women came from the Equal Pay Act of 1963 and Title VII of the Civil Rights Act of 1964, prohibiting discrimination on the basis of sex in employment ([7], [19]). But women's rights activists continued to work for stronger

[2] *Table 1 presents data from the annual Survey of Earned Doctorates, formerly conducted by the National Research Council and now under the aegis of the National Center for Science and Engineering Statistics of the NSF. These numbers often differ somewhat from statistics compiled in the annual survey of doctorates published annually in the* Notices ([1]). *The discrepancy between different sets of statistics stems from shifting definitions of what constitutes a PhD in mathematics, particularly with the rise of separate departments of statistics, computer science, and other mathematical sciences.*

[3] *For a recent, accessible discussion of the impact of the GI Bill on higher education in the US, see* [4, Chapter 2]. *For more nuanced discussion of the impact of the GI Bill on women, see* [12] *and* [19].

protections. In the arena of higher education, their efforts culminated in Title IX of the Educational Amendments Act of 1972, which effectively prohibits discrimination on the basis of sex in any educational program receiving federal funding ([24], [25], [26]). I would argue that the passage (and subsequent enforcement) of Title IX was the single most potent driver of increased representation of women, not only in mathematics, but across all STEM fields.

Thus, although American women who earned their PhDs in mathematics during the 1950s started their careers in an inauspicious time, the women's movement of the 1960s and 1970s and after—in which many of these women played key roles—helped to open up professional opportunities for them and (especially) for generations of women that followed. Indeed, in contrast to the data in Table 1, in the 1990s women's share of US PhDs in mathematics exceeded 20% for the first time, and that share has remained consistently near 30% in the first two decades of the 21st century ([1], [8, p. 118]). Thus the mathematics PhDs of the 1950s, despite their comparatively small numbers, represent a decisive generation in the history of American women in mathematics.

The List

Since the early 1990s I have been working on a database of all the women who earned mathematics PhDs from US and Canadian universities during the 1940s and 1950s. I have been able to identify 106 such women who earned their PhDs in the 1950s; basic information about these women is presented in Tables 2–4 below. For each individual, I've provided the year in which her doctorate was awarded, the most complete version of her name that I've been able to identify, her years of birth and death, and the institution from which she earned the PhD.

As of early September 2022, twelve of the 106 women are apparently still living, ranging in age from 87 to 100. Among those who have died, the median age at death was 87; across the entire group, at least 43 individuals lived into their nineties. Three of these have lived to age 100 and beyond: Elizabeth Wetherell Ferentz (Syracuse 1959), still living at age 100; Susan Gerber Hahn (NYU 1957), who lived to age 101; and Evelyn Kinney Wantland (Illinois[4] 1958), who lived to age 104. Ferentz, a student of Lipman Bers, worked at NYU, IBM, and Hunter College. Hahn, a postwar refugee from Hungary, earned her PhD under Peter Lax and had a lengthy career at IBM. Wantland, a student of Waldemar Trjitzinsky, taught for a number of years at Illinois Wesleyan University. As with the women PhDs of the 1940s, this is a surprisingly long-lived group.

Proceeding chronologically through Table 2 (1950–1953), we encounter individuals, both familiar and unfamiliar, whose lives and careers are historically significant. Marjorie Lee Browne (Michigan 1950) was the third African-American woman to earn a PhD in mathematics—and the only Black woman to do so in the 1950s. A student of George Y. Rainich, she served for 30 years on the faculty of North Carolina Central University, including 19 years as department chair. Margaret Alice Waugh Maxfield (Oregon 1951), a student of Ivan Niven, appears to have been the first woman to earn a PhD in mathematics from the University of Oregon; she enjoyed a long and varied career as both mathematician and statistician, inside and outside academia. The distinguished applied mathematician Cathleen Synge Morawetz (NYU 1951), a student of Kurt O. Friedrichs, enjoyed a lengthy career at the Courant Institute; among her many honors, she was the first female mathematician to receive the National Medal of Science, in 1998. Mary-Elizabeth Hamstrom (Texas 1952), a student of R.L. Moore, supervised 10 PhD students during her 38 years on the mathematics faculty at Illinois. Gertrude Ehrlich (Tennessee 1953) escaped from Vienna after the *Anschlüss* and settled with her family in Atlanta. Ehrlich, who studied ring theory with J. Wallace Givens, became the first woman to earn a PhD in mathematics from the University of Tennessee and, for nearly four decades, served on the mathematics faculty at the University of Maryland.

The first name we encounter in Table 3 (1954–1956) is Lida Baker Kittrell Barrett (Pennsylvania 1954), whose varied career included numerous leadership positions, including chair of the mathematics department at the University of Tennessee (1973–1980); dean of the College of Arts and Sciences at Northern Illinois University (1987–1991); and president of the MAA (1989–1991), the second woman to lead that organization. Proceeding through the table, we come to Phyllis Ann Fox (MIT 1954) who, when denied admission to MIT's doctoral program in electrical engineering, pursued a doctorate in applied mathematics (fluid dynamics) with C.C. Lin instead. Fox subsequently had a long and varied career as a computing pioneer.[5] Elvy Lennea Fredrickson (Oregon State 1954), a native of Oregon and the first woman to earn a PhD in mathematics at Oregon State University, chaired the mathematics department at Lewis & Clark College in Portland for nearly three decades. Anneli Cahn Lax (NYU 1955), who escaped her native Germany in 1935, completed her PhD with Richard Courant and went on to an interesting and varied career at NYU's Washington Square College. From 1958 to 1999, Lax was the technical editor of the MAA's New

[4] *Here and in the sequel, the term* Illinois *refers to the University of Illinois Urbana-Champaign.*

[5] *I corresponded with Phyllis Fox but was, regrettably, unable to interview her myself. Fortunately, however, Thomas Haigh conducted a fascinating interview with Fox in 2005 (*[10]*).*

Year	Name	PhD-granting institution
1950 (11)	Felice Hilda Davidson Bateman (1922–2013)	Michigan
	Marjorie Lee Browne (1914–1979)	Michigan
	Geraldine Alma ("Jerry") Coon (1913–2008)	Rochester
	Helen Frances Cullen (1919–2007)	Michigan
	Joanne Elliott (1925–)	Cornell
	Violet Grace Hachmeister Larney (1920–2016)	Wisconsin
	Alice Winzer Lytton (1923–2001)	Brown (Appl)
	Margaret Owchar Marchand (1925–2014)	Minnesota
	Sister Mary Ferrer McFarland RSM (1913–1993)	Notre Dame
	F(lorence) Virginia Rohde (1918–2008)	Kentucky
	Wanda Montlak Szmielew (1918–1976)	California/Berkeley
1951 (11)	Leila Ann Dragonette Bram (1927–1979)	Pennsylvania
	Elizabeth ("Betty") Heinemann Cuthill (1923–2011)	Minnesota
	Flora Dinkines (1910–2006)	Chicago
	Sister Mary Seraphim Gibbons CSJ (1913–2003)	Minnesota
	Violet Bushwick Haas (1926–1986)	MIT
	Margaret Alice Waugh Maxfield (1926–2016)	Oregon
	Cathleen Synge Morawetz (1923–2017)	NYU
	Sister Marie Jesse (Sister Edith) Morrison SSJ (1915–2008)	Catholic
	Elsie Temple Church Ozley (1918–1982)	Kentucky
	Jean Marie Boyer Porter (1923–2013)	Maryland
	Ruth Bernice Lind Potter (1922–1990)	Washington/St. Louis
1952 (5)	Mary-Elizabeth Hamstrom (1927–2009)	Texas
	Emilie Virginia Haynsworth (1916–1985)	North Carolina
	Augusta Louise Schurrer (1925–2015)	Wisconsin
	Sister Mary Stephanie Sloyan RSM (1918–2007)	Catholic
	Joan Elizabeth Robinson Westlake (1925–)	Bryn Mawr
1953 (13)	Margaret Frances Conroy (1923–2005)	Brown (Appl)
	Alice Elisabeth Braunlich Dickinson (1921–1987)	Michigan
	Jean Lee Blaney (Feidner) Durfee (1926–1988)	Buffalo
	Gertrude Ehrlich (1923–)	Tennessee
	Herta Taussig Freitag (1908–2000)	Columbia
	Lillian Gough (1918–2006)	Buffalo
	Lila Peck Walker McRae (1922–2001)	North Carolina
	Mabel Delores Montgomery (1919–2007)	Buffalo
	Marian Alease Moore (1907–1995)	Purdue
	Anne Barbara Carples (Davis) Morel (1919–1984)	California/Berkeley
	Pauline Chisholm Mann Nachbar (1927–)	Brown (Appl)
	Anne Elizabeth Scheerer (1924–2013)	Pennsylvania
	Rosedith Sitgreaves (Bowker) (1915–1992)	Columbia

Table 2. Women Mathematics PhDs from American institutions, 1950–1953.

Mathematical Library, now known as the Anneli Lax New Mathematical Library and jointly published by MAA and AMS. Also from the class of 1955, Jean Estelle Hirsh Rubin (Stanford 1955) studied with J.C.C. McKinsey (and, after his 1953 suicide, Patrick Suppes); she published widely in mathematical logic—with a particular focus on set theory and the Axiom of Choice—right up until her death in 2002.

Table 4 lists the doctoral classes of 1957–1959; again, every name has a story to tell. For example, Vera Stepen Pless (Northwestern 1957), who wrote her dissertation in ring theory with Alex Rosenberg, became a leading expert in error-correcting codes during her time at the Air Force Cambridge Research Laboratory in the 1960s and, not long after Title IX, was hired as a full professor of mathematics at the University of Illinois at Chicago—her first serious academic position, which she held for over 30 years.

Year	Name	PhD-granting institution
1954 (12)	Lida Baker Kittrell Barrett (1927–2021)	Pennsylvania
	Elsa Emilie Keitzer Boyce (1929–2019)	Carnegie Tech
	(Kumari) Anna Chandapillai (1912–?)	Wisconsin
	Jacqueline Lydie Penez Criscenti (1924–2015)	Minnesota
	Jacqueline Pascal ("Jill") Evans (1923–2006)	Harvard/Radcliffe
	Phyllis Ann Fox (Sternlieb) (1923–2017)	MIT
	Elvy Lennea Fredrickson (1921–2008)	Oregon State
	Sister Mary Andrea Johnson SCL (1921–1911)	Catholic
	Sister Christine Françoise Marie (Sister Irène) Morvan DHS (1912–1995)	Catholic
	Rose Mary Ring-Carroll (1924–2002)	Brown (Appl)
	Evelyn Mary Bender Vaskas (1928–2015)	MIT
	Joyce Lenore White Williams (1929–)	Illinois
1955 (13)	Barbara Jean Beechler (1928–2003)	Iowa
	Helen Bozivich (1916–2008)	Iowa State
	Thelma Mae Chaney (1926–2004)	Washington/Seattle
	Ruth Margaret Davis (Lohr) (1928–2012)	Maryland
	Patricia Ramsay James (Ruml Wells) Eberlein (1923–1998)	Michigan State
	Kathryn Lucille Powell Ellis (1921–2011)	Iowa
	Lillian ("Lila") Rose Elveback (1915–2004)	Minnesota
	(Anna) Elizabeth Vaughan Holmes (1910–2001)	Stanford
	Anneli Cahn Lax (1922–1999)	NYU
	Jean Estelle Hirsch Rubin (1926–2002)	Stanford
	Ruth Rebekka Struik (1928–2022)	NYU
	Sister M. Ferdinand Torline CSJ (1902–1995)	St. Louis
	Eleanor Sylvia Weiss (1915–1977)	Harvard/Radcliffe
1956 (9)	(Olive) Jean Hoover (Ballou) Dunn (1915–2008)	UCLA
	Sister Barbara Ann (Catherine H.) Foos SSJ (1926–2016)	Notre Dame
	Marguerite Josephine Straus Frank (1927–)	Harvard/Radcliffe
	Vivienne Esta Brenner Morley (1930–2013)	Chicago
	Barbara Ann Cotter Morrison (1924–2016)	Brown (Appl)
	Mary Frances Muskoff Neff (1930–2021)	Florida
	Elvira Rapaport Strasser (1913–1998)	NYU
	Joan Eliot Raup Rosenblatt (1926–2018)	North Carolina
	Lily Hanna Matthew Seshu (1925–1986)	Illinois

Table 3. Women Mathematics PhDs from American institutions, 1954–1956.

Marian Boykan Pour-El (Harvard/Radcliffe 1958) studied logic at both Harvard and Berkeley, creating a thesis largely independent of any advisor; a world-renowned expert in computability theory, she spent much of her career on the faculty of the University of Minnesota. And Alexandra Bagdasar Bellow (Yale 1959), a student of Shizuo Kakutani and an expert in ergodic theory and probability, spent most of her career at Northwestern University. There, in the late 1960s, she became the first female full professor in the mathematics department.

Patterns in the PhDs of the 1950s

Perhaps the most striking difference in the demographic makeup of the American women mathematics PhDs of the 1950s, as compared to previous decades, is in their national origins. Of the 106 women listed in Tables 2–4, 17 were born in Europe; the majority of these came to the US as refugees, reflecting the spectacular dislocations brought about by anti-semitism, the Holocaust, and global war. In addition, five were born in Canada, three in India, and one in the Philippines. While some of these women eventually returned to their countries of origin, most did not.

A total of 42 schools are included among the degree-granting institutions in Tables 2–4. In a notable change from previous decades, the institutions that awarded the greatest number of PhDs to women in the 1950s were NYU (10) and Brown (6), displacing older institutions like the University of Chicago and Catholic University ([14, Chapter 2]). Their emergence as the top grantors of PhDs to

Year	Name	PhD-granting institution
1957 (10)	Betty Jane Gassner (1934–)	NYU
	Susan Gerber Hahn (1914–2015)	NYU
	May Ella Risch Kinsolving (1926–2022)	Syracuse
	Edith Hirsch Luchins (1921–2002)	Oregon
	Vera Stepen Pless (1931–2020)	Northwestern
	Mary I. Hanania Regier (1926–2020)	California/Berkeley
	Sara Louise Ripy (1924–2020)	Kentucky
	Lena Sharney (1909–1975)	NYU
	Mary Catherine Bishop Weiss (1930–1966)	Chicago
	Maria Josefa Wonenburger Planells (1927–2014)	Yale
1958 (9)	Ellen Correl Lehner (1930–2020)	Purdue
	Judith Ann Richman (Blankfield) Gumerman (1931–1972)	Illinois
	Dorothy Evelyn Mussman Levy (1932–)	NYU
	Christina Perlas Parel (1917–2011)	Michigan
	Marian Boykan Pour-El (1928–2009)	Harvard/Radcliffe
	Doris Grove Skillman Stockton (1924?–2018)	Brown
	Sister Mary Paul James Villemure OP (1928–)	Notre Dame
	Evelyn Douglas Kendrick (Kinney) Wantland (1917–2022)	Illinois
	Dorothy Jean Christensen Williams (1930–2003)	Washington/Seattle
1959 (13)	Kathleen Ethel Baxter (O'Keefe) (1923–2012)	California/Berkeley
	Alexandra Bagdasar (Ionescu Tulcea) Bellow (1935–)	Yale
	Yvonne Germaine Marie Ghislane Cuttle (1932–)	Oregon
	Diane Mary Johnson Dowling (1935–2005)	Toronto
	Elizabeth Harriet Wetherell Ferentz (1922–)	Syracuse
	Sister Catherine Josephine Gillis SND (1900–1988)	Boston University
	Carol Ruth Vander Velde Karp (1926–1972)	Southern California
	Halina Ladavicias Montvila (1917–1997)	NYU
	Sister Mary Rose Rauen OSB (1929–2021)	St. Louis
	Elizabeth Anna Shuhany (1925–2010)	Boston University
	Rosemarie Margarete Gabriele Scheerer Stemmler (1930–2011)	Illinois
	Sister Mary Alberta Uzendoski OSF (1911–2001)	St. Louis
	Tilla Savanuck (Klotz Milnor) Weinstein (1934–2002)	NYU

Table 4. Women Mathematics PhDs from American institutions, 1957–1959.

women reflects the emerging significance of applied mathematics in the postwar years. At Brown, five of the six degrees were awarded by the relatively new Graduate Division of Applied Mathematics, established in 1946 ([3]). At NYU, the degrees were evenly split between pure and applied mathematics, reflecting the dual mission of the Institute for Mechanics and Mathematics, also established in 1946 and later to be known as the Courant Institute for the Mathematical Sciences ([6]).

In keeping with this increased emphasis on applied mathematics, many of the 1950s women pursued PhDs and careers in the statistical sciences, including Rosedith Sitgreaves (Columbia 1953), Helen Bozivich (Iowa State 1955), Lila Rose Elveback (Minnesota 1955), Elizabeth Vaughan Holmes (Stanford 1955), Olive Jean Dunn (UCLA 1956), Joan Raup Rosenblatt (North Carolina 1956), Christina Perlas Parel (Michigan 1958), and Elizabeth Anna Shuhany (Boston University 1959). In particular, Christina Parel served as Dean of the Statistical Center at the University of the Philippines, where she developed the graduate program in statistics. Sitgreaves (1960), Rosenblatt (1967), and Parel (1971) were all elected Fellows of the American Statistical Association ([2]).

A number of women of the 1950s generation took advantage of career opportunities outside academia made possible by increased federal investment in mathematics. In 1955, for example, one year before completing her PhD, the statistician Joan Rosenblatt joined the National Bureau of Standards as a mathematician. After several promotions—and the rechristening of the agency as the National Institute for Standards and Technology—Rosenblatt retired 40 years later as NIST's Director of the Computing and Applied Mathematics Laboratory ([17]).

Elizabeth Vaughan Holmes, the first woman awarded a PhD in statistics at Stanford, subsequently worked as a statistician for the Department of Fisheries, the Department of the Interior, and the US Navy.

Outside the statistical sciences, Leila Dragonette Bram (Pennsylvania 1951) began work for the Office of Naval Research upon completing her PhD and, at her untimely death in 1979, was director of ONR's mathematics program. Elizabeth Heinemann Cuthill (Minnesota 1951) had a lengthy career with the US Naval Surface Warfare Center. Ruth Margaret Davis (Maryland 1955) held a succession of positions in the federal government, culminating in her appointment as Assistant Secretary of the Department of Energy in the Carter administration (1979–1981). And Phyllis Fox did much of her groundbreaking work in computing while on the staff of Bell Laboratories.

Even so, the most common profession for women of the 1950s generation was teaching. This was the career route for all twelve of the Roman Catholic sisters (commonly referred to as nuns) whose names appear in Tables 2–4. The letters that appear after each sister's name denote the religious order to which she belongs (e.g., OSF denotes the Order of St. Francis; OP denotes the Order of Preachers, *Ordo Praedicatorum*, founded by St. Dominic and also known as Dominicans). Many of these sisters were already teaching—sometimes at the college level—when their order sent them to graduate school to pursue PhDs. As a result, the sisters' average age at completion of the PhD, 39, is somewhat older than the rest of the group. Indeed, the oldest PhD recipient among the 1950s women, Sister Catherine Josephine Gillis SND (Boston University 1959) was born in 1900, joined the Sisters of Notre Dame de Namur in 1919, began teaching at Emmanuel College in Boston in 1942, earned her PhD in the same year she turned 59, and immediately thereafter became chair of the Emmanuel math department.

Of course, the vast majority of the 1950s women who pursued careers in teaching had to deal with the vicissitudes of the academic job market. While postsecondary teaching positions were plentiful in the 1950s and 1960s, many women—especially married women—experienced discrimination in pursuit of these jobs. As in previous generations, a large share of the 1950s women math PhDs married fellow academics, often fellow mathematicians. In the years prior to Title IX, the anti-nepotism practices of colleges and universities posed the greatest barrier to employment for these women. Anti-nepotism practices, ostensibly designed to prevent conflicts of interest in employment, often prevented women from working in the same department or even the same college as their spouses (see [14], [24]). But both women and departments benefitted when an institution was willing to set aside these practices, as Purdue University did in 1967 when they offered tenured full professorships to both Jean Rubin and her husband, statistician Herman Rubin.

Not surprisingly, many of these 1950s PhDs played an active role in advocating for greater equality for women—in mathematics, in academia, and in society at large. Many of these women were among the founding members of the Association for Women in Mathematics (AWM)—most notably, Vera Pless, who was among a small group of Boston-area mathematicians who helped establish the AWM in 1971. Lida Barrett—who'd been barred from a regular faculty position in the mathematics department at Tennessee while her husband, mathematician John Barrett, was alive—was named full professor and head of the department within just a few years of his death in 1969. She was among the first women to head a department of mathematics at a major research university. As a result of her own, highly personal experiences of discrimination, Lida Barrett became an outspoken advocate for women and other underrepresented groups—at Tennessee, in her subsequent administrative positions, and in the wider community of academia.

Certainly, many of the women PhDs of the 1950s experienced family-related career disruptions and blatant discrimination, most acutely in the first couple of decades post-PhD. But—thanks to individual and collective advocacy, and the prospects opened up by federal legislation like the NDEA, the Civil Rights Act, and Title IX—some were also able to benefit from a wider range of professional opportunities, especially later in their careers. By virtue of their efforts, the 1950s generation of women mathematics PhDs have served, and continue to serve, as role models for subsequent generations of women in mathematics.

A Repository for the Future

Much of what I've written here relies on the database I've compiled on the 192 women who earned mathematics PhDs from American institutions during the years 1940–1959. Some of the material from the database is currently published on the *Women Becoming Mathematicians* website ([18]), to which I continue to add information on a regular basis. I plan to make all my research materials—not only the database, but all the documents associated with my research on the women PhDs of the 1940s and 1950s—accessible to other researchers. To this end, I am currently in conversation with the Archives of American Mathematics at the Dolph Briscoe Center for American History at the University of Texas at Austin. It is my hope that the lives and experiences of these pioneering mathematicians will provide inspiration and guidance as we move into an uncertain and perilous future.

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Margaret A. M. Murray

Credits

Author photo is courtesy of Douglas Slauson.

From the Till to the International Space Station: Josephine Jue and Spaceflight Computing

Jennifer Ross-Nazzal

Mathematician and aerospace technologist Josephine Jue spent more than thirty years working behind-the-scenes on computers and software at NASA, making giant leaps for humankind possible. One of the few professional women working at the Manned Spacecraft Center (MSC) in the 1960s, Jue literally changed the face of computing at the Houston Center as the first Asian-American woman in her division. Fresh out of college with a bachelor's degree in mathematics, she applied to work at NASA in 1963, a time when most MSC female employees worked in the clerical field, as they did in other government agencies and across the aerospace industry.[1] There was a general belief that women were better suited for these careers and tasks rather than other professional positions. For instance, when a male flight controller requested a typewriter to type up his notes, he was told, "engineers don't get typewriters. *The girls* get the typewriters."[2]

Although few in number, there were professional opportunities for women. NASA needed employees with technical training, and women with math degrees found exciting opportunities with the space agency. Jue became active in the Federal Women's Program (FWP) and in the American Federation of Government Employees.[3] As the lunar program wound down in the early 1970s, her work helped to usher in a new era of spaceflight with the development and testing of the avionics for the Space Shuttle Orbiter. Later, she oversaw the Center's efforts to provide all employees with workstations and desktop training. Jue's groundbreaking career is just one example of the types of contributions professional women made at the space agency in the field of computer science and beyond.

Jue was the child of Chinese immigrants. Like many Chinese people who immigrated to the Mississippi Delta, her parents ran a grocery store. Born in the back of that Vance, Mississippi, store in 1940, she was only three months old when her parents chose to relocate to Houston. They hoped for greater opportunity in Texas and

Jennifer Ross-Nazzal is the human spaceflight historian at NASA. Her email address is jennifer.m.ross-nazzal@nasa.gov.

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[1] *Jue recalls arriving at MSC in 1963, but the telephone directories do not show her arriving until two years later. The Manned Spacecraft Center became the Lyndon B. Johnson Space Center (JSC) in 1973. For more information about the female experience at JSC, see Jennifer M. Ross-Nazzal,* Making Space for Women: Stories from Trailblazing Women of NASA's Johnson Space Center *(College Station: Texas A&M Press, 2022).*

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purchased a store in Houston's Third Ward and renamed it Far East Grocery. They later opened a second market.[4]

It was here, at her parent's store, that Jue developed an interest in mathematics. Her parents expected her to help with the store when she came of age. Before she was ten, she worked at the cash register and learned how to count by twos, tens, and twenties. In an age when clerks had to add the price of items and compute the total sale, including sales tax, "math came in handy" Jue said, especially when customers only had a couple of items so she did not have "to use the adding machine." She also learned how to correctly count the change to give people. While in high school, Jue signed up for all the math classes her school offered including algebra, geometry, infinite series, and trigonometry.[5]

After high school graduation, she attended the University of Houston so she could continue to help her parents when she was not attending classes. She chose to major in math and minor in chemistry, not knowing what she might do with her degree. Jue did know one thing, however, she did not want to be a teacher, a nurse, or a secretary. In addition to her required courses, she also completed a computer programming class in FORTRAN (FORmula TRANslator) and another in MAD (Michigan Algorithm Decoder). "That was my start in computers," Jue recalled.[6]

While Jue worked on her degree, NASA announced it would be building a Center about twenty-five miles south of downtown Houston to achieve President John F. Kennedy's dream of sending a man to the moon and returning him safely by the end of the 1960s. The Manned Spacecraft Center would be home to the astronauts and Mission Control. Houstonians were thrilled with the decision to locate a Center in their community, and they welcomed employees with open arms. Newspapers, radio, and television reporters hailed the decision and covered the site construction.[7] To meet the president's ambitious goal, NASA needed engineers, scientists, technicians, and mathematicians, so after graduation Jue applied. "NASA," she explained, "opened up a new world" for her and the other women who worked outside of the pink-collar workforce.[8]

NASA needed employees with technical training, and women with math degrees like Jue found exciting opportunities with the space agency. Poppy Northcutt held a math degree from the University of Texas and worked for aerospace contractor TRW on trajectories to safely return the astronauts home from the moon. Ivy Hooks and Dottie Lee became engineers and were the only women to serve on the original design team for the Space Shuttle Orbiter in 1969.[9]

Given her background and their need, NASA offered Jue a position, and personnel asked about her interests before they placed her. She told them she liked math, so they offered her a job as a mathematician in the Computation and Analysis Division. Located in the Engineering and Development Directorate, the division handled computation and data reduction for the entire Center, and she served as one of eighty computer programmers and analysts. With the facility still under construction, Jue reported to one of NASA's fourteen temporary sites: Site 11 at the University of Houston. The building housed NASA's digital computers and would later become the site for KUHT, Houston's PBS station. Given the choice of scientific or business programming, Jue chose to focus on the latter. She recalled, "That [option] sounded better."[10] Later that year the division moved onsite when their building opened.

Her assigned branch, the Data Systems Development Branch, handled three systems: financial, procurement, and logistics as well as applied data analysis techniques and general management applications. Her supervisor knew that she had experience working with computers and offered her the choice of using either the IBM 1401 or the 7094. She chose the 1401, a much smaller computer specifically designed for processing and handling administrative and business tasks such as payroll, procurement, and managing of contracts. Jue learned to program in COBOL (Common Business Oriented Language) at NASA and designed a program for the library to check out books to employees. As NASA came closer to landing on the lunar surface, she helped provide "resource, time, [and] cost analysis" for the Gemini and Apollo Programs. Over the years she received promotions and eventually became a project lead.[11]

Merv Hughes, the coordinator of MSC's Federal Women's Program, credited women's promotions at the

[4]*Josephine Jue, interviewed by Daisy Chan Gee, circa 1980, audio,* `https://hdl.handle.net/1911/63493`; *Charles Reagan Wilson, "Chinese in Mississippi: An Ethnic People in a Biracial Society,"* `https://www.mshistorynow.mdah.ms.gov/issue/mississippi-chinese-an-ethnic-people-in-a-biracial-society`; *Josephine Jue, interview with author, August 4, 2021, JSC Oral History Project, transcript.*

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[7]*Henry C. Dethloff,* Suddenly, Tomorrow Came: A History of the Johnson Space Center *(Houston: NASA, 1993), 44–45, 50–51.*

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[10]*Jue interview with author; Major Test Facilities of the Engineering and Development Directorate, April 1966, Apollo Series, Box 066-64, JSC History Collection, University of Houston-Clear Lake.*

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Figure 1. Josephine Jue in front of one of the Center's mainframe computers.

Center to a lack of discrimination in the agency. "When you've got your eyes on the stars," he said, "you haven't time for prejudice." *Los Angeles Times* reporter Ursula Vils highlighted the opportunities for women working in the space program in a series of articles featuring women who worked at some of the NASA Centers in 1969, the year the United States accomplished the nation's goal of landing on the moon. Jue was one of several professional women she interviewed. In her opinion, the benefits offered by NASA were excellent. "Our salaries are comparable to industry or better and we get twenty days leave after three years and thirteen days sick leave," Jue reported.[12]

Generous compensation did not mean improvements could not be made, however. In 1975 she joined the Federal Women's Program Committee to assist the Center's FWP coordinator and Equal Employment Opportunity Officer with problems faced by female employees.[13] Recognizing that women lacked access to certain career paths at NASA, the program set out to recruit and hire women and place them in fields for which they were qualified but underrepresented. The coordinator and committee also offered guidance about career advancement as well as training. Jue recalled that the committee met to "talk about what we could do" to ensure the Center avoided prejudice in the hiring or promotion of women employees and remembered they discussed how to move secretaries into positions with higher pay and greater authority.[14]

Jue, like many other professional women at the Center, insisted that she did not face any discrimination because of her sex. Other women involved in computing agreed. "If you write a computer program," Northcutt said, "either it works or it doesn't. There's no opportunity for anyone to be subjective about your work." Jue admitted she "was just glad to have a job and working and earning money and making more money than [her] parents in the grocery store." NASA might not have had many professional female employees, but the agency did offer job opportunities that women did not have elsewhere.[15]

As part of the 1975 International Women's Year and the FWP week activities, Jue and the committee put together a program for Center employees to be held in the Center's large Teague Auditorium. The "Space for Women" program included a series of talks with Harriett Jenkins, NASA's Assistant Administrator for Equal Opportunity, Secretary Renee I. Hall from nearby Ellington Air Force Base, and two professors from the University of Houston. University professor Bette Stead spoke about how organizations could utilize female employees "to their fullest potential" so that they would be "accepted on an equal basis." Dr. Dale Hill addressed the issue of women's visibility in the workplace and programs designed to help women become more assertive. Hall spoke directly to the secretaries in attendance, encouraging them to set goals early in their career and to work toward accomplishing them. Center Director Christopher C. Kraft called the day an important step in advancing women's opportunities within NASA and emphasized the importance of using the talents of female employees "to the maximum extent for the benefit of the space effort."[16]

Throughout the 1970s, more women were increasingly involved in NASA's follow-on human spaceflight vehicle, the Space Shuttle, the world's first reusable spacecraft. Jue was one of those who found herself immersed in the

[12]*Ursula Vils, "NASA Women Behind Man on the moon,"* Los Angeles Times, *October 6, 1969.*

[13]*President Lyndon B. Johnson signed Executive Order 11375 in 1967, which created the Federal Women's Program. His order added sex to other prohibited forms of discrimination in the federal government. In 1972, NASA hired its first agencywide Federal Women's Program Manager to advance opportunities for women in the space agency.*

[14]*Jue interview with author.*

[15]*Jue interview with author; "The Personal Views of Eight Women Who Succeeded in It,"* Life Magazine, *September 4, 1970, 20.*

[16]*"FWD Program Committee Will Sponsor 'Space for Women,'"* Roundup, *May 9, 1975, 2; "Goals of FWP emphasized,"* Roundup, *May 23, 1975, 1–2; Bette Ann Stead, "Women in Management,"* Vital Speeches of the Day *(July 1975) 41, no. 19: 589.*

Figure 2. Jue sits in the front row at the "Space for Women" program. She is the third woman on the left.

program when, in 1975, she turned her attention to the development and testing of Shuttle software; that year she became part of the Spacecraft Software Division headed by Richard Parten. At the time the Shuttle "was one of the most complex software systems ever produced," an expensive and time-consuming endeavor. Compared to the relatively simplistic Mercury capsule, which had no onboard computer, the Shuttle could not have been developed or operated without onboard computers and their associated software. As John Garman, an engineer who monitored the Apollo Guidance Computer software and managed the Space Shuttle software development, explained, "virtually no component in [the vehicle] could be checked out without some form of software in the computers. And that's a first."[17]

As development of the Space Shuttle ramped up, Jue found herself no longer writing computer programs, because NASA opted to contract out the work. "We [civil servants] were out of the programming business," she said. In 1975, when she joined the Spacecraft Software Division, she served as a contract monitor, chaired the HAL/S Language Definition and User Coordination Group, and managed operations in the Software Development Laboratory (SDL), one of several facilities that tested Shuttle software.[18] For its latest spacecraft, NASA chose to program the Shuttle's Primary Avionics System using a high-order language known as HAL/S, and Jue came on board just before the software's configuration inspection in Cambridge, Massachusetts. While the bulk of the vehicle was built by Rockwell International, the agency let another contract for the design, development, and testing of the Shuttle's flight software. IBM received that contract, with Intermetrics supplying the compiler for the software. (The compiler translated the code used by the Orbiter's General Purpose Computers.) The Spacecraft Software Division closely monitored the software's development and Jue specifically approved any modifications to HAL/S as chair of the HAL/S Language Definition and User Coordination Group.[19]

As a contract monitor, she evaluated and graded IBM and their Intermetrics counterparts. Jue reviewed their ability to meet the technical requirements as required by NASA and evaluated their performance. Did they submit quality work? Did they meet all deadlines and come in at or under cost?

Managing operations in the SDL involved overseeing the design and testing of the software needed before the Space Shuttle could even take flight. Established in 1972, the lab used mainframes, old IBM-360s from Mission Control, to simulate and run early versions of test software. During simulations, three General Purpose Computers plugged into a specially designed Flight Equipment Interface Device (FEID) which allowed IBM programmers and NASA employees to test, start, stop, and view the development code's variables and parameters to look for software bugs. The SDL also ensured that the software met all of NASA's requirements. From 1975 to 1978, the lab was instrumental in providing the test software for the Center's crew trainers; simulators used by the astronauts and flight controllers to train for Shuttle missions; Rockwell's Palmdale facility, where the Orbiter was being manufactured and tested; the Shuttle Avionics and Integration Laboratory; and the Kennedy Space Center in Florida. Once technicians, engineers, and flight crews tested the hardware and software together to demonstrate their compatibility, changes to the software were necessary, which required hundreds of programmers to come up with multiple versions of software.[20]

In October 1977, about four years before the launch of *Columbia*, the first Orbiter to fly in space, the lab began releasing new flight software to be verified and certified in flight simulators. Change requests grew substantially between 1977 and 1980, and Jue was busy juggling changing requirements, overseeing the change orders, and monitoring the IBM team. Continual evaluation and testing of the

[17]*John Garman interview by Adam L. Gruen, May 9, 1988, transcript, NASA Headquarters History Division, Washington, DC; James E. Tomayko,* Computers in Spaceflight: The NASA Experience *(Washington, DC: NASA, 1988), 3, 86.*

[18]*The name HAL is a bit of a mystery. Jack Garman attributed the name to one of the early software developers named Hal; some say it stands for Higher Avionics Language. Jue said, "Some think it stands for Houston Aerospace Language." Tomayko,* Computers in Spaceflight, *92; Jue interview by author.*

[19]*Jue interview with author; John R. Garman, interview with Kevin M. Rusnak, March 27, 2001, JSC Oral History Project, transcript,* https://historycollection.jsc.nasa.gov/JSCHistoryPortal/history/oral_histories/GarmanJR/GarmanJR_3-27-01.htm; *Tomayko,* Computers in Spaceflight, *92.*

[20]*Garman interview by Rusnak, March 2001.*

software were important because the first National Space Transportation System mission, STS-1, would be the first test flight of the Space Shuttle in space. Called the boldest test flight in history, this was the first time NASA chose to test a new spacecraft on its inaugural flight with astronauts onboard. Everything had to work so that the crew could safely return home. The Shuttle's Primary Avionics Software System, "the most complex flight computer program ever developed," had to be free of bugs and went through an intensive testing program that involved the release of seventeen interim versions over a thirty-one-month period before NASA deemed it flight worthy.[21]

Richard Feynman, the famed physicist and later a member of the Rogers Commission investigating the Space Shuttle *Challenger* accident, described the meticulous process of software testing by IBM that Jue was monitoring:

> The software is checked very carefully in a bottom-up fashion. First, each new line of code is checked, then sections of code or modules with special functions are verified. The scope is increased step by step until the new changes are incorporated into a complete system and checked. This complete output is considered the final product, newly released. But completely independently there is an independent verification group, that takes an adversary attitude to the software development group, and tests and verifies the software as if it were a customer of the delivered product. There is additional verification in using the new programs in simulators, etc. A discovery of an error during verification testing is considered very serious, and its origin studied very carefully to avoid such mistakes in the future.[22]

Once all the subsystems had been successfully certified, NASA was ready to launch the Space Shuttle and the two-man crew.

Flight controllers scrubbed the first launch attempt when there was a software glitch. Two days later the crew successfully launched into orbit where they spent nearly fifty-five hours in space, completed a lengthy list of systems tests, and returned home safely. In February 1982, after two successful Shuttle missions, Jue became the operations manager for the new Software Production Facility (SPF), pronounced "spiff." Like the SDL, the new facility continued to develop, verify, and produce flight software

Figure 3. Jue in the Software Production Facility.

for upcoming Shuttle missions. NASA anticipated flying multiple flights a month, up to fifty-two times a year, so using the old Apollo-era IBMs would be woefully inadequate as the agency moved from its first four test flights into operational missions. New technology was required. The 360s were updated to a single processor, the IBM 3033N, which had more capacity than those it replaced.[23]

In 1985, Jue learned of an opening in the Data Processing Systems Division and applied. NASA, like other federal agencies, recognized that employees needed personal computers, and Jue became JSC's User Workstation Manager. Computers were necessary to increase efficiency at the Center, and Jue was responsible for determining the user requirements and then overseeing the procurement and set up of the workstations across the Johnson Space Center. If machines required maintenance, her group handled those issues as well as training or troubleshooting. The staff also offered a Security Expo to help new computer users learn how to protect their hardware from viruses.[24]

In 1993, Jue heard the Space Station Program Office was "looking for somebody to head up the computer world

[21] *William A. Madden and Kyle Y. Rone, "Design, Development, Integration: Space Shuttle Primary Flight Software System,"* Communication of the ACM *(September 1984) 27, no. 9: 914–925; Gene D. Carlow, "Architecture of the Space Shuttle Primary Avionics Software System,"* Communication of the ACM *(September 1984) 27, no. 9: 926.*

[22] *R.P. Feynman, "Personal Observations on Reliability of Shuttle," in the* Report of the Presidential Commission on the Space Shuttle *Challenger* Accident, *Vol. II Appendix F,* https://history.nasa.gov/rogersrep/v2appf.htm.

[23] *Garman interview by Rusnak, March 2001; "Software Production Facility Now Operational,"* Roundup, *February 23, 1982, 3.*

[24] *Jue interview by author; "Users learn to protect, disinfect,"* Space News Roundup, *September 22, 1989, 1;U.S. Congress, Office of Technology Assessment,* Federal Government Information Technology: Management, Security, and Congressional Oversight *(Washington, DC: GPO, 1986), 30; Michael Schrage, "U.S. is Now Biggest Buyer of Computers,"* Washington Post, *September 11, 1985,* https://www.washingtonpost.com/archive/business/1985/09/11/us-is-now-biggest-buyer-of-computers/e1b12703-196f-49ba-bf27-953ddf8eada0/.

over there," so she decided to apply. The work was similar to what she had been doing for the Data Processing Systems Division, but she did not have to offer classes for Center employees or manage the help desk. She did, however, receive a promotion to a GS-15, the top of the pay scale for civil servants. Located in the brand-new home of the expanding Astronaut Office, Building 4 South, Jue found herself busy making certain the Space Station Program offices had the necessary computer connections, and she traveled to Japan to speak with the National Space Development Agency of Japan about their involvement in the International Space Station.

In 1997, she chose to retire from NASA when they offered her a buyout.[25] Having worked for thirty-four years, Jue had witnessed numerous changes in the workforce and in technology since her arrival. Women were no longer just secretaries or stenographers. They were astronauts and flight directors, and some served as branch or division chiefs. Just three years earlier, in 1994, Carolyn L. Huntoon, who also started at the Center in the sixties, became NASA's first female Center Director. Technology evolved over the years, and the mainframe computers employees once relied on were replaced with individual desktop computers. Jue made important contributions to the space agency in a mission support role, the type of work that if left undone, NASA would been unable to function. She provided vital services in the field of computer programming and as the technology evolved, in the procurement, distribution, and training of employees on personal workstations. Most importantly, she oversaw the dynamic development and testing of Shuttle software as manager of operations in the Software Development Lab and the Software Production Facility. Both were critically important to demonstrating and proving the reliability of the avionics system in flight.

Jennifer Ross-Nazzal

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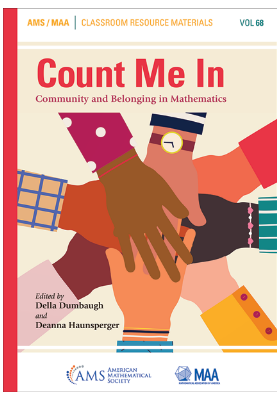
All figures and author photo are courtesy of NASA.

[25] *Jue interview by author.*

BOOK REVIEW

Count Me In: Community and Belonging in Mathematics

Reviewed by Emily J. Olson



Count Me In
Community and Belonging in Mathematics
Edited by Della Dumbaugh and Deanna Haunsperger
AMS/MAA Press, CLRM/68, 2022.

What are the biggest open problems you know in mathematics today? Perhaps your first thought was something quite famous, such as the Riemann hypothesis or P versus NP. Perhaps you have an open problem in your area of mathematics that you have thought about for a few months (or a few years). There are very few open problems that affect almost every mathematician. *Count Me In* highlights one in particular: How can we increase diversity among mathematicians?

The issue of diversity isn't new; in fact, mathematicians, academics, and students alike have noticed for a long time there aren't many people from [insert minority[1] group here] among their colleagues. I intentionally call this question "a problem" to imply there may be a solution.

Emily J. Olson is assistant professor of mathematics at Millikin University in Decatur, IL. Her email address is ejolson@millikin.edu.

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[1] *Throughout this review, I use the term "minority" to refer to any group of people that feel part of their identity is underrepresented in mathematics. I acknowledge I cannot discuss every group and intersection of identity in this review.*

Through its chapters, *Count Me In* outlines a variety of communities that exist or have existed in mathematics, many of which desire to enhance diversity in the mathematics profession. Some of these communities began specifically to address a need in mathematics, while others grew more organically and enhanced diversity as a happy byproduct of their activities. In this review, I will consider the "problem" of the lack of diversity in mathematics and discuss various communities presented in the book, focusing on why community is an important step toward increasing diversity.

The editors of the volume are themselves members of various communities. Della Dumbaugh is a professor at the University of Richmond and in 2022 became the second female editor in chief of *The American Mathematical Monthly*, a publication of the MAA. Deanna Haunsperger is a professor at Carleton College in Northfield, MN. She is currently the editor of the MAA's *Math Values* blog, and from 1995–2014 she codirected the Carleton Summer Mathematics Program for Women.

Throughout their careers, it seems likely that both of these women asked themselves why there isn't more diversity in mathematics. Identifying as a female myself, I have seen firsthand the effects of too few women in the room/panel/department/institution/program. In addition, I have benefited from the work of other female mathematicians who endeavored to welcome more women into the field.

Whether some of your identity intersects with a minority in mathematics or not, you might be wondering what this book has to offer you. Through many chapters about various communities, it offers a big-picture view of the experiences of mathematicians. You may not have had the same experiences as others, but that does not nullify anyone's experiences. We should listen, appreciate, and learn from each other; these stories indicate that the problem of diversity truly is a problem. We can only solve this problem together.

When you pick up this book, you will find easy-to-digest chapters that are not much longer than this review. They are perfectly engaging one at a time or completely bingeworthy as a group. The book is divided into two main sections: "Communities for undergraduate and secondary-school mathematics students" and "Communities for graduate students and professional mathematicians." No matter your job title, institution, or career path, it has something of interest for you.

Perhaps you teach. Have you intentionally made a community in your classroom? Chapter 1 describes how Uri Treisman at UC Berkeley builds community in his classrooms. These ideas seem fairly easy-to-implement in any classroom. For instance, use your students' names and talk to them about activities on campus or other courses they are in. Also, make them learn the name of at least one other student in the classroom; this can help build a community of learners.

Perhaps you are employed in a department at an institution of higher education. How can you effectively advise students whose backgrounds may be very different from your own? This book contains information about programs you can suggest to current undergraduate or graduate students, such as EDGE (Enhancing Diversity in Graduate Education) in Chapter 6, the Nebraska Conference for Undergraduate Women in Mathematics in Chapter 5, the Smith College Center for Women in Mathematics in Chapter 9, the Math Alliance in Chapter 10, the Infinite Possibilities Conference in Chapter 16, and the Spectra organization in Chapter 21.

Perhaps you are a student, and you would like to organize events in your department to build community among your peers. Chapter 11 discusses events held by the AWM Chapter at Youngstown State University. You would not need to host an official chapter of a professional organization to hold some of these same events (or suggest potential events to some of your professors!).

Perhaps your institution does not have as many mathematics majors as you would like. In Chapter 2, we learn about how St. Olaf in Northfield, MN, went from 18 majors to 100+ majors in the 1970s and 1980s. The professors did this by making mathematics seem like the most desirable major on campus. They did not let the coursework get stale, and they made it easy for students to add math as a second major. The institution let faculty focus on professional engagement opportunities rather than traditional research, which benefited the students overall.

Many communities include members who come from a variety of institutions. At the Duluth Undergraduate Research Program, discussed in Chapter 13, students come from all over the country to work on research for 10 weeks over the summer. Interacting with like-minded students while working on math research for a summer creates an unforgettable community that occurs at a pivotal time in a student's career. Similarly, the post-baccalaureate program at Smith College, discussed in Chapter 9, gives students the tools they need to be successful before attending graduate school the following year. Arguably, the support the participants at Smith receive is just as important as the educational component of the program.

There are communities that provide a view into graduate school for undergraduates, such as at the Nebraska Conference for Undergraduate Women in Mathematics in Chapter 5. Vertical integration of the communities is a common theme, since the members benefit from the experiences of others. Vertical integration is the mixing of mathematicians at a variety of levels, such as undergraduates working with graduate students or junior faculty working with senior faculty. This happened at the Carleton Summer Mathematics Program (Chapter 4), the EDGE program (Chapter 6), the Women in Numbers research community (Chapter 14), and many others. Even at a younger age, it seems that vertical integration is effective; the Prepare2Nspire program (Chapter 7) partners 8th graders with 11th graders at a tutoring program in Minnesota. Along with an undergraduate mathematics major, the 11th grade students serve as near-peer tutors for the 8th grade students to help them learn mathematics through accountability and community.

Much like using an approximation method to determine a numerical value, some communities have used a "try it and see if it works" approach. These communities tend to make changes over time, and their iterations better approximate the solution to their problem. The MAA Project NExT program (Chapter 18) uses this approach, especially as the community has grown larger over the years. While Project NExT focuses their year-long program on young professionals, they recently added a mentorship component to the program. It turns out that sharing ideas and learning innovative teaching techniques can occur at all stages of one's career; this vertical integration has enhanced the teaching potential of both the younger and older members of the community. Other programs similarly saw benefits for all of their members, junior and senior alike, when interactions were mixed. The SIGMAA on Research in Undergraduate Mathematics Education (RUME) in Chapter 25, the AIM SQuaREs program in Chapter 22, the Women in Numbers program in Chapter 14, the Women in Mathematics (WAM) program at IAS in Chapter 15, and MSRI in Chapter 26 all describe intentionally mixing junior and senior researchers at their conferences, workshops, and events. In fact, the presence of senior researchers is a vital component of a program's success; as Karen Uhlenbeck explains about the WAM at IAS program on page 138, "The biggest lesson I learned is that the senior women benefited even more than the participants."

While some communities grow organically, some begin with very clear intentions. The WAM at IAS program (Chapter 15) aims to see more women fill tenure and tenure-track jobs at Tier 1 research institutions. MSRI (Chapter 26) offers full funding to support women and other underrepresented groups with their research, which can influence the participant's academic job prospects. The chapters in *Count Me In* discuss the success that programs have had achieving their goals. While it may be impossible to attribute an individual's successes to a particular community, it is clear that the support, be it financial, social, emotional, academic, or a combination of these, is valuable.

Of course, a community should expect to encounter challenges. For instance, we are warned against a "savior mindset" in teaching mathematics to those in a drastically different culture from our own. This is especially true in Chapter 12 which outlines Indigenous and Latinx communities in mathematics. In Chapter 17, we learn about communities of female mathematicians in Africa and the challenges the culture and physical distances bring. Women mathematicians tend to be very spread out across Africa, making it hard to plan conferences which many researchers are able to attend.

Chapter 14 about the Women in Numbers community was especially explicit about the difficulties in establishing and maintaining a community. Funding is hard to obtain and organizing workshops and conferences is very time-consuming. As I read this and other chapters, I noticed that many of the groups were started and maintained by members of the community themselves. Of course, it makes sense for leadership to come from within the community. However, I can easily see how a member of a minority group in mathematics only has so much bandwidth to be a leader on top of all their other duties as a mathematician. I don't mean to imply that leadership should come from outside the community either; rather, my observation makes me wonder what I can do to support the valuable work these leaders are doing to maintain their communities. In the end, these leaders serve as role models and are a vital component of their communities. For instance, in Chapter 8 we learn that the success Bryn Mawr College has had at graduating female PhD students is attributed in part to the presence of strong role models at their institution.

Another challenge is: who do we invite to the community? Sometimes a group is focused on a common idea while other times, on a common identity. In Chapter 19, the founder of the Math Mamas Facebook group grappled with wanting to include other mathematicians who are parents, but ultimately decided that those who are navigating motherhood and a mathematical career needed a community. In addition, some of the chapters ask readers to face uncomfortable truths, no matter one's identity. Reading about the lengths taken to keep Black students, professors, and researchers out of mathematics in Chapter 20 is a difficult yet important part of learning about our larger mathematical community.

Juxtaposed with difficulties are celebrations. Truly, communities are built around a common idea and thrive due to the personalities of its members. Many communities offer a chance for their members to be themselves. This is an essential aspect of humanity; Brené Brown, a researcher who studies belonging, explains, "True belonging doesn't require us to change who we are; it requires us to be who we are" [1]. Members of ECCO (Chapter 3), Spectra (Chapter 21), and the Infinite Possibilities conference (Chapter 16) described how these communities helped them feel welcome and "feel less alone" (p. 149). The Infinite Possibilities conference, originally created to celebrate Etta Falconer and the graduates of Spelman College, asked a particularly illuminating question on page 151, which is

> "How, then, can we create a mathematical community where we don't have to check some part of our identity at the door in order to enter?"

There is a real need for community. Brené Brown addresses this by saying, "We want to be a part of something-to experience real connection with others - but not at the cost of their authenticity, freedom, or power" [1]. Therefore, to have a sincere community, it's essential that everyone is true to themselves.

What is the cost of exclusion? On a professional level, someone might be overlooked for a promotion or opportunity based on their identity. Even worse, Chapter 21 describes members of the LGBTQ+ community who are fearful of losing their jobs if they reveal their true selves.

Why do we need the communities featured in *Count Me In* and others? It would be a beautiful, equitable world if individuals were valued for their contributions and experiences, rather than if they are a "good fit" for a group or not. However, we are far from this equitable paradise. Progress comes when we listen to those most vulnerable. For instance, in Chapter 3 we learn sidewalk ramps benefit many people, not just those in wheelchairs for whom they were installed. Until the sidewalks of mathematics are truly available to everyone, we need to listen to the stories of minority groups and work to build "ramps" to help everyone in mathematics thrive.

Reading a book like *Count Me In* allowed me the chance to see a big-picture view of many mathematical communities and the problems that no one community addresses on its own. I often describe mathematicians as "pattern finders" to my students, and I couldn't help but notice some patterns here. I noticed many stories of individuals who initially felt like they didn't belong in math; in the Math Alliance program in Chapter 10, many participants self-described their experience as "unique." In the communities of ECCO (Chapter 3) and Spectra (Chapter 21) and so many others, I read about individuals who were worried they weren't on the "right" path in mathematics. They felt excluded until they found a community that welcomed them. These communities continue to evolve, just as humans continue to evolve. We aren't ever done learning. Even if you think diversity in mathematics isn't a problem you created, it's an issue that we all are responsible for solving.

Many research questions, when pondered, lead to more questions. As you think about the communities to which you belong, you might ask yourself,

- What makes you feel included (or not) in that community?
- What works in that community and what needs to be improved?
- What about those who feel like they don't fit into any community? How can we bring them in?

Chris Stevens summarizes the important aspects of many communities when discussing MAA Project NExT in Chapter 18:

> "The communities that we build should be organic, growing out of their local situation and reflecting local needs. They should be integrated, with all features working harmoniously together. They should be diverse, because diversity makes a community more useful and thus more valuable. Finally, their essence should be defined by the people who participate in them."

Stevens's words are a helpful call to action. In reading them, I realize that when I see a need, I have the power to build a community, perhaps using ideas from this book. And you do, too.

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[1] Brené Brown, *Atlas of the heart*, Random House, New York, NY, 2021.

Emily J. Olson

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BOOKSHELF

New and Noteworthy Titles on our Bookshelf
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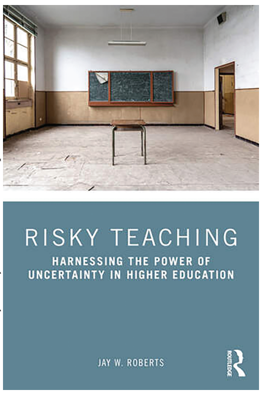

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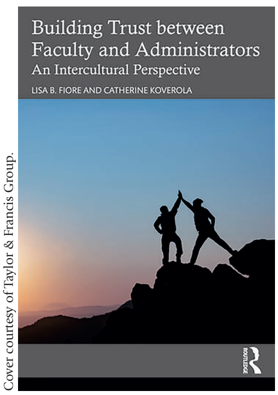
Risky Teaching
Harnessing the Power of Uncertainty in Higher Education
By Jay W. Roberts
Routledge, 2021, 174 pp.

Higher education is facing a plethora of new challenges from changing student demographics to financial strains brought on by declining enrollment and Covid related expenditures. Faculty members are finding themselves pivoting to meet the needs of a new student body who are also adjusting to life back in the classroom after pandemic learning. How can we use this period of adjustment and turmoil to help make us better educators that provide all students with a more impactful learning experience? *Risky Teaching* encourages us to think differently about our classrooms and how we use class time to make this transition.

In the first part of *Risky Teaching*, uncertainty and risk are explored at the broad level of higher education and then again as it pertains to changing student profiles and the recent challenges facing faculty. The second part of the book introduces ideas for how to embrace uncertainty and attempt to build opportunities for uncertainty into a course, using it to create a more student-centered learning environment. A chapter of the book is dedicated to how to design hands-on moments of uncertainty that will hopefully result in meaningful learning opportunities for students. Roberts acknowledges throughout that this can be daunting and encourages the reader to start with small tasks and build your new course over time. While redesigning content delivery, it is natural to think about more effective ways to assess student learning. Roberts dedicates another chapter to this, drawing on his long career as an educator throughout. This book provides an excellent, user-friendly guide for faculty members in any field, including mathematics, looking to rethink how they deliver content to an ever-changing student body.



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Building Trust between Faculty and Administrators
An Intercultural Perspective
By Lisa B. Fiore and Catherine Koverola
Routledge, 2021, 218 pp.

The relationship between administrators and faculty is often filled with mistrust. While higher education is an institution that promotes education, it is also a business, and as a result faculty and administrators can have different experiences and priorities. Finding commonalities and learning how to communicate is crucial for helping to bridge the divide that exists. *Building Trust between Faculty and Administrators* aims to help in this process.

The book is broken into three parts: The first part highlights the differences in the administrator and faculty cultures which can lead to a lack of understanding and mistrust. It discusses the need for better communication, working together and pooling resources to achieve common goals, and listening without judgement. The second part highlights the different priorities each group has that can contribute to tense relationships and the third suggests practicing inclusive leadership and leading with compassion to help foster trust.

Written by long-time faculty members, one of whom has ample experience as an administrator, several case studies are used to dive deeper into the tensions that exist between faculty members and administrators. Discussion questions that can help promote thoughtful dialogue between the two groups and feedback from faculty and administrator focus groups are incorporated throughout. With the uncertainty facing higher education, it is becoming increasingly important that we work collectively to help promote the growth and health of our institutions. This book can help foster the relationships and understanding required to achieve such a task.

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AMS BOOKSHELF

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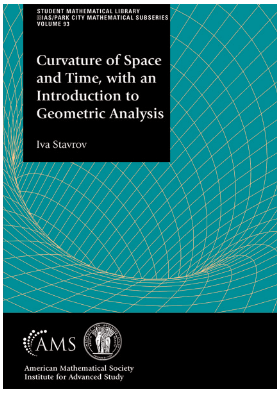
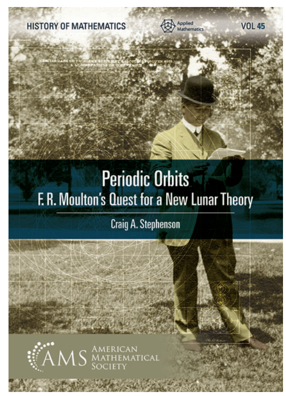


Curvature of Space and Time, with an Introduction to Geometric Analysis
By Iva Stavrov
AMS, 2020, 243 pp., STML/93.

This book introduces advanced undergraduates to Riemannian geometry and mathematical general relativity. The strategy of the book is to explain the concept of curvature via the Jacobi equation which, through discussion of tidal forces, further helps motivate the Einstein field equations. The material in this book was first developed for a summer program at Park City and later modified and expanded to reflect the author's experience of teaching mathematical general relativity at Lewis & Clark College. Stavrov's experience is that of de-facto converting strong physics majors into math majors through the use of this text.

Before she wrote her own book, Stavrov would send her students to Woodhouse's book on general relativity. Like Woodhouse, she likes to present an undergraduate-friendly tidal-force narrative, which is then used to motivate the Einstein equations. In that sense these two books are similar in spirit. However, their goals are different. Woodhouse discusses relativity and uses geometry and curvature as tools and interprets them exclusively through physics. By contrast, Stavrov's book is primarily mathematical and introduces its readers to differential and Riemannian geometry as well as to geometric analysis.

This small book contains a wonderful collection of results, arranged in what seems to be the perfect order. Ideas are always illustrated with simple but nontrivial examples. It is the ideal reference for students of elementary differential geometry who want to familiarize themselves with the mathematical ideas of relativity theory.



Periodic Orbits: F. R. Moulton's Quest for a New Lunar Theory
By Craig A. Stephenson
AMS, 2021, 255 pp., HMATH/45.

To determine the gravitationally produced motions of the Sun-Earth-Moon system was the first, most important, and still unsolved example of the three-body problem. Poincaré believed that periodic solutions were dense in the set of all solutions; nonperiodic solutions could be viewed, at least locally, as perturbations of periodic solutions. Thus, early twentieth-century celestial mechanics expended considerable effort in finding periodic orbits. American astronomer Forest Ray Moulton created analytic tools and discovered entire new classes of such orbits. This book provides the first in-depth account of Moulton's research by describing his 1920 *Periodic Orbits*, which arose from his ambitious goal of creating an entirely new lunar theory.

After providing the motivation for the study of the three-body problem and its close connection with the lunar theory, the author provides some background on the pioneering work of others, such as George William Hill, Ernest William Brown, Henri Poincaré, and George Howard Darwin. Stephenson then focuses on the contribution to the subject made by Moulton and his students at the University of Chicago, much of which is showcased in Moulton's *Periodic Orbits*. He also includes some information about Moulton's travels and correspondence that paints a newly enriched picture of him as a person, administrator, and scientist.

Stephenson's comprehensive and precise exposition is a masterly contribution to the history of celestial mechanics that highlights the centennial publication of Moulton's *Periodic Orbits*, preserves it from oblivion, and gives it the wider audience it deserves. Readers will need a solid background in celestial mechanics to appreciate some of the more sophisticated ideas and techniques.

The AMS Bookshelf was contributed by AMS Book Acquisitions Consultant Ina Mette. For questions or comments, please email acquisitions@ams.org.

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COMMUNICATION

Tackling the Gender Gap from an Institutional Perspective

The Example of IHES

Giulia Foffano, Claire Lenz, and Emmanuel Ullmo

Gender imbalance remains high in mathematics and theoretical physics. This is particularly true at IHES, where only 10% of the visiting researchers are women. IHES is fully intent on reducing that gap and attracting more women researchers.

Without claiming to set an example, this article intends to be a conversation starter about how research institutes in mathematics can attract more women scientists, encourage more women to become mathematicians, and ensure that women get fair and equal treatment when they do so.

To reinforce gender balance, and more generally reinforce diversity and inclusion, IHES is tackling the issue from several angles, with a special focus on:

- Building key partnerships to strengthen diversity and inclusion;
- Celebrating and supporting women scientists at IHES.

Building Key Partnerships to Strengthen Diversity and Inclusion

When reflecting on the topic of gender imbalance, as well as the more general diversity and inclusion issues in STEM, a useful first step is to turn toward dedicated networks and communities, with experts working in the field locally, nationally, and globally. Benchmarking and looking at what similar institutions are doing is also an excellent way to think about relevant initiatives and projects to implement. Finding financial support and partners willing to fund these projects is essential, too, when developing a targeted strategy and aiming for long-term improvements.

Identifying key local partners. As a founding member of Université Paris-Saclay, located in the Paris region with its unique hub for mathematics and related sciences, IHES is at the heart of a vibrant scientific ecosystem. Given its limited size (a dozen faculty members, on top of which the Institute welcomes about 20 postdocs and PhD candidates, as well as 200 visiting researchers each year), it seems important for a research institute like IHES to reach out to local partners for stronger impact, especially when it comes to diversity.

Together with IHES, Université Paris-Saclay brings together ten constituent faculties and institutes, four *grandes écoles*, two associate institutions, and laboratories shared with national research organizations, representing 48,000 students, 8,100 academic staff, and 8,500 administrative and technical staff. Specialists working specifically on diversity and inclusion issues cover the entirety of all the university's faculties and institutes. Connecting with them immediately gives IHES access to a variety of materials and relevant resources, and ensures a better reach when promoting the Institute's initiatives. For example, in order to take part in the May12 initiative celebrating women in mathematics in 2022, IHES benefited from the support of several local partners. IHES borrowed the exhibition "Infinités plur*ielles*" (a play on "plural" and the pronoun "she") by photographer and artist **Marie-Hélène Le Ny** from Université Paris-Saclay. It showed several portraits of women scientists, with texts presenting their research. The

Giulia Foffano is development officer at IHES. Her email address is foffano@ihes.fr.

Claire Lenz is director of development and communications at IHES. Her email address is claire.lenz@ihes.fr.

Emmanuel Ullmo is director of IHES. His email address is dir@ihes.fr.

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public could visit the exhibit freely for a month at the Institute and on May 17, 2022, a dedicated event was organized with another key local partner, the Fondation Mathématique Jacques Hadamard (FMJH). On that occasion, **Bertille Follain**, a PhD student at Ecole Normale Supérieure and laureate of the Junior Maryam Mirzakhani 2021 prize in applied mathematics awarded by the FMJH, introduced the speakers. Marie-Hélène Le Ny presented her project and **Indira Chatterji**, professor of mathematics at Université Côte d'Azur, gave a talk: "Institutional bias, explained by a mathematician." Reminding the audience that in statistics a bias is the discrepancy between the theoretical value of a variable and the observed value, she used neutral examples to explore the ways biases could be created. Watch the replay of this event here.

Figure 1. Indira Chatterji and Bertille Follain at the event organized at IHES on May 17, 2022.

Working toward a more inclusive language is another subtler example of how diversity and inclusion at IHES might benefit from its partnerships with larger organizations. In France and elsewhere, the way language can be used to make it more inclusive is a source of debate. Meetings on this specific issue are time-consuming and often lead to heated conversations. That being said, organizations of a certain size have by now typically agreed on clear guidelines and rules to produce communication materials with fewer implicit biases. It is easier for an institute like IHES to then adopt these guidelines, given that they have already been approved by partner institutions. Indeed, as IHES regularly has to produce communication materials with partners, it becomes necessary to take these considerations into account. For example, when referring to a woman professor, both "professeur" and "professeure" are available options in French. The latter is often regarded as being more inclusive, as it underlines the fact that the professor is a woman. This is also the choice recommended both by Université Paris-Saclay and CNRS, and it was therefore adopted by IHES.

A more global conversation with dedicated organizations. At an international level, research institutes can take advantage of the experience, hindsight, and know-how of several organizations created over the years to raise awareness and promote equal opportunities for women in mathematics—and actually contribute to promoting their work in doing so. For example, IHES has been collaborating closely with the Association for Women in Mathematics as well as the European Women in Mathematics, both of which offer numerous activities and provide several tools to empower female mathematicians. As the Institute continues to think of ways in which to improve on diversity and inclusion, several experts from these networks have given some of their time to discuss ideas, exchange opinions, and suggest concrete initiatives.

This in turn led to the organization of a very fruitful panel discussion in October 2021 with the following speakers:

Eva Bayer-Fluckiger is a mathematician whose interests include algebraic number theory, Galois cohomology of algebraic groups, quadratic forms, Hermitian forms and algebras with involution, knot theory, and the application of algebra and number theory to algebraic codes. She was invited to IHES several times. She is currently professor emeritus at École Polytechnique Fédérale de Lausanne.

Kathryn Leonard is a mathematician and computer scientist whose research interests include geometric models for computer graphics, computer vision, and data analysis, with an emphasis on explainability. She is a professor of computer science and director of the Center for Undergraduate Research in Mathematics at the Occidental College, Los Angeles. She was the president of the Association for Women in Mathematics from February 2021 through February 2023.

Andrea Walther is an applied mathematician working on nonlinear optimization, non-smooth optimization, and scientific computing. She is known in particular for her work on automatic differentiation. She is professor of mathematical optimization at the Institute for Mathematics at Humboldt University of Berlin. Walther has been convenor of the European Women in Mathematics since July 2020.

By sharing their experiences and their expertise, speakers helped identify relevant initiatives to support and promote women researchers in mathematics and to ensure that they are better represented at institutions like IHES. Watch the discussion here.

Some main take-home messages were identified:

- It is important to be proactive in advertising the call for applications, for example by partnering

with associations such as EWM and AWM to promote the available positions and to identify female talents who will be offered particularly attractive visiting packages;
- It is important to provide occasions for women to network and create a community that can foster a sense of belonging. Sessions dedicated to topics to improve gender equality—mentoring sessions, discussion on work/life balance—might be included;
- It is important to support women at the early stages of their careers, especially since what is regarded as the most productive time for a researcher often coincides with the period in which women have children.

Figure 2. Kathryn Leonard and Méline Ruel at the panel discussion Promoting Women in Mathematics on October 4, 2021.

IHES has already started implementing some of these points. For example, the latest calls for applications were advertised on the EWM website and newsletter. However, the number of women applicants remains far too low, and this is a good opportunity to remind all readers that the Institute reviews applications for research visits twice a year, in June and December.

Best practices in other organizations and research institutes. When reflecting on how to improve diversity and inclusion in an institution, another approach naturally consists in looking for the best practices implemented by other organizations. Benchmarking is always an interesting way to challenge the status quo and look for possible improvements. Each institution has its own identity and its particular constraints, but the gender imbalance is a widespread concern in STEM research institutes, as well as in a number of organizations, including in private companies recruiting STEM candidates. Learning from the experience of others is very enriching. There are few organizations worldwide with which IHES can easily compare itself. However, in the US, two institutions in particular operate in a similar way.

The first is the Institute for Advanced Study in Princeton. The IAS actually served as the initial model on which IHES was founded and there are naturally similarities between the two institutes. It is therefore interesting to study the initiatives taken by the IAS to promote gender diversity, especially at the School of Mathematics. For example, the IAS has developed its annual Women and Mathematics Program, whose mission it is to recruit and retain more women in mathematics. To get a sense of what initiatives worked and why, as well as what didn't work out so well, it is useful to share feedback with both the scientists who experienced them and the staff involved in setting them up and monitoring the results. Special thanks should be addressed to **Nicole Maldonado**, Administrative Officer for the School of Mathematics at the IAS, who serves on the IAS Diversity, Equity, and Inclusion Committee, and on the IAS Title IX Misconduct Panel, who helped fuel the conversation on these issues at IHES.

The second is the Simons Laufer Mathematical Sciences Institute in Berkeley. Some of the initiatives implemented could be replicated at IHES. For example, a research visit far from home can be challenging, not only for the researchers themselves, but also for the family members who accompany them. To address this, the Simons Laufer Mathematical Sciences Institute provides extensive family services, including grants for childcare for researchers with children under the age of 17. Knowing that these services are available gives an extra incentive to researchers who might consider a visit, especially women scientists requiring childcare, as they could be discouraged from applying to a research institute. At IHES, we also recognize that the type of family support made available can play a significant role for women not only when deciding to accept an offer for a research visit, but also when deciding to apply for one.

Financial support for lasting change. These services and programs obviously require funding, which often takes time to secure. Such funds are necessary to ensure that the Institute's strategy on those issues are long-term oriented. IHES already benefits from some financial support to improve diversity and inclusion. James and Marilyn Simons, major IHES donors and current cochairs of Friends of IHES, the Institute's partner organization in the US, have made a significant ten-year contribution to the Institute's operational budget. Some of the funds have been specifically earmarked to support initiatives in that direction at IHES, so that it can plan ahead and get a chance to properly assess the results of its efforts. The Agilent

Technologies Foundation, which also aims to bring about lasting change, has chosen to make its gift to the Friends of IHES' endowment fund so that the proceeds generated by the donation support women scientists visiting the Institute, not just on a one-off basis, but also in the long run. The funding already secured makes it easier to approach new donors willing to accompany IHES further in this direction. We are therefore currently working on building the necessary funding to develop family support for our visiting researchers at IHES, which would certainly be a very significant step forward. More specifically, the Institute wishes to offset the cost of childcare for its visiting researchers, reserving specific funding at first for women scientists with children under the age of ten, who prove to be the most likely not to apply for a visit at IHES because of concerns related to childcare. In the longer term, the goal is to extend child support to all researchers with young children to help them focus more easily on their research during their stays at the Institute.

Celebrating and Supporting Women Scientists at IHES

Inspiring role models at IHES. One of the obstacles that young women in research face is a sense of isolation due to a lack of role models. Research institutes such as IHES have an important role to play in that regard by celebrating women scientists as an example and an inspiration to younger generations. This means a commitment to better gender balance within the organization, and the implementation of a communication strategy that aims to provide women researchers with greater visibility, for example by showcasing success stories and presenting their experiences.

No matter how underrepresented women have been at IHES, several female scientists have been associated with the Institute over the years. It seemed important to start by highlighting their experience and making it available to younger generations of researchers.

Far from providing a comprehensive record of all the women who have been affiliated with IHES over the years, here we present a list of the researchers whose interviews have been published between 2020 and 2022, as part of the Institute's communication efforts to increase the visibility of women scientists at IHES.

Remarkable female scientists have marked the history of IHES and significantly contributed to its development. Two of the women who have had the most lasting impact on IHES are Cécile DeWitt-Morette and Yvonne Choquet-Bruhat. Their life histories are intertwined and IHES was where they worked together and developed a very close friendship. They also coauthored a book on mathematical physics, *Analysis, Manifolds and Physics*.

Figure 3. Cécile DeWitt-Morette and Yvonne Choquet-Bruhat at IHES.

Cécile DeWitt-Morette was a physicist, best known for having founded Les Houches School of Physics, where generations of physicists have learned from some of the greatest scientists of the 20th and 21st centuries. It is she who introduced the Institute's founder, Léon Motchane, to Robert Oppenheimer, then director of the Institute for Advanced Study, thus playing a key role in the Institute's founding. She was also a member of the Board of Directors.

Yvonne Choquet-Bruhat is both a mathematician and a physicist. She was the first woman to be elected to the prestigious French Academy of Sciences. She arrived at IHES late in her career, when she retired, but she kept on working. Being at the Institute allowed her to continue to have an active role in the physics and mathematics community.

Cécile DeWitt-Morette passed away in 2017, and it was not possible to communicate directly with Yvonne Choquet-Bruhat, who lives in a care home. However, their respective children, **Christiane** and **Nicolette DeWitt**, and **Daniel Choquet**, agreed to help us retrace their connections with the Institute. They also shared some thoughts on how their mothers experienced their role as women researchers. DeWitt-Morette and Choquet-Bruhat were among the few women of their generation to lead very successful careers in research, which is why their children's testimonies are particularly poignant. The article that resulted from those memories can be found here.

The Institute also contacted former IHES visiting researchers, as well as senior women scientists associated with the Institute. A number of them kindly agreed to answer questions about their experience at IHES. Conversations led to learning more about how the Institute could improve the experience of visiting scientists and represented an opportunity to pay tribute to women researchers who have taken part in the Institute's scientific activity.

Some of the scientists contacted gave video interviews; others preferred to express their thoughts in an article. Most of them left a short testimony. This resulted in a series of interviews, *Voices of Women at IHES*, published between May 2021 and January 2022. The women whose testimonies were published as part of this initiative are (appearing in the order of publication):

Sylvie Retailleau, the current French Minister of Research and Higher Education, who was then president of Université Paris-Saclay. Being a physicist herself with a multifaceted career, first as a researcher and as a university professor, and then as a top manager at one of the best-ranked universities worldwide, she shared her vision of IHES and gave her take on the importance of gender diversity and inclusion in science. Her interview can be found here.

Erica Flapan, editor-in-chief of the *Notices* and professor emerita of mathematics at Pomona College, California. She is an inaugural Fellow of the American Mathematical Society. In her testimony, available here, Flapan shared her memories of working at IHES and stressed the importance of mentoring, which she has been actively doing throughout her career. In recognition of her particular dedication to mentoring women, she was awarded the M. Gweneth Humphreys Award by the Association for Women in Mathematics.

Motoko Kotani, professor at the Mathematical Institute of Tohoku University, executive vice president of Tohoku University for Research, and current president of the International Science Council. In addition to sharing her experience as a mathematician, she described the difficulties experienced by women scientists who want to have a career in Japan and the type of initiatives she has set up to promote gender diversity in this context. You can read her full testimony here.

Alessandra Buonanno, director at the Max-Planck Institute for Gravitational Physics in Potsdam, also called the Albert Einstein Institute. Alessandra Buonanno spent two years at IHES from 1997 to 1999 as a postdoctoral researcher. The work she carried out there in collaboration with Professor **Thibault Damour**, won her the Galileo Galilei Medal, the Dirac Medal, and the Balzan Prize in 2021. In her testimony, which you can find here, she talks about the importance of her work as a postdoc.

Delaram Kahrobaei, dean for research at the City University of New York, and chair of cyber security at the University of York (UK) at the time of the interview. She is also the founder of the New York Women in Mathematics and Computing Network. Her testimony can be found here.

Jennifer Chayes, mathematician and physicist, associate provost of the new Division of Computing, Data Science, and Society at the University of California, Berkeley, dean of the School of Information, and professor in mathematics, electrical engineering and computer sciences, information, and statistics. In her interview, she enthusiastically recalled her time as a postdoc at IHES and explained how it benefited her career. She also shared her views on what a leader should think about when making new hires and her advice to women scientists. Her interview can be found here.

Figure 4. Alessandra Buonanno at the conference Damour Fest: Adventures in Gravitation organized at IHES on October 12–15, 2021.

These, and further testimonies from **Caterina Consani**, professor at John Hopkins University, **Fanny Kassel**, CNRS research director at IHES, and **Susan Friedlander**, director of the Center for Applied Mathematical Sciences at University of Southern California, contributed to *Towards More Women Scientists at IHES*. This is a video collection of their experiences as researchers in a male-dominated environment at IHES, as well as what they think are important steps toward making the Institute more welcoming to women researchers.

In a separate video, *Cultivating diversity in STEM*, women scientists familiar with IHES, namely **Alessandra Buonanno**, **Jennifer Chayes, Caterina Consani**, **Delaram Kahrobaei**, **Kathryn Leonard and Sylvie Retailleau**, express their powerful messages to younger generations of researchers, by sharing their advice and experience.

In addition to focusing on former visiting researchers, it is important to draw special attention to the three women scientists currently associated with IHES: **Fanny Kassel**, a CNRS research director who joined IHES in 2016, **Laure Saint-Raymond**, who joined IHES in 2021 as a permanent professor, and **Yilin Wang**, who joined IHES in 2022 as the first junior professor. As Laure Saint-Raymond was the first female permanent professor at the Institute, IHES wanted to particularly highlight her research and the relevance and impact of her work as a mathematician. IHES has so far published three videos aimed at a broad audience, in which Saint-Raymond explains the background to her work and her clear vision of research. The three videos were produced with other institutions.

An animated video generously produced by the Simons Foundation for IHES retraces the foundations of Saint-Raymond's contributions to solving one of the major mathematical challenges formulated by the great mathematician David Hilbert at the beginning of the 20th

Figure 5. Laure Saint-Raymond at the conference Large-Scale Limits of Interacting Particle Systems organized at IHES on October 4–8, 2021.

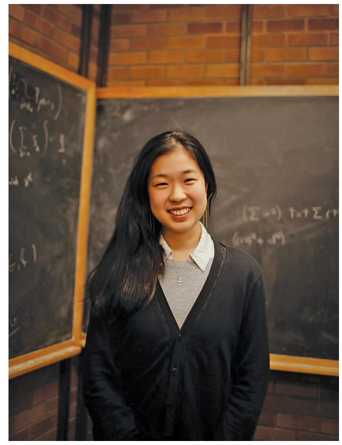


Figure 6. Yilin Wang, first junior professor at IHES.

century. The video starts from the theory developed by physicist Ludwig Boltzmann in the 19th century, and explains its connection to Hilbert's sixth problem. Saint-Raymond has helped get a step closer to solving it. The video provides some context to her work, and an appreciation of its impact and importance.

A conversation between Saint-Raymond and **Nathalie Ayi**, assistant professor at Sorbonne Université, Laboratoire Jacques-Louis Lions (CNRS/Sorbonne Université/Université Paris-Cité), one of her collaborators and her former PhD student, is at the heart of a second video. In it, the two researchers discuss the way they see research and the important role that scientists, particularly mathematicians, can play in talking about their work and making it understandable by as wide an audience as possible. This conversation was recorded in January 2022, in preparation for the 2022 International Congress of Mathematicians, where Saint-Raymond was a plenary speaker, the video was produced with CNRS.

In a third, shorter video interview, also made with CNRS, Laure Saint-Raymond answers three questions about what it means to be a mathematician and the qualities that are required to succeed in mathematics. The interview, in French, is aimed at an audience of middle and high school students, with the purpose of showing them that mathematics is a very creative discipline, where conversations and informal discussions lead to new thoughts and ideas.

In June 2022, **Yilin Wang** joined IHES as the first junior professor, a prestigious position for researchers at the beginning of their career who have distinguished themselves by their remarkable talent in the discipline. An interview with Wang was featured in the IHES annual newsletter, published in the Fall of 2022.

Taking a stand for women in fundamental research and other male-dominated fields. In addition to women academics, important role models can be found among women outside universities. Many have managed to make themselves heard in largely male-dominated environments, or have simply spoken their truth and expressed themselves. Two events organized in 2021 were important opportunities to celebrate women's work in a wide range of fields including a presentation of the book *Do Not Erase: Mathematicians and their Chalkboards*, by photographer Jessica Wynne, and the 2021 Friends of IHES Gala on Women in Fundamental Research.

On September 15, IHES and Institut Henri Poincaré (IHP) organized a joint event based on the book ***Do Not Erase: Mathematicians and their Chalkboards***, by photographer Jessica Wynne. The book is a collection of more than one hundred photographs of mathematical chalkboards, gathered from a diverse group of mathematicians around the world. The photographs are accompanied by essays from each mathematician, reflecting on their work and processes. The event took place at IHP, in the presence of author Jessica Wynne and mathematician Amie Wilkinson, a regular visiting researcher at IHP and a former visitor at IHES, whose work is portrayed in the book.

Jessica Wynne is an associate professor of photography at the Fashion Institute of Technology. Her photographs are in collections at the Morgan Library & Museum and the San Francisco Museum of Modern Art and have been exhibited at the Whitney Museum of American Art and the Cleveland Center for Contemporary Art.

Amie Wilkinson is a professor of mathematics at the University of Chicago. She works in ergodic theory and smooth dynamics with a focus on the interplay between dynamics and other structures in pure mathematics—geometric, statistical, topological, and algebraic.

Wynne presented her project, which was inspired by her conversations with mathematician Amie Wilkinson. In turn, Wilkinson described the importance of blackboards in her work, as well as the experience of having her work as a mathematician portrayed by a photographer and friend. In addition to inviting reflections on the unique relationship between mathematics, art, and creativity, the event was an opportunity to hear two professional women talk about their work and their collaborative experience.

For IHES, the 2021 Friends of IHES Gala, dedicated to women in fundamental research, was an important celebration of women researchers as well as of other strong advocates for diversity and inclusion.

The gala featured a keynote speech on black holes by theoretical astrophysicist **Janna Levin**. The emcee for the evening was **Gioia De Cari**, an actress, singer, and playwright, who defines herself as a "recovering mathematician" and whose critically acclaimed award-winning play ***Truth Values*** is based on her experiences as a woman in the male-dominated world of mathematics. The gala was also an occasion to celebrate honored guest **Marilyn Simons**, in recognition of her incredible contribution to supporting and advancing fundamental research.

The gala featured a virtual component that included a recorded message by the president of the European Central Bank, **Christine Lagarde,** in which she highlighted the importance of encouraging women to thrive in any discipline. She also stressed how more gender-balanced working groups benefit from richer perspectives and a more diverse environment.

Dominique Senequier, President of Ardian, one of the sponsors of the gala, also recorded a personal message in which she talked about her experience as one of the first women graduating from the prestigious Ecole Polytechnique in France, and how she became a successful business woman, with a commitment to making Ardian a diverse and inclusive workplace. In her message, Senequier also shared what she thinks are some of the main actions that can be taken to make science more inclusive:

- Attention to inclusivity should be given from early education by organizing initiatives in schools that can show young women that science is exciting and that they have a role to play;
- Traditional structures in workplaces and institutions should be changed to ensure that they do not hold women back, for example by adopting charters in which they commit to welcome and embrace diversity "in all shapes and forms";
- Clear targets should be set and progress should be tracked and made publicly available.

All the videos recorded for and during the gala are available online here.

Listening to women scientists at IHES. Although the first female permanent professor was recruited in 2021, several prominent women scientists have been affiliated with the Institute over the years. Some of them have played major roles in decision-making in different ways.

Most recently, **Claire Voisin**, CNRS Research Director at IHES from 2007 to 2009, was the editor in chief of *Publications Mathématiques de l'IHES* from 2010 to 2019. The three women scientists who are currently members of IHES' Scientific Council, actively shape and guide the Institute's scientific strategy: **Hélène Esnault**, professor at Freie Universität Berlin (since 2016); **Fanny Kassel**, CNRS research director at IHES (since 2019); **Laure Saint-Raymond**, permanent professor at IHES (since 2021). **Yilin Wang**, junior professor at IHES, is also invited to participate in the Scientific Council when selecting visiting researchers. Not only are these women mathematicians familiar with the Institute, they also have a significant say in its development and future.

In 2019, both the IHES Board of Directors and the Scientific Council adopted a Code of Ethics for Research, based on the work of the Council for Research Ethics and Scientific Integrity of Université Paris-Saclay and the CNRS Ethics Committee. Among various other points, it sets out a principle of fairness, which applies to selection procedures for scientific programs, the invitation of scientists, recruitment procedures, and the selection of the director. For any scientific event organized at IHES, this Code of Ethics states that a "reasonable number of women should be involved at all levels: organizing committee, scientific council, speakers, and participants." To ensure that this fairness principle is regularly assessed, quantitative data is presented at the meetings of IHES' Scientific Council, giving all members a chance to appreciate both the progress made and the aspects that still need to be improved. There are two such meetings of the Scientific Council every year, ensuring that this topic is brought to everyone's mind on a regular basis.

Raising awareness of diversity and inclusion in STEM disciplines. IHES' community is encouraged to consider the existence of unconscious biases as well as the impact they can have on one's career. To that end, the Institute regularly promotes communication materials raising awareness of diversity and inclusion, and organizes specific events on that topic. For instance, the Institute draws attention to important dates related to gender diversity in STEM, such as the International Day of Women and Girls in Science on February 11, the International Women's Day on March 8, and the May12 global initiative which celebrates women in mathematics more specifically. These are all opportunities to talk about past and current women scientists at IHES, be it through a simple post on social media, a video, an article on the website, a conference, or a talk at the Institute, highlighting the existing challenges for women in STEM while also celebrating the progress already achieved.

As for organizing events, being in tune with what expert organizations – such as those mentioned earlier – are putting out, is a great way to access materials as well as to find spokespeople familiar with these issues. This is how some of the IHES staff heard about the 2021 movie, *Picture a Scientist*, a documentary directed by Ian Cheney and Sharon Shattuck, and produced by **Manette Pottle, Ian Cheney**, and **Sharon Shattuck**. It shows a variety of points of view on how to make science more diverse, equitable, and open to all. IHES decided to make it available to the larger community: people could watch it online on request and the Institute also offered a public screening. Later, **Martine Liautaud**, founder and president of the Women Initiative Foundation, was interviewed on the role of women in science and in the corporate world by **Bruno Dranesas,** director of editorial strategies at ELEPHANT. The purpose of that discussion was to reflect on the lessons that higher education and research can learn from certain initiatives implemented by companies, emphasizing the importance of certain aspects such as:

- Mentoring and networking;
- Acknowledging and taking into account the difficulties and obstacles encountered at an institutional level by women throughout their careers;
- Creating occasions for women to share their experience and actively highlighting their success, thus helping to create role models for younger generations.

Watch the replay of this interview here.

Figure 7. Martine Liautaud and Bruno Dranesas discuss the movie *Picture a Scientist* at the event organized at IHES on September 15, 2021.

IHES also believes that it is very important to reach out to younger generations. They need to be presented with diverse role models in STEM, in order to envision themselves having a career in these fields. Every year, IHES opens its doors to some of the most promising high schoolers in the area, giving them a chance to learn more about the Institute and its history and to meet with researchers. These visits include a majority of girls, as high schools are typically invited to send three students, one boy and two girls. In 2022, the students, who were all passionate about science and mathematics, got a chance to meet a Fields Medalist, **Cédric Villani** and hear two talks. The first talk was by **Blandine Galiay**, Laureate of the Junior Maryam Mirzakhani 2021 Prize in Pure Mathematics awarded by the FMJH, who was pursuing a master's degree at École Normale Supérieure Paris-Saclay and doing her research internship at IHES under Fanny Kassel's supervision. The second was by **Mendes Oulamara**, who was then a PhD student from Université Paris-Saclay working at IHES. An event like this helps make research, and science in general, more accessible. Although this is obviously not IHES' primary mission, we believe that this type of outreach can also play a major inspirational role.

Figure 8. Blandine Galiay presents her work to high school students on May 21, 2022.

This article has presented the two main angles of what IHES is working on to reinforce gender diversity in STEM, but there are several other aspects that could have been discussed as well, including ways to keep track of the progress made or to ensure that the experience of women scientists at the Institute is fully satisfactory. By organizing these events and collecting testimonies, IHES has already identified areas of improvement needed to attract more women scientists. For example, the Institute is planning to organize a conference where the members of the scientific committee are all women. This is one of the ideas that has been put forward in discussions of initiatives to encourage gender diversity. This will give participants a great opportunity to be in contact with several established female researchers, who can play a role as mentors. Funding to organize such a conference has already been secured thanks to the generosity of donors and sponsors at the gala organized by Friends of IHES on Women in Fundamental Research.

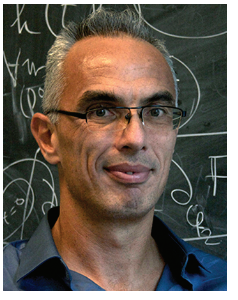
In order to assess the impact of its strategy, the Institute looks first at quantitative data, such as the number of women applicants for postdocs and visits, and the number of women participants in conferences. It is more difficult to obtain clear qualitative indicators and in that regard, the feedback provided by women scientists who spent time at IHES is very valuable. As testimonies are naturally always subjective, the more we collect, the better chance we have of understanding what more could be done.

It is even more important to get feedback for longer visits, and that is particularly true for PhD candidates and postdocs who often spend several years at IHES. Keeping track of the women scientists at IHES, working on a PhD or as a postdoc, is quite informative, as this is often a crucial period for their future career. The Institute was of course particularly pleased to learn that **Maryna Viazovska**, who was a postdoc at IHES between 2012 and 2013, was awarded the Fields Medal. However, in addition to great successes like this one, it is also important to gain insight into the choices of those who decide not to pursue a career in academia, contributing to the "leaky pipeline" phenomenon. What responsibilities does the institution have in that case? Not all PhD candidates and postdocs aspire to stay in academia, and that is certainly very positive for the corporate world, who also need to recruit scientists. It is however essential for institutions like IHES to play a part in making sure that these choices are not a reaction against an environment where they do not feel truly welcome. Ensuring that all young promising researchers feel included, showing them that that they fully belong, and guiding and mentoring them are all key elements needed to improve diversity in academia and in STEM in the long run.

If you have any comments or ideas that you would like to share, or if you are interested in learning more about diversity and inclusion at IHES, please do not hesitate to contact us at `comdev@ihes.com`.

Giulia Foffano

Claire Lenz



Emmanuel Ullmo

Credits

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The Societies Consortium on Sexual Harassment in STEMM: An Update

Michael Pearson and Jamie Lewis Keith

> Throughout my career, it has seemed that the men were having more fun when the women weren't around. You could simply feel the atmosphere change as women came and went. – Autumn Kent, from *Living Proof: Stories of Resilience along the Mathematical Journey*

While we may not consider "fun" to be central to our profession, the continuing underrepresentation of women and other historically marginalized groups is driven by both overt harassment and discrimination, and by the subtle ways in which our culture signals who belongs.

In 2018, the National Academies produced the Consensus Panel Report, *Sexual Harassment of Women: Climate, Culture, and Consequences in Academic Sciences, Engineering, and Medicine*.[1] While noting that the participation of women in STEMM (Science, Technology, Engineering, Mathematics, and Medicine) has increased significantly over recent decades, biases and barriers—and in particular gender and sexual harassment remain—a persistent feature of women's experience both as students and professionals.

With substantial leadership from AAAS, the American Geophysical Union, the Association of American Medical Colleges, and EducationCounsel LLC (the three "sponsoring societies" and a law, policy, and strategy consulting group that together led the initial formation of the Consortium), in late 2018 initial meetings of leaders from STEMM societies led to the formation of the Societies Consortium on Sexual Harassment in STEMM.[2] There is now a total of more than 120 members in the Consortium, with membership open to all STEMM professional societies. Inaugural members include all four members of the Joint Policy Board for Mathematics (AMS, ASA, MAA, and SIAM). The mission of the Consortium is to support academic and professional disciplinary societies in fulfilling their mission-driven roles as standard bearers and standard setters for excellence in STEMM fields, addressing sexual harassment in all its forms and intersectionalities, so that all talent can participate and thrive.

While the work of the Consortium was inspired specifically by issues of sexual/gender harassment, including climate/norms-based inequity affecting women and members of the LGBTQ+ community, intersectionality has always been core to its mission. Member societies almost immediately noted that sexual harassment intersects with racial, ethnic, and other bases for harassment and discrimination, exacerbating the adverse effect on those individuals targeted on multiple bases, and that the work of the Consortium should explicitly recognize and address concerns across these dimensions. A 2021 Strategic Plan sets four strategic goals, reflecting member input:

1. Catalyze members to drive systemic change that leads to full gender and intersecting racial equity;

Michael Pearson serves as executive director of the MAA. His email address is `pearson@maa.org`.

Jamie Lewis Keith is the Distinguished Senior Law and Policy Fellow of EducationCounsel LLC. Her email address is `jamie.keith@educationcounsel.com`.

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[1] `https://www.nationalacademies.org/our-work/sexual-harassment-in-academia`

[2] `https://societiesconsortium.com/`

2. Create informal affinity groups of member societies that are working on similar issues, and facilitate members' ability to share information and resources on their Consortium mission-related priorities;
3. Expand the Consortium's leadership footprint;
4. Position the Societies Consortium for longer-term sustainability and affordability to meet societies' and fields' needs.

By creating model policies as a foundation for action, as well as implementation tools and law-attentive policy guidance, the Consortium creates efficiencies and helps build member societies' capacity to develop their own policies. Collectively, we seek to remove structural barriers to ethical, professional, and inclusive operations, awards, and other professional recognition. We fully embrace the goal of improving conditions for those who are harmed by such barriers. Also, while the Consortium cannot eliminate the difficulty of this work, its members are not alone in their efforts. In addition to resources, the Consortium creates a community of committed leaders to ease the burden and share valuable experience, including through members-only Hot Topics webinars, informal virtual "chat and collaborate" sessions, and in a new Inclusive Journals Policies Affinity Group[3] as well as on the new Consortium LinkedIn Platform (CLIP).[4]

For many years, the mathematics community has recognized the need to improve the professional environment for historically marginalized groups. With the civil rights and women's rights movements in the mid-20th century as background, both the National Association of Mathematicians and the Association for Women in Mathematics were established (in 1969 and 1971, respectively) to build awareness and support for minorities and women in our profession. Also in 1971, the Joint Committee on the Participation of Women was established. The MAA committees on minority participation and on participation of women in mathematics were also established to address these issues within the MAA. Over the years, the level of attention to what are now broadly referred to as DEI issues has ebbed and flowed, with mixed results, as documented in the 2021 AMS report *Towards a Fully Inclusive Mathematics Profession*.[5] Publications from the AMS and MAA, such as *Living Proof: Stories of Resilience along the Mathematical Journey*,[6] remind all of us that we have yet to meet our aspiration to welcome participation regardless sex, gender identity, sexual orientation, race, or ethnicity.

As members of the Consortium, our organizations are part of a larger effort to improve conditions for historically marginalized groups across all STEMM fields, which we believe will strengthen our nation's scientific enterprise and ultimately benefit society as a whole. Further, our advocacy for an inclusive culture can serve as an example inside and outside of STEMM fields, making it clear that we stand for an open, inclusive, and welcoming society that values the full participation of all its members.

Over the last three years, much progress has been made toward development of a full suite of model policies and implementation guidance. In fact, the MAA recently revised our Code of Conduct[7] based on Consortium models (currently available to Consortium members only). While the MAA has had a "welcoming environment" policy in place for some years, the new Code of Conduct is much broader and includes provisions to withhold or retract awards, leadership positions, and even membership. The Consortium's work, while aspirational, is also grounded in the realities of professional societies' limitations (we can only take action in our "spaces" but our influence can be broad) and the legal landscape, recognizing the importance of protecting the confidentiality and rights of all parties. Professional societies cannot, for example, launch an investigation of workplace behavior of our members.

In addition to the model policies, other resources, and members' engagement activities, the Consortium has recently launched three new field-facing initiatives.

1. Ethical Transparency Tool. This is aimed to lead to the creation of a norm of transparency across the higher ed-research ecosystem by making it easier for societies, institutes of higher education, and other research organizations to access sexual, gender, and racial misconduct findings wherever they occur in the ecosystem (and doing so in a way that treats everyone ethically, retains each entity's independence in policy- and decision-making, reduces legal and enterprise risks, and increases efficiency).[8]
2. Case Studies Library. This is aimed to help societies, institutions of higher education, and members of their communities develop tools to prevent harmful conduct at meetings, and in isolated and other high-risk settings, while being sensitive to marginalized populations (women, people in the LGBTQ+ community, people of color, students, and early career folks). It is also aimed to provide guidance on how to respond effectively when such conduct occurs and on self

[3] *This is a members-only group—any society that is a Consortium member and wants to join the journals affinity group can let us know by emailing* societiesconsortium@educationcounsel.com.

[4] https://www.linkedin.com/groups/9169225/

[5] https://www.ams.org/about-us/Towards-a-Fully-Inclusive-Mathematics-Profession.pdf

[6] https://www.maa.org/press/ebooks/living-proof-stories-of-resilience-along-the-mathematical-journey-2

[7] https://www.maa.org/about-maa/policies-and-procedures/maa-code-of-conduct

[8] https://societiesconsortium.com/ett/

protection (without excusing those whose responsibility it is to prevent and respond).[9]

3. Library of Interviews on Courageous and Transformative Leadership toward gender and intersecting racial equity and inclusion. This includes short, topical experience-sharing videos from CEOs and Board Presidents.[10]

The Consortium has made substantial progress towards our initial goals of developing model policies and establishing mechanisms for members to learn from our perspectives and experiences, while also expanding efforts to develop a suite of tools and resources to guide and support implementation and improvement of policies to diminish discrimination and harassment across the STEMM sector. The Consortium has created a space for people in a wide range of leadership and DEI roles to become part of a community of peers with whom to share ideas, challenges, and successes. The mathematics community is a visible part of these efforts, and, through sharpening our own knowledge and policies, our participation in the Consortium is supporting our efforts to be the welcoming community we aspire to become.

Michael Pearson

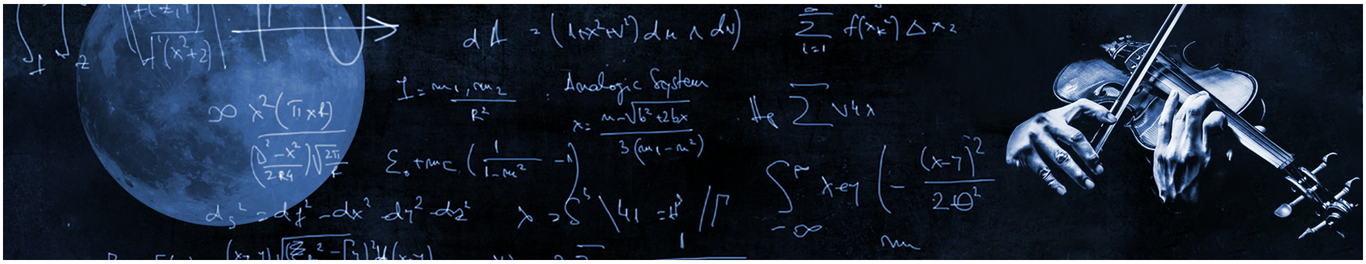
Jamie Lewis Keith

Credits

Photo of Michael Pearson is courtesy of Michael Pearson.
Photo of Jamie Lewis Keith is courtesy of Jamie Lewis Keith.

[9] https://societiesconsortium.com/library/case-study-library

[10] https://societiesconsortium.com/library/courageous-leadership/



Escape from the Ivory Tower

Sophia D. Merow

Figure 1. Joshua Palmatier signs copies of *Shattering the Ley*, the first book in his third fantasy trilogy.

In the early aughts, then PhD candidate Joshua Palmatier was expending a lot of time and mental energy trying to fathom an algebraic structure called the m-zeroid. When he tired periodically of cataloging properties or developing visualizations, got frustrated in his efforts to outline generation methods or prove equivalences, Palmatier escaped into a realm of his own creation: Amenkor, a once marvelous metropolis devastated by a mysterious force called the White Fire. There Palmatier spun the tale of Varis, a homeless orphan with a gift for perceiving people's true natures.[1]

Sophia D. Merow is a freelance writer and editor. Her email address is sdmerow@gmail.com.

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"It was what kept me sane," Palmatier says of the fiction he wrote while in graduate school. "Every student should have something completely different from their PhD subject to give them something to turn to when the PhD work itself becomes too stressful."

Most of Palmatier's professors and fellow students at SUNY Binghamton knew he was writing on the side; they mostly regarded his verbal world-building as a hobby and harmless. One professor did take Palmatier aside, though, and advise him to abandon his avocation. There was no way, he told the yet unpublished Palmatier, that he could write and get his PhD at the same time. "But that just motivated me more," Palmatier recalls.

Juggling math and writing, see, was no idle whim for Palmatier; he had been planning the balancing act since middle school. Palmatier caught the writing bug in the eighth grade when he completed—with relish and to much acclaim—an assignment to compose a *Twilight Zone*-ish short story. It was fine to be a writer, his mom allowed when he came home abuzz, but he also needed a more dependable way to pay the bills. Luckily Palmatier had decided in the fifth grade, swayed by his apparent knack for explaining mathematical concepts to his peers, to teach math when he grew up.

[1] *And an explicitly stated hatred for math. "Recall I wrote this book while getting my PhD," says Palmatier, "so that explains that."*

"I decided there was no reason I couldn't do both," Palmatier remembers. The two pursuits would pair well, he figured, since teaching would leave him with summers free to focus on writing.

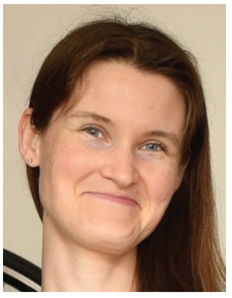
So, as Palmatier started racking up math degrees, he wrote too, honing his craft as he earned first a BS and then an MA from Penn State. The tomes he drafted during those years remain "in a trunk collecting dust," but the product of the breathers Palmatier took from his draining contemplation of m-zeroids—*The Skewed Throne*, the first installment of a trilogy set in Amenkor—sold the same semester he got his PhD.

Doing math and writing epic fantasy complement each other nicely, Palmatier has found. A mathematician's proclivity for logic and coherence comes in handy when framing fictional worlds, for one thing. "The magic has to follow a certain set of rules in order for it to make sense," Palmatier explains. "Readers will suspend their disbelief for magic, but not if it doesn't have its own consistency." And the fanciful habits of mind one cultivates as an author of speculative fiction can facilitate the flashes of creative insight often required to push the boundaries of mathematical knowledge. "Being able to imagine and attempt things that aren't 'typical' helped me get through my PhD," Palmatier says. And his colleagues at SUNY Oneonta—who, Palmatier reports, tap him to "do any heavy-duty administrative write-ups for the department"—also benefit from his way with words. "Everyone knows I can probably produce something in a half hour that might take someone else an hour or two."

Palmatier's focus on the Oneonta campus is math (though he has overseen a few independent studies in the English department for students expressing an interest in writing fantasy novels), but in his off hours... He edits themed anthologies of fantasy and science fiction stories for his small press, Zombies Need Brains. He manages his lately launched online magazine, *ZNB Presents*. And he writes. Palmatier aims for 1000 words per day when classes are in session, 2500 when they're not.

He doesn't think the authors he deals with—members of his local writers group, say, or those submitting work for his editorial review—pay his math teaching much heed (if, indeed, they're even aware of it). "Almost every writer has a day job," he says. "We all understand that a day job is basically a necessity and it's just a matter of figuring out how to work the writing into the schedule without burning yourself out." For Palmatier, this means not letting the math side of his life take over. He sets aside a few hours a day to write, fiercely guards those daily windows against encroachment from other commitments, and leverages willpower to ensure that he sits down and actually does the creative work. "It's really all about organizing your time and sticking to your plan," he says.

Although math has been largely absent from Palmatier's fiction to date, his latest series—due out later this year—realizes his long-percolating idea of using the m-zeroid (the subject of his dissertation research) as the basis for a system of magic. The protagonist of the Crystal Cities trilogy—a mathematician—discovers over the course of the story the mathematical structure—inspired by the m-zeroid—underlying the mages' wizardry. Two tetrahedrons are joined along one face to form a triangular bipyramid.[2] Nodes corresponding to particular aspects of a magical phenomenon are selected in the bottom tetrahedron to pick out a unique node in the upper tetrahedron corresponding to the specific magic to be implemented.[3] Palmatier does not bog down the novels' flow with the "gory details" of the m-zeroid structure. "It's just mentioned and used," he says. "After all, this is supposed to be an escape from the real world!"



Sophia D. Merow

Credits

Figure 1 is courtesy of Joshua Palmatier.
Author photo is by Igor Tolkov.

[2] *"Basically it looks like a diamond shape," Palmatier says.*

[3] *"So say they want a fireball," Palmatier explains. "They'd select the nodes for fire and shape and size and intensity and direction, etc., and when they've formulated exactly what they want, they end the 'spell' and the diamond implements the magic. Obviously there's a power source behind all of this, but that's the basic idea behind the 'math' side of the magic."*

Calls for Nominations & Applications

AMS Prizes & Awards

NEW! Ivo and Renata Babuška Thesis Prize

The Ivo and Renata Babuška Thesis Prize is awarded annually ($3,000) to the author of an outstanding PhD thesis in mathematics, interdisciplinary in nature, possibly with applications to other fields.

About this Prize

Ivo Babuška is a Czech-American mathematician whose honors include five doctorates honoris causa, the Czechoslovak State prize for Mathematics, the Leroy P. Steele Prize, the Birkhoff Prize, the Humboldt Award of Federal Republic of Germany, the John von Neumann Medal, the Neuron Prize Czech Republic, the ICAM Congress Medal (Newton Gauss), the Bolzano Medal, and the Honorary Medal De Scientia Et Humanitate Optime Meritis. Asteroid 36060 Babuška was named in his honor by the International Astronomical Union.

Renata Babuška (nee Mikulášek) was Ivo's wife and partner for 63 years. Renata grew up in Prague, Czechoslovakia and graduated from Charles University in 1953 with a degree in Mathematical Statistical Engineering. Upon graduation, she was assigned to the Education Department as an administrator evaluating universities and technical schools. Two years later she became an Assistant Professor of Mathematics at the Czech Technical University. After moving to the US, Renata worked as a data and computing management consultant for different government agencies in Washington, DC. She liked to point out that behind every successful man is a strong woman and he often said that without Renata, he would not have accomplished all that he did.

Babuška was a Distinguished Professor at the University of Maryland at College Park and then the Robert B. Trull Chair in Engineering, TICAM Senior Research Scientist, Professor of Aerospace Engineering and Engineering Mechanics, and Professor of Mathematics at the University of Texas, Austin. He is a Fellow of SIAM, ACM, and ICAM, a member of the US National Academy of Engineering, the Academy of Medicine, Engineering, and Sciences of Texas, the European Academy of Sciences, and an honorary Foreign Member of the Czech Learned Society.

Babuška's work spans the fields of theoretical and applied mathematics with emphasis on numerical methods, finite element methods, and computational mechanics. His interest in fostering collaboration among mathematicians, engineers, and physicists led him to establish this prize to encourage and recognize interdisciplinary work with practical applications.

The Ivo and Renata Babuška Thesis Prize is awarded in line with other AMS Prizes and Awards, according to governance rules and practice in effect at that time.

Next Prize: Inaugural Prize January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure:

1. The prize will recognize a thesis for a PhD granted between July 1 of year -1 and June 30 of year 0 and will be presented at the Joint Mathematics Meetings in January of year +1.
2. The nominating institution will be a PhD-granting institution that is either a. located in the United States of America (USA), or b. located outside the USA and an institutional AMS member at the time of the nomination.
3. One PhD thesis may be nominated by a nominating institution.
4. The nominating institution will submit a copy of the thesis along with a letter in support of the nomination, and both will be written in English.
5. A selection committee will be appointed by the AMS President.

To make a nomination go to `https://www.ams.org/babuska-prize`.

NEW! The Elias M. Stein Prize for New Perspectives in Analysis

The Elias M. Stein Prize for New Perspectives in Analysis is awarded for the development of groundbreaking methods in analysis which demonstrate promise to revitalize established areas or create new opportunities for mathematical discovery. The current prize amount is US$5,000 and the prize is awarded every three years for work published in the preceding six years.

About this Prize

This prize was endowed in 2022 by students, colleagues, and friends of Elias M. Stein to honor his remarkable legacy in the area of mathematical analysis. Stein is remembered for identifying many deep principles and methods which transcend their original context, and for opening entirely new areas of research which captivated the attention and imagination of generations of analysts. This prize seeks to recognize mathematicians at any career stage who, like Stein, have found exciting new avenues for mathematical exploration in subjects old or new or made deep insights which demonstrate promise to reshape thinking across areas.

Next Prize: Inaugural Prize January 2024

Nomination Period: 1 February – 30 June 2023

Nomination Procedure: Nominations can be submitted between February 1 and June 30. Nominations should include a letter of nomination and a brief citation to be used in the event that the nomination is successful.

To make a nomination go to `https://www.ams.org/stein-prize`.

Leroy P. Steele Prize for Lifetime Achievement

About this Prize

The Steele Prize for Lifetime Achievement is awarded for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students. The amount of this prize is US$10,000.

Next Prize: January 2024

Nomination Period: 1 February – 31 March 2023

Nomination Procedure: `https://www.ams.org/steele-prize`

Nominations can be submitted between February 1 and March 31. Nominations for the Steele Prizes for Lifetime Achievement should include a letter of nomination, the nominee's CV, and a short citation to be used in the event that the nomination is successful. Nominations will remain active and receive consideration for three consecutive years.

Leroy P. Steele Prize for Mathematical Exposition

About this Prize

The Steele Prize for Mathematical Exposition is awarded for a book or substantial survey or expository research paper. The amount of this prize is US$5,000.

Next Prize: January 2024

Nomination Period: 1 February – 31 March 2023

Nomination Procedure: `https://www.ams.org/steele-prize`

Nominations can be submitted between February 1 and March 31. Nominations for the Steele Prizes for Mathematical Exposition should include a letter of nomination, a complete bibliographic citation for the work being nominated, and a brief citation to be used in the event that the nomination is successful. Nominations will remain active and receive consideration for three consecutive years.

Leroy P. Steele Prize for Seminal Contribution to Research

About this Prize

The Steele Prize for Seminal Contribution to Research is awarded for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research.

Special Note: The Steele Prize for Seminal Contribution to Research is awarded according to the following six-year rotation of subject areas:

1. Open (2025)
2. Analysis/Probability (2026)
3. Algebra/Number Theory (2027)
4. Applied Mathematics (2028)
5. Geometry/Topology (2029)
6. **Discrete Mathematics/Logic (2024)**

Next Prize: January 2024

Nomination Period: 1 February – 31 March 2023

Nomination Procedure: `https://www.ams.org/steele-prize`

Nominations can be submitted between February 1 and March 31. Nominations for the Steele Prizes for Seminal Contribution to Research should include a letter of nomination, a complete bibliographic citation for the work being nominated, and a brief citation to be used in the event that the nomination is successful.

Chevalley Prize in Lie Theory

The Chevalley Prize is awarded for notable work in Lie theory published during the preceding six years; a recipient should be at most twenty-five years past the PhD.

About this Prize

The Chevalley Prize was established in 2014 by George Lusztig to honor Claude Chevalley (1909–1984). Chevalley was a founding member of the Bourbaki group. He made fundamental contributions to class field theory, algebraic geometry, and group theory. His three-volume treatise on Lie groups served as standard reference for many decades. His classification of semisimple groups over an arbitrary algebraically closed field provides a link between Lie's theory of continuous groups and the theory of finite groups, to the enormous enrichment of both subjects.

The current prize amount is US$8,000, awarded in even-numbered years, without restriction on society membership, citizenship, or venue of publication.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Submit a letter of nomination, complete bibliographic citations for the work being nominated, and a brief citation that might be used in the event that the nomination is successful.

To make a nomination go to `https://www.ams.org/chevalley-prize`.

Frank Nelson Cole Prize in Algebra

This prize recognizes a notable research work in algebra that has appeared in the last six years. The work must be published in a recognized, peer-reviewed venue.

About this Prize

This prize (and the Frank Nelson Cole Prize in Number Theory) was founded in honor of Professor Frank Nelson Cole upon his retirement after twenty-five years as secretary of the American Mathematical Society. Cole also served as editor-in-chief of the *Bulletin* for twenty-one years. The original fund was donated by Professor Cole from moneys presented to him on his retirement, and was augmented by contributions from members of the Society. The fund was later doubled by his son, Charles A. Cole, and supported by family members. It has been further supplemented by George Lusztig and by an anonymous donor.

The current prize amount is US$5,000, and the prize is awarded every three years.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Submit a letter of nomination, a complete bibliographic citation for the work being nominated, and a brief citation that explains why the work is important.

To make a nomination go to `https://www.ams.org/cole-prize-algebra`.

Levi L. Conant Prize

This prize was established in 2000 in honor of Levi L. Conant to recognize the best expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years.

About this Prize

Levi L. Conant was a mathematician and educator who spent most of his career as a faculty member at Worcester Polytechnic Institute. He was head of the mathematics department from 1908 until his death and served as interim president of WPI from 1911 to 1913. Conant was noted as an outstanding teacher and an active scholar. He published a number of articles in scientific journals and wrote four textbooks. His will provided for funds to be donated to the AMS upon the death of his wife.

Prize winners are invited to present a public lecture at Worcester Polytechnic Institute as part of their Levi L. Conant Lecture Series, which was established in 2006.

The Conant Prize is awarded annually in the amount of US$1,000.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Nominations with supporting information should be submitted online. Nominations should include a letter of nomination, a short description of the work that is the basis of the nomination, and a complete bibliographic citation for the article being nominated.

To make a nomination go to `https://www.ams.org/conant-prize`.

Ulf Grenander Prize in Stochastic Theory and Modeling

The Grenander Prize recognizes exceptional theoretical and applied contributions in stochastic theory and modeling. It is awarded for seminal work, theoretical or applied, in the areas of probabilistic modeling, statistical inference, or related computational algorithms, especially for the analysis of complex or high-dimensional systems.

About this Prize

This prize was established in 2016 by colleagues of Ulf Grenander (1923–2016). Professor Grenander was an influential scholar in stochastic processes, abstract inference, and pattern theory. He published landmark works throughout his career, notably his 1950 dissertation, *Stochastic Processes and Statistical Interference* at Stockholm University, *Abstract Inference*, his seminal *Pattern Theory: From representation to inference,* and *General Pattern Theory*. A long-time faculty member of Brown University's Division of Applied Mathematics, Grenander received many honors. He was a Fellow of the American Academy of Arts and Sciences and the National Academy of Sciences and was a member of the Royal Swedish Academy of Sciences.

The current prize amount is US$5,000, and the prize is awarded every three years.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: To make a nomination go to `https://www.ams.org/grenander-prize`.

Bertrand Russell Prize

About this Prize

The Bertrand Russell Prize of the AMS was established in 2016 by Thomas Hales. The prize looks beyond the confines of the profession to research or service contributions of mathematicians or related professionals to promoting good in the world. It recognizes the various ways that mathematics furthers fundamental human values. Mathematical contributions that further world health, our understanding of climate change, digital privacy, or education in developing countries are some examples of the type of work that might be considered for the prize.

The current prize amount is US$5,000, awarded every three years.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Include a short description of the work that is the basis of the nomination, including complete bibliographic citations. A curriculum vitae should be included.

To make a nomination go to `https://www.ams.org/russell-prize`.

Albert Leon Whiteman Memorial Prize

The Whiteman Prize recognizes notable exposition and exceptional scholarship in the history of mathematics.

About this Prize

This prize was established in 1998 using funds donated by Mrs. Sally Whiteman in memory of her husband, Albert Leon Whiteman.

The US$5,000 prize is awarded every three years.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Include a short description of the work that is the basis of the nomination, including complete bibliographic citations. A curriculum vitae should be included.

To make a nomination go to `https://www.ams.org/whiteman-prize`.

Award for Distinguished Public Service

The Award for Distinguished Public Service recognizes a research mathematician who has made recent or sustained distinguished contributions to the mathematics profession through public service.

About this Award

The AMS Council established this award in response to a recommendation from its Committee on Science Policy.

The US$4,000 award is presented every two years.

Next Award: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Submit a letter of nomination describing the candidate's accomplishments, a CV for the nominee, and a brief citation that explains why the work is important.

To make a nomination go to `https://www.ams.org/public-service-award`.

Award for an Exemplary Program or Achievement in a Mathematics Department

This award recognizes a department which has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Examples might include a department that runs a notable minority outreach program, a department that has instituted an unusually effective industrial mathematics internship program, a department that has promoted mathematics so successfully that a large fraction of its university's undergraduate population majors in mathematics, or a department that has made some form of innovation in its research support to faculty and/or graduate students, or which has created a special and innovative environment for some aspect of mathematics research.

About this Award

This award was established in 2004. For the first three awards (2006–2008), the prize amount was US$1,200. The prize was endowed by an anonymous donor in 2008, and starting with the 2009 prize, the amount is US$5,000.

This US$5,000 prize is awarded annually. Departments of mathematical sciences in North America that offer at least a bachelor's degree in mathematical sciences are eligible.

Next Award: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: A letter of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements which make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). Where possible, the letter and documentation should address how these successes 1) came about by systematic, reproducible changes in programs that might be implemented by others, and/or 2) have value outside the mathematical community. The letter should not exceed two pages, with supporting documentation not to exceed an additional three pages.

To make a nomination go to `https://www.ams.org/department-award`.

Award for Mathematics Programs that Make a Difference

The Award for Mathematics Programs that Make a Difference was established in 2005 by the AMS's Committee on the Profession to compile and publish a series of profiles of programs that:

1. aim to bring more persons from underrepresented backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to advanced degrees in mathematics and professional success, or retain them once in the pipeline;
2. have achieved documentable success in doing so; and
3. are potentially replicable models.

About this Award

This award brings recognition to outstanding programs that have successfully addressed the issues of underrepresented groups in mathematics. Examples of such groups include racial and ethnic minorities, women, low-income students, and first-generation college students.

One program is selected each year by a Selection Committee appointed by the AMS President and is awarded US$1,000 provided by the Mark Green and Kathryn Kert Green Fund for Inclusion and Diversity.

Preference is given to programs with significant participation by underrepresented minorities. Note that programs aimed at pre-college students are eligible only if there is a significant component of the program benefiting individuals from underrepresented groups at or beyond the undergraduate level. Nomination of one's own institution or program is permitted and encouraged.

Next Award: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: The letter of nomination should describe the specific program being nominated and the achievements that make the program an outstanding success. It should include clear and current evidence of that success. A strong nomination typically includes a description of the program's activities and goals, a brief history of the program, evidence of its effectiveness, and statements from participants about its impact. The letter of nomination should not exceed two pages, with supporting documentation not to exceed three more pages. Up to three supporting letters may be included in addition to these five pages. Nomination of the writer's own institution or program is permitted. Non-winning nominations will automatically be reconsidered for the award for the next two years.

To make a nomination go to `https://www.ams.org/make-a-diff-award`.

Award for Impact on the Teaching and Learning of Mathematics

This award is given annually to a mathematician (or group of mathematicians) who has made significant contributions of lasting value to mathematics education.

Priorities of the award include recognition of:

(a) accomplished mathematicians who have worked directly with pre-college teachers to enhance teachers' impact on mathematics achievement for all students, or

(b) sustainable and replicable contributions by mathematicians to improving the mathematics education of students in the first two years of college.

About this Award

The Award for Impact on the Teaching and Learning of Mathematics was established by the AMS Committee on Education in 2013. The endowment fund that supports the award was established in 2012 by a contribution from Kenneth I. and Mary Lou Gross in honor of their daughters Laura and Karen.

The US$1,000 award is given annually.

Next Award: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: Letters of nomination may be submitted by one or more individuals. The letter of nomination should describe the significant contributions made by the nominee(s) and provide evidence of the impact these contributions have made on the teaching and learning of mathematics. The letter of nomination should not exceed two pages, and may include supporting documentation not to exceed three additional pages. A brief curriculum vitae for each nominee should also be included. The non-winning nominations will automatically be reconsidered, without further updating, for the awards to be presented over the next two years.

To make a nomination go to `https://www.ams.org/impact`.

Fellowships

Fellows of the American Mathematical Society

The Fellows of the American Mathematical Society program recognizes members who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics.

AMS members may be nominated for this honor during the nomination period which occurs in February and March each year. Selection of new Fellows (from among those nominated) is managed by the AMS Fellows Selection Committee, comprised of twelve members of the AMS who are also Fellows. Those selected are subsequently invited to become Fellows and the new class of Fellows is publicly announced each year on November 1.

Learn more about the qualifications and process for nomination at `https://www.ams.org/ams-fellows`.

Joint Prizes & Awards

George David Birkhoff Prize in Applied Mathematics (AMS-SIAM)

The Birkhoff Prize is awarded for an outstanding contribution to applied mathematics in the highest and broadest sense.

About this Prize

The prize was established in 1967 in honor of Professor George David Birkhoff, with an initial endowment contributed by the Birkhoff family and subsequent additions by others. The American Mathematical Society (AMS) and the Society for Industrial and Applied Mathematics (SIAM) award the Birkhoff Prize jointly.

The current prize amount is US$5,000, awarded every three years to a member of AMS or SIAM.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: To make a nomination go to `https://www.ams.org/birkhoff-prize`.

Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student (AMS-MAA-SIAM)

The Morgan Prize is awarded each year to an undergraduate student (or students for joint work) for outstanding research in mathematics. Any student who was enrolled as an undergraduate in December at a college or university in the United States or its possessions, Canada, or Mexico is eligible for the prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be completed while the student is an undergraduate. Publication of research is not required.

About this Prize

The prize was established in 1995. It is entirely endowed by a gift from Mrs. Frank (Brennie) Morgan. It is made jointly by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.

The current prize amount is US$1,200, awarded annually.

Next Prize: January 2024

Nomination Period: 1 February – 31 May 2023

Nomination Procedure: To nominate a student, submit a letter of nomination, a brief description of the work that is the basis of the nomination, and complete bibliographic citations (or copies of unpublished work). All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research.

To make a nomination go to `https://www.ams.org/morgan-prize`.

JPBM Communications Award

This award is given each year to reward and encourage communicators who, on a sustained basis, bring mathematical ideas and information to non-mathematical audiences.

About this Award

This award was established by the Joint Policy Board for Mathematics (JPBM) in 1988. JPBM is a collaborative effort of the American Mathematical Society, the Mathematical Association of America, the Society for Industrial and Applied Mathematics, and the American Statistical Association.

Up to two awards of US$2,000 are made annually. Both mathematicians and non-mathematicians are eligible.

Next Prize: January 2024

Nomination Period: open

Nomination Procedure: Nominations should be submitted on `mathprograms.org`. Note: Nominations collected before September 15th in year N will be considered for an award in year $N+2$.

AMS Primer on Open Access

Robert M. Harington

Introduction

Open Access (OA) refers to published scholarly content (such as journal research articles, and books) made openly available in online digital form. This content is free of charge at point of use, free of most copyright and licensing restrictions, and free of technical or other barriers to access (such as digital rights management or requirements to register to access). Typically, OA is associated with reuse licenses, primarily, Creative Commons[1] licenses that allow for reuse of the content.

Public Access (PA) refers to access mandated by the White House Office of Science and Technology Policy (OSTP). PA differs from OA in that content is made freely available under terms defined by OSTP and federal funding agencies. Articles and data are made freely available to all, but are not associated with reuse licenses.

Communicating and sharing discoveries is an essential part of the research process. Any author of a research paper wants it to be read. Therefore, the fewer restrictions placed on access to those papers means that more people may benefit from the research. In many ways, the OA movement is very much in line with the shared mission of researchers, scholarly societies, and publishers.

Journal publishing programs perform many services for researchers including peer review, communication, and career advancement. In society publishing programs, revenue from journal publishing directly supports the important work societies do on behalf of their scholarly communities.

How do we maximize the dissemination of knowledge while at the same time maintain both a high level of quality and a sustainable financial future for our professional society, the AMS?

The OA movement can be traced to a letter from the year 2000, signed by some 34,000 researchers, demanding publishers make all content free after 6 months. The signatories of the letter said they would boycott any journals refusing to comply. In 2002, the accepted definition of OA was encapsulated in the Budapest Open Access Initiative[2] declaration.

While the threatened boycott never materialized, an antagonistic tone was set, and this has marked much of the discussion around OA to the present day. There has been a lot of unproductive argument and invective, rather than efforts to find common ground. Unfortunately, advocacy for balanced discourse has floundered, resulting in societies, such as the AMS, contemplating how to tread independently with a posture of balance and reason.

In 2013, the OSTP issued policy guidance to federal agencies in the form of the OSTP memorandum on "Increasing Access to the Results of Federally Funded Scientific Research" (the "2013 Memorandum," also widely referred to as the "Holdren Memo"[3] because it was issued by John Holdren, at the time the Director of OSTP). The Holdren Memo directed federal agencies in the U.S. with annual research and development budgets of more than $100 million to develop policies to ensure public access to federally funded research. The Holdren Memo mandated a 12-month post-publication embargo period for papers stemming from federally funded research. By "post-publication embargo period," the Holdren Memo was referring to the period between publication of an article resulting from funded research in a journal and the freely accessible public release of that journal article in the form of either the author accepted manuscript (AAM) or the final published version of record (VOR).

On 25th August, 2022, Alondra Nelson (Interim Director of OSTP) issued a memorandum titled "Ensuring Free, Immediate, and Equitable Access to Federally Funded Research."[4] The memo is known as the "Nelson Memorandum." It is accompanied by an impact statement titled

Robert M. Harington is the Associate Executive Director, Publishing at the American Mathematical Society. His email address is rmh@ams.org.

[1] https://creativecommons.org/licenses/

[2] https://www.budapestopenaccessinitiative.org/read/

[3] https://obamawhitehouse.archives.gov/sites/default/files/microsites/ostp/ostp_public_access_memo_2013.pdf

[4] https://www.whitehouse.gov/ostp/news-updates/2022/08/25/ostp-issues-guidance-to-make-federally-funded-research-freely-available-without-delay/

"Economic Landscape of Federal Public Access Policy"[5] (the "2022 Impact Statement"), which was submitted to Congress pursuant to the Consolidated Appropriations Act, 2022. The memorandum directs federal agencies to develop policies that will require free public release of research articles upon publication, and that all supporting research data behind the articles be similarly made immediately and freely available.

In this update to the original AMS Primer on Open Access released in 2019, we discuss potential implications of the "Nelson Memorandum" public access memo for the mathematics community. We also consider developments in Europe from cOAlition S[6] funders, and other European funding agencies, as they also have a potential impact on the AMS publishing program and the global mathematics community.

Open Access (OA) Categories

The AMS recognizes openness as important, but also wants to balance this ideal with pragmatic business models that allow the AMS to continue providing services to the mathematical community.

Approaches to OA may be considered in terms of *what* is made open, *when* it is made open, and *where* it is made open. Taking a peer-reviewed journal article as an example, three stages may be distinguished:

Stage 1—Author's original: this is the author's unrefereed draft manuscript intended for consideration by a journal, also called a preprint. Authors usually retain copyright over this version of the manuscript.

Stage 2—Author accepted manuscript: this is the author's final refereed manuscript accepted for publication by a journal, containing all changes required as a result of peer review. From this stage, copyright is typically assigned to the publisher, or an exclusive license is granted.

Stage 3—Version of record: this is the final published, citable article available from the journal's website. Copyright is now retained by the publisher as an exclusive license.

Bearing in mind these OA stages, there are a number of OA models in use today:

1. **Gold OA** is usually meant to describe when journal articles (*version of record*) are freely available to the reader immediately upon publication because the author has paid an article processing charge (APC). It is worth noting that this changes the business model for a journal from being reader-centric (reader or library pays to access the article) to author-centric (author pays for OA). Most publishers who offer Gold OA do so either as pure Gold, in other words, all articles are supported by APCs; or as hybrid Gold, in other words, a journal contains some OA articles supported by APCs and other articles that are behind a paywall. Either of these scenarios allows for deploying a transformational "read & publish/publish & read" model, which will be described in more detail later in this document.
2. **Green OA** is when articles (most often the final *author accepted manuscripts*) are made freely available at no cost to the author or reader. This can happen when the author deposits the article in an external repository (e.g., personal web page, their institution's noncommercial repository, arXiv.org) or the article (*version of record*) becomes open in the online version of the journal after a time delay known as an embargo period.
3. **Diamond/Platinum OA** is when articles (*versions of record*) are freely available immediately upon publication with no cost to the author or the reader. The majority of fully open access journals listed in the Directory of Open Access Journals[7] follow this model. These journals tend to be small ones funded by private grants or by a host institution or library.

Journals that publish articles exclusively under the pure gold or platinum methods are called "fully OA." Journals that employ a combination of the traditional subscription model along with some OA options are "hybrid" journals. A hybrid journal has some content that requires a subscription ("behind the paywall") and some that is freely available ("open access"). A "mirror" journal shares the same editorial board as a traditional subscription model journal but is itself a separate, Gold OA publication. The difference between "hybrid" and "mirror" journals matters, as there is currently some controversy from certain funding agencies pertaining to this distinction. This will be explained more below in the discussion of new funder-driven OA initiatives.

OSTP Public Access (PA) Mandate August 2022 (the "Nelson Memorandum")

Key points to note are:

1. The OSTP is mandating zero embargo public access, not open access—in other words, free to read to all, but not necessarily associated with Creative Commons reuse licenses.
2. For now, this is an unfunded mandate—in other words, no extra funds have been provided to federal agencies to incorporate publications costs.
3. The zero embargo policy applies to all authors on a publication, not just corresponding authors.
4. The memo does not clearly specify what version of research papers this policy will apply to.
5. Immediate public access to underlying data is required.
6. Metadata is a focus—in other words, the policy requires collecting and making publicly available metadata

[5] `https://www.whitehouse.gov/ostp/reports/`

[6] `https://www.coalition-s.org/`

[7] `https://doaj.org/`

associated with publication such as author names, affiliations, dates of publication, sources of funding etc. This is done via the use of persistent identifiers.

7. The policy goes into effect by the beginning of 2026. It is not yet clear if this means the policy will take effect with grants issued after January 1, 2026, which seems likely.

Current State of AMS Open Access (OA) and Public Access (PA)

The AMS currently publishes its new flagship journal, *Communications of the AMS* (CAMS) in a Diamond OA model.

AMS currently has two Gold OA journals with APCs discounted to $750 (currently waived for AMS members)—*Proceedings of the American Mathematical Society, Series B* and *Transactions of the American Mathematical Society, Series B*. These journals were launched in 2014 as spin-offs ("mirror" journals) from their parent journals, sharing the same editorial board.

Articles published in AMS Diamond and Gold OA journals are distributed under the terms of one of the following licenses:

- The Creative Commons Attribution-Noncommercial License[8] **(CC BY-NC)**, which excludes commercial use. This license lets others remix, tweak, and build upon the article noncommercially; the new work must credit the original article and be noncommercial.
- The Creative Commons Attribution License[9] **(CC BY)**. This license lets others distribute, remix, tweak, and build upon the article, even commercially, as long as credit for the original article is given.

Authors retain copyright to their articles under both CC BY-NC and CC BY.

The AMS is fully compliant with green OA. In all of our journals, AMS authors may post an accepted, pre-publication version of a journal article on their personal web page, on their institution's noncommercial repository, and on arXiv.org. In this model, there are no processing fees, and the article is included in a subscription journal. All AMS subscription research journals[10] are green OA journals.

The embargo period for AMS journals is five years. This means all AMS version of record articles are publicly available ("open") five years after publication.

Some funding agencies require their authors to publish green OA articles within one year of publication. In these cases, the AMS works with the non-profit organization CHORUS[11] to facilitate free access to the accepted manuscript version of the article (not the version of record) following the allowable embargo period of one year. Through CHORUS we can track these articles and this may be seen on the CHORUS Dashboard.[12]

AMS staff looked at AMS journal articles published from 2018–2022 (ytd) in our research journals: *Journal of the American Mathematical Society* (JAMS), *Proceedings of the American Mathematical Society* (PROC), *Proceedings of the American Mathematical Society, Series B* (BPROC), *Transactions of the American Mathematical Society* (TRAN), *Transactions of the American Mathematical Society, Series B* (BTRAN), *Mathematics of Computation* (MCOM), *Memoirs of the American Mathematical Society* (MEMO), *Conformal Geometry and Dynamics* (ECGD), *Representation Theory* (ERT). *Communications of the American Mathematical Society* (CAMS) was excluded as it has only just been launched.

BPROC and BTRAN are Gold OA "mirror" journals with the same editorial boards as their parent journals.

- Total articles: 4,512
- Grant supported: 2,180—42%
- U.S. federal supported: 838—18.5%
- cOAlition S supported: 572—13%

Assumptions: In cases where multiple funders are listed, it is possible that they are listed in order of level of support, but this is just an assumption. Included in the count is any article listing U.S. federal (or cOAlition S) funding, regardless of position in the authors' listing order.

Any AMS response to the OSTP memo's requirements will need to be considered in context of the U.S. federal funding agencies response to the OSTP mandate. Most federal funds for mathematics research are provided through the NSF. The NSF supports more basic research in mathematics—done at colleges and universities—than any other federal agency. NSF's annual budget represents approximately 24% of the total federal budget for basic research conducted at U.S. colleges and universities; in mathematics, the percentage is much higher at 71%.

Although NSF grant guidelines permit grant writers to request funds for APCs, this has not been widely used by mathematicians. There is a concern that if the OSTP memo's requirements remain an unfunded mandate, precious grant dollars reallocated to APCs will lead to other research elements of the grant proposals becoming unfunded.

We are communicating with funding agencies, focusing on NSF, using the following talking points:

1. As funding agencies develop their policies, societies should be kept in mind. Cultural differences in academic communities such as mathematics should be considered.

[8] https://creativecommons.org/licenses/by-nc/3.0/

[9] https://creativecommons.org/licenses/by/2.0/

[10] https://www.ams.org/journals/

[11] https://www.chorusaccess.org/

[12] https://dashboard.chorusaccess.org/publishers/ams#/summary

2. How might the AMS comply with zero embargo PA, without relying on Gold OA?
3. The AMS has equity concerns about choices of where to publish for those who may have access to APC funds and those who do not.
4. At present this is an unfunded mandate. Are there likely to be funds appropriated to this policy?
5. Reuse licenses (Creative Commons licenses) are potentially an issue—will the author's peer reviewed accepted manuscript, without reuse licenses attached, be acceptable?
6. We worry about society sustainability, given that 70% of our revenue is derived from publishing product sales, including 22% from journal sales.

Benefits and Risks of Open Access

The AMS publishes a broad portfolio of high-quality journals. When considering the benefits and risks of adopting OA models for journals, it is important to note there are significant costs associated with publishing journals. These costs include, for example, expenses related to editorial office management, peer review management, technical manipulation of LaTeX files, workflows for tools such as AMS Math Viewer, systems for processing, presentation and sale of print and online products, marketing and strategic journal development, public awareness, and in-house expertise on best-publishing practices.[13]

Benefits of OA

There have been few rigorous studies on the benefits of OA. One such study is (Davis P. M. (2011) vol. 25 no. 7 2129–2134).[14] This research shows that OA articles received significantly more downloads than subscription articles and reached a broader audience—basically twice as many full-text html views, a 62% increase in PDF downloads and more than 30% more unique visitors. Citations, however, remained flat—despite wider readership there was no similar increase in citation of OA articles.

The results indicate that OA is a valuable tool for broadening access to the research literature. OA publishing extends information beyond the research community.

A recent report from (Digital Science—January 2019)[15] indicates countries that have invested in OA publishing over the last decade have typically increased their level of international collaboration.

Risks of OA

Gold OA

Gold OA appears to be effective for increasing access to journal articles, yet it also introduces some unintended consequences. The business model for Gold OA shifts financial responsibility from the readers (who currently pay, or have their institutional libraries pay, to access articles) to the authors (who now must pay, or have their institutions or funders pay, to make their articles available to be read).

OA costs land on the author, rather than being distributed across a large number of readers. This creates a system where you have to pay to publish. In our discipline of mathematics, the percentage of federally funded researchers is relatively low at around 25%. Other scientific fields rely more heavily on federal funds, as seen in this chart from the National Science Foundation.[16]

Much of the drive toward Gold OA comes from the biomedical world, where research funding is more prevalent. Although many societies and publishers are launching new Gold OA journals, with relatively fewer mathematics researchers having access to funding, one size cannot fit all.

For established journals, the equation is different. A society such as the AMS relies on subscription revenues to generate income, which enables it to provide services to its members and the global mathematical community. ***For the AMS, publication revenues account for approximately 70% of its operating income, including 22% from journal sales***. It is naïve to anticipate that APCs could adequately replace subscription revenues. Indeed, the modest APC we now have for our Gold B-series journals does not fully cover the costs associated with publishing an article, let alone generate revenue that could be made available to the general AMS operating budget.

While many researchers consider current APC rates too high, the reality is that they would likely need to be a lot higher for most journals to remain sustainable. For a Gold OA journal, the only revenue comes from the articles you accept. The more stringent the quality control, the greater the effort (and expense) of reviewing and rejecting articles.

A significant problem with the Gold OA path is that Gold OA introduces a lack of equity and inclusiveness into the research ecosystem. A recent editorial in *Science* describes this well (*Science*, Vol. 377, No. 6613, "Public Access is not Equal Access," Sudip Parikh, Shirley M. Malcom, Bill Moran).

"Public access should foster a diverse universe of authors and readers regardless of their economic circumstances. This drives scientific excellence and public understanding. Some models for public access are bad for inclusivity. Gold OA journals, for which authors pay publication charges, work for senior scientists who are well-funded, tenured, and overwhelmingly male and white, but not so much for early-career scientists who may be poorly funded, not yet tenured, and much more diverse. Also disadvantaged are scientists at smaller schools, including historically Black colleges and universities, and in underfunded disciplines like math and the social sciences. Although it enables 'open

[13] https://publicationethics.org/

[14] https://faseb.onlinelibrary.wiley.com/doi/abs/10.1096/fj.11-183988

[15] https://www.digital-science.com/blog/2019/01/the-ascent-of-open-access-report/

[16] https://www.nsf.gov/statistics/2016/nsb20161/uploads/1/8/at05-20.pdf

access' to readers, this model can be inequitable for many scientists and institutions."

The *Science* editorial goes on to say: *"As a scientific membership society, AAAS seeks the best path forward for the enterprise it serves. We are actively seeking to balance the tensions between equitable access for readers and equitable access to publishing. As such,* Science *is made available through progressively priced licenses whereby larger, more research-intensive institutions pay more. We will soon provide immediate public access to all taxpayer-funded research through a policy called 'Green OA-zero day,' which allows* Science *authors to post their 'author accepted manuscript' (a fully peer-reviewed and revised version), without delay or incurring additional fees, in a public repository of their choice. This approach allows immediate public access without requiring authors to pay a publication charge, while maintaining the ability of* Science *to fulfill its mission of communicating groundbreaking research discoveries and illuminating the impact of research on society."*

Green OA

Green OA comes with a different set of risks. Green OA relies on the continuing existence of subscription journals, yet requires those journals to give away their content. To balance the risk of lost subscriptions, most Green OA journals have an embargo period, a period of time that allows the journal exclusive rights to the article. This gives the journal a chance to earn some revenue before it opens up access to all. At the AMS, this revenue not only pays the costs of producing the journals, it also provides additional revenue for the AMS operating budget. For AMS journals, the embargo period is 5 years (although as described above, we work with authors whose funding prescribes shorter embargo periods). Default embargo lengths across all scientific journals seem to be 6 or 12 months.

Platinum OA

Platinum OA has seen some success in scenarios where funds are available to ensure continued publication of a journal with no charge to the author or reader. A successful example is seen in ACS Central Science[17] from the American Chemical Society. It is unclear if this model can scale across societies, given that significant resources are needed to publish such a journal in perpetuity.

Copyright

The AMS offers Creative Commons licenses for its Gold OA journals (our Series B journals) of CC BY, or CC BY-NC.

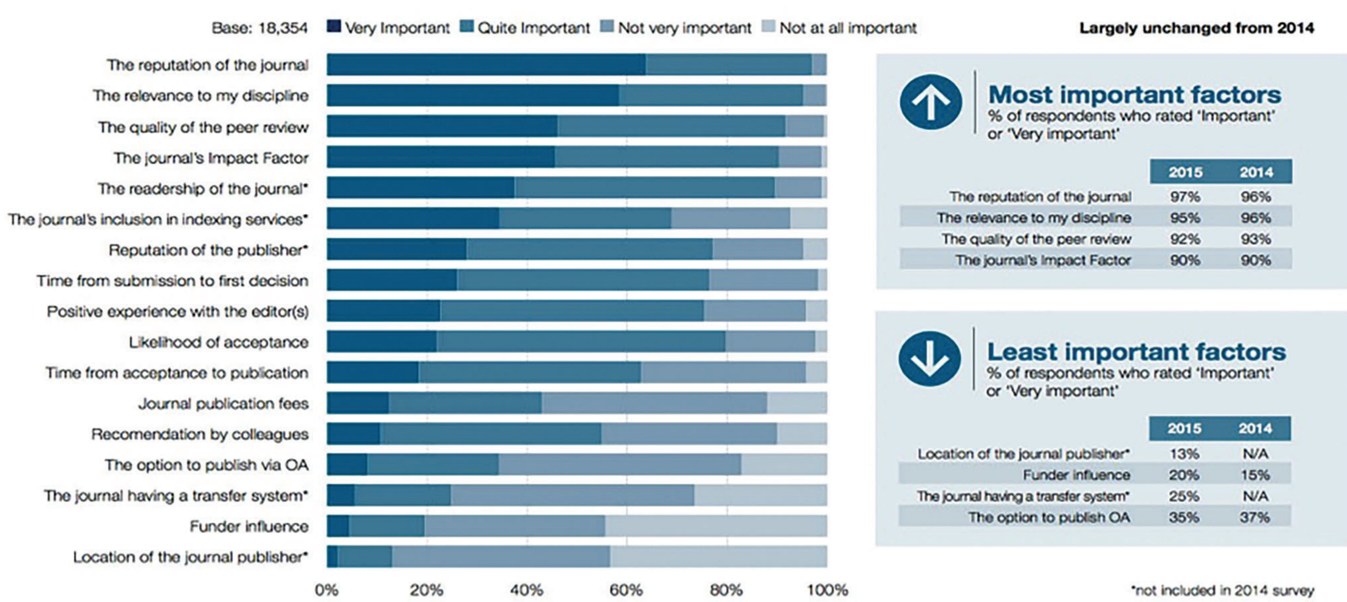
Most researchers are unaware that when publishing under a CC BY license they are giving away their intellectual property. When an author pays an APC, they are paying for a publisher to not only publish their article, but also allowing their content to be monetized in new ways, by both the chosen publisher, and also by other commercial entities, as long as there is attribution to the author, potentially at high prices. Indeed, it is unclear what recourse an author has if a commercial entity fails to credit content to the author. As artificial intelligence and machine learning tools grow more sophisticated, the ability to monetize content in new ways will be of increasing importance commercially.

Priorities of Researchers

Below are some of the results of a 2015 survey from *Nature*. This chart shows responses of science researchers in how they choose which journal to submit their article to. Top reasons are journal reputation and relevance, quality of peer review and impact factor. OA is fourth from the bottom.

In general, OA remains a low priority for most researchers, mathematicians included. In fact, one can make the argument that in mathematics—with virtually all

[17] https://pubs.acs.org/page/acscii/about.html



mathematical research existing on arXiv—we already have OA. This survey suggests that researchers are more likely to think of themselves as authors, rather than as readers. As authors, OA is not a primary concern.

Emerging Business Models

The forces behind the expansion of OA are economic. Library budgets are flat, if not declining. Commercial publishers must continually increase their earnings. In a flat economy, they can do this by acquiring more and more of the current market. This is why we're seeing so many mergers and acquisitions, such as Springer and Nature recently becoming one company. Gold OA publishing has become a priority for many of the larger publishers. Gold OA fits here because costs for launching a new journal, with significant APCs, mean that each article is paid for as it is published. There is no need to consider how to replace existing subscription revenues as these are new journals, and additional to the current market. Indeed, many of the larger commercial publishers are seeking other sources of revenue, such as with author workflow tools.

"Big Deals"

For many years, large commercial publishers have successfully engaged the market through "Big Deals." In these deals, libraries, groups of institutions and even countries subscribe to large packages of journals. These big deals offer significant discounts over a la carte journal subscriptions. Institutions have recently been pushing back on such deals because of their cost.

Transformative "Read and Publish" / "Publish and Read" Deals

Adding to the complexity of the sales environment for journals is the emergence of a new type of deal, reflecting the shift of many of the larger publishers to Gold and hybrid OA publishing. The deals are called "Read and Publish" (RAP) agreements, with a more recent iteration called "Publish and Read" (PAR). In such models, institutions pay a fee to a publisher to make all of their own authors' work immediately OA. The deals are based around agreed levels of APCs as well as access to reading the content.

RAP and PAR deals shift the economics—whereas the "Big Deals" sell journal content, these new arrangements are dependent on publishing OA articles and determining who pays for the APCs. RAP and PAR deals differ on whether the institution is paying primarily for its own authors' APC costs, or for the ability to access content.

While commercial publishers led the way on creating these deals, more transformative deals are appearing, e.g., from Cambridge University Press (publisher of 400+ journals) providing access to no-cost APCs for authors in a range of disciplines, including mathematics for many U.S. universities and colleges.

Some small to medium-sized societies are experimenting with transformative publishing deals. It is not possible for small publishers/societies to replicate the scale seen in larger publishers. Instead, societies are engaging in deals that base the cost to an institution on existing subscription payments plus a fee to allow publishing in that society's OA journals. This approach allows an institution's faculty to publish with no-cost APCs in that publisher's OA/hybrid OA journals.

For the AMS, this would mean that growth in revenues would no longer rely on subscription growth, but in the number of articles published OA, resulting in a higher fee to the institutions for faculty to publish OA. While this may be a model that the AMS could embrace, there are questions of sustainability and equity. Only wealthy institutions that can afford to participate in such deals will benefit from no-cost APCs for their faculty. This raises equity concerns in terms of the inability of mathematicians at smaller institutions to publish in OA journals, thus restricting their choice compared to researchers in wealthy institutions. In addition, it seems questionable whether larger institutions would be willing, long-term, to subsidize OA for global access.

The effect of moving to a RAP/PAR deal on the AMS may be profound, and a deep analysis of potential outcomes, both in terms of access and revenues would need to be undertaken.

Subscribe to Open

Another model for AMS to analyze is Subscribe to Open, initially developed by the non-profit publisher, Annual Reviews.[18] It is designed to motivate collective action by libraries that are asked to continue to subscribe even though the content will be published OA. A 5% discount off the regular subscription price is offered to existing customers. If all current customers continue to subscribe, then that year's content is made available OA and all the back files are also made available OA. None of this content is opened if the number of subscribers decline, which discourages free riding. Libraries are not locked in to these arrangements, with a new arrangement negotiated annually. Any institutions that do not renew and that later return, do so at the list subscription price and do not receive the 5% discount. This model uses the conventional subscription process and existing library budgets, avoids the need to invest in transactional payment infrastructure, minimizes customer disruption by using routine library accounts payable processes, and avoids the prohibition some libraries face in paying for things that would otherwise be free.

Unfortunately, this model may not be sustainable. How many institutions, long-term, will be willing to subsidize OA to other institutions/researchers around the world? In addition, once subscriptions are lost, they would be hard to win back. Another complicating factor is that when a

[18] https://www.annualreviews.org/

journal is made OA in this model, there is no potential revenue growth to the journal as a publisher is unable to grow its subscriber base.

Politics of Open Access

Funders and Government

Another factor in the developing OA landscape is the role of funders, be they private or governmental. Funders in the U.S. are taking a different approach than funders in Europe, as noted above with the "Nelson Memorandum."

As described in a recent Scholarly Kitchen article (Quantifying the Impact of the OSTP Policy, Chris Petrou, 13th September, 2022)[19]:

"The U.S. published 630k papers in 2021 according to the Web of Science, based on the affiliations of all authors on an article. The chart below shows how these papers can be narrowed down to those in scope of the OSTP policy (197k federally-funded papers), then those that are currently paywalled and will be affected by the policy (132k papers), and finally those that are likely to be published in a Gold-OA format (99k papers based on a working assumption that ¾ of papers will be published Gold-OA as a result of the policy)."

This assumption by Petrou reflects the prevailing publishing community view that the OSTP policy will drive papers to Gold OA, rather than Green OA.

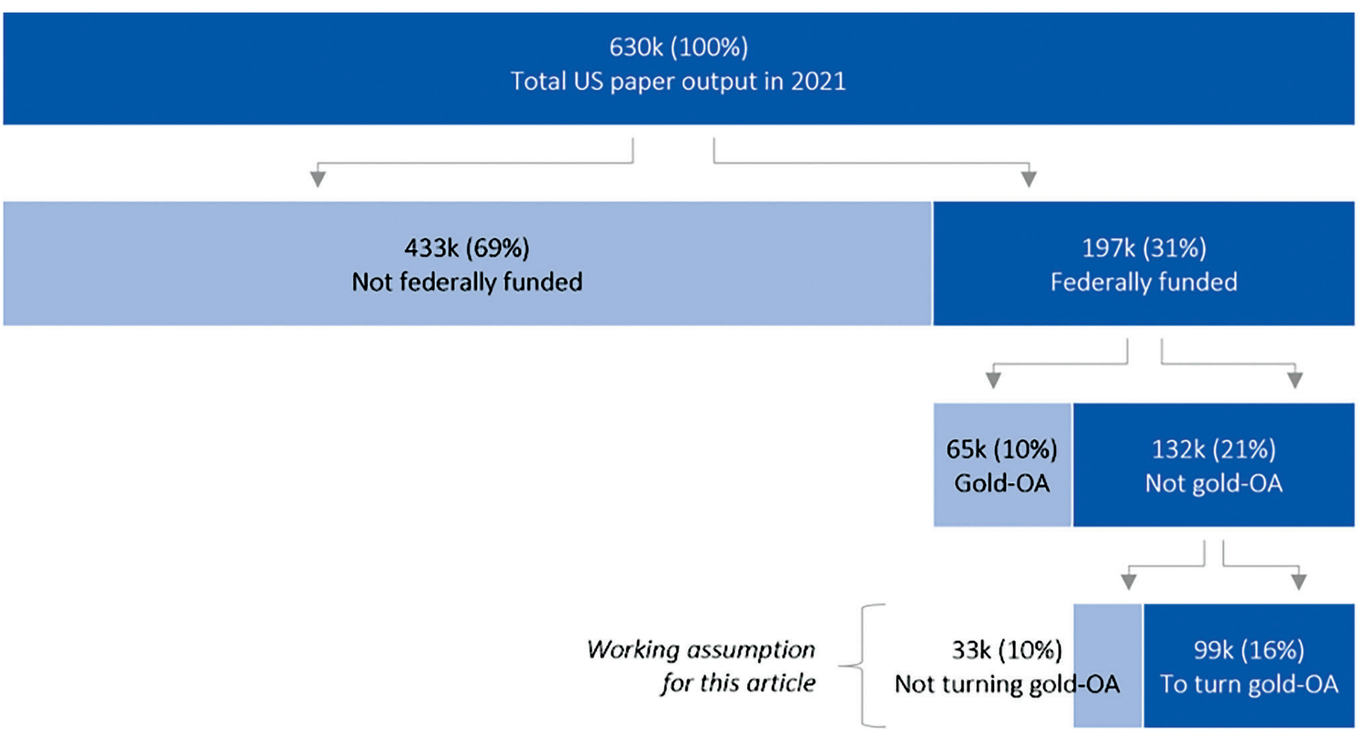
[19] https://scholarlykitchen.sspnet.org/2022/09/13/guest-post-quantifying-the-impact-of-the-ostp-policy/

Europe

Europe is taking a different approach. The most recent and highly publicized funder initiative comes from a group of 16 funders in 13 countries called cOAlition S. This coalition has released a set of principles, called Plan S, for authors funded by any of the cOAlition S funders, which it intends to push forward as a mandate.

For the AMS, it is worth noting that 15% of articles published in AMS primary journals are funded by cOAlition S funders. The basics are as follows:

- **Open Access Journals or Open Access Platforms:** Authors publish in a Plan S compliant OA journal or on a Plan S compliant OA platform with a CC BY license.
- **Deposition of Scholarly Articles in Open Access Repositories:** Immediately upon publication, authors deposit the final published version of a scholarly publication (version of record), or an author's accepted manuscript in a Plan S compliant repository. The document is immediately OA (with no embargo) under a CC BY license.
- **Transformative Agreements:** Authors publish OA with a CC BY license in a subscription journal that is covered by a transformative agreement that has a clear and time-specified commitment to a full OA transition.



U.S. paper output split by funding and OA status. Articles and Review Articles in 2021 per the Web of Science (SCIE, SSCI, AHCI, and ESCI indexes); all federal agencies with >1,000 papers and some with fewer papers were identified via the 'Funding Agencies' filter of the Web of Science.

Compliance with Plan S is quite complicated. The highlights are:

- The journal/platform must be registered in the Directory of Open Access Journals[20] (DOAJ) or in the process of being registered.
- All scholarly content must be openly accessible (journal website or dedicated platform), and free to read and download immediately upon publication, without any kind of technical or other form of obstacles.
- The journal/platform must enable authors to publish under a CC BY license.
- The journal/platform must offer authors/institutions the option of full copyright retention without any restrictions (i.e., no copyright transfer or license to publish that strips the author of essential rights).
- The journal/platform must have a solid system in place for review, according to the standards within the relevant discipline; and according to the Core Practices of the Committee on Publication Ethics[21] (COPE), details on this must be openly available through the website.
- The journal/platform must provide automatic APC waivers for authors from low-income countries and discounts for authors from middle-income countries.
- The journal must not have a mirror/sister subscription journal with substantial overlap in editorial board to avoid business models charging for both access and publication. Such journals will de facto be considered hybrid journals (see 'Transformative Agreements" below).
- Transformative Agreements: cOAlition S acknowledges that some publishers have established mirror journals with one part being subscription based, and the other part being OA. Such journals are not compliant with Plan S unless they are a part of a transformative agreement (an agreement to eventually move a publisher's entire journal portfolio to Gold OA) since they de facto lead to charging for both access and publishing in the same way as a hybrid journal does. Funding for publishing in such journals will only be supported under a transformative agreement. It is worth noting that AMS OA journals are defined by cOAlition S funders as "mirror" journals.

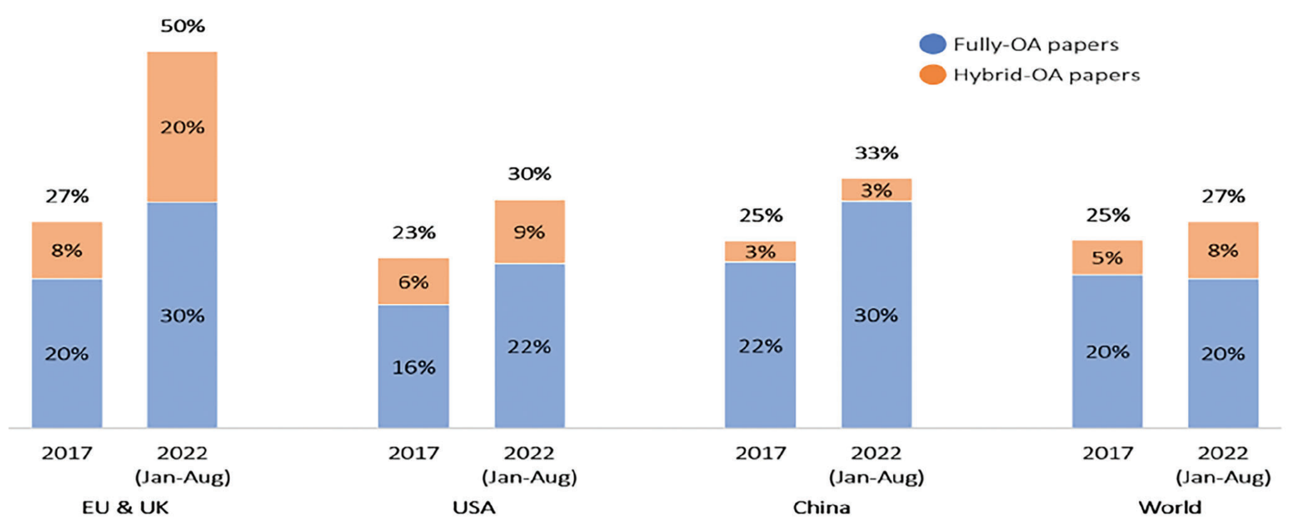
The chart below shows that the EU and the UK publish 50% of their papers in a Gold-OA format. Europe's performance places it well ahead of the USA and China, which publish 30% and 33% of their content, respectively, in a Gold-OA format.

Reactions to Plan S

Looking further afield, a number of countries are expressing tacit support for Plan S. China and India, for example, are expressing public support for the European Plan S approach, yet there is little or no funding available to participate in such arrangements.

A recent proposal called Plan U has been suggested by the founders of bioRxiv,[22] which is the biology equivalent to arXiv. This plan suggests that true OA could be achieved if funders mandate that all their funded researchers deposit their accepted manuscripts in the appropriate preprint repository—arXiv, bioRxiv, chemRxiv for example.

[20] https://doaj.org/
[21] https://publicationethics.org/core-practices
[22] https://www.biorxiv.org/



Gold-OA uptake by region. Articles and Review Articles per the Web of Science (SCIE, SSCI, AHCI, and ESCI indexes)

Summary

There is enormous complexity in the range of OA models available to researchers, funders, institutions, societies and publishers. Some are optional models and others are mandated.

Through the activities of Robert Harington (Associate Executive Director, Publishing), Karen Saxe (Associate Executive Director, Government Relations) and Catherine Roberts (Executive Director), the AMS will continue to engage in relevant discussions among funders, institutions, publishers, and researchers. Saxe and Harington have visited the OSTP multiple times for meetings on this issue. On behalf of the AMS and scholarly societies more broadly, Harington participated in the previous administration's OSTP roundtables on a potential zero embargo policy. Also planned are visits to the NSF to discuss the "Nelson Memorandum," and open publishing models in context of the discipline of mathematics. Harington is a board member of CHORUS, and is a "Chef" at the widely read publishing blog (6,000–8,000 daily readers) called The Scholarly Kitchen.[23] The AMS also participates in the Alliance for Open Scholarship[24] (All4OS), a cohort of societies and associations collaborating to identify, articulate and socialize open scholarship norms within their disciplines. The intent of these myriad efforts is to influence how funders, such as the NSF, interpret the "Nelson Memorandum," as they determine NSF public access policies, as well as to inform the mathematical community about important publishing developments, while ensuring the culture of mathematics is well-understood in the publishing world.

The range of OA initiatives that may be forced on society publishers clearly represents risk to AMS's ability to generate operating income. Many societies are feeling pressured to move into publishing arrangements with larger publishers in order to benefit from the economies of scale, infrastructure, and access to consortia benefits that only larger publishers currently provide.

For the AMS publishing program to remain financially healthy and independent, a balance of new and existing publishing models will need to be deployed that allow for compliance with federal public access mandates, as well as European open access funder mandates, while maintaining financial sustainability and access equity for all mathematicians.

For the AMS, it is worth remembering that in 2022 relatively few of our authors are directly funded by federal or cOAlition S funders.

The AMS position on OA is to take a balanced approach to journal publishing, while recognizing that whatever the AMS does, it needs to serve the mathematical community and ensure equity and inclusiveness in publishing scholarly mathematical content. Whatever we do, we do not want to place the burden of public access (PA) or open access (OA) on individual researchers.

The AMS needs to take account of the inexorable drive towards openness, be it open research, open-source software development, and open access publishing models. In considering how to move forward, we need to take account of the AMS mission, the nature of mathematical research and the cultural differences between math and other disciplines, as well as perceptions among mathematicians in the U.S. and beyond.

The AMS should place equity at the core of its approach to PA and OA models. Of primary importance is to develop models that do not place existing publishing revenue streams at risk. Perhaps the most interesting response to the "Nelson Memorandum" is from *Science*, described earlier. The AMS already offers immediate Green OA on final, peer-reviewed author manuscripts. While there is clearly risk to subscriptions with this model, it may be that this approach reflects best an AMS balanced approach to complying with funder PA and OA mandates.

The AMS should continue to explore appropriate OA business models, while maintaining subscription models as appropriate for a balanced offering that provides options for all authors—funded, or not funded. Further analysis of a range of OA model scenarios will need to be performed, including Gold OA models such as transformative arrangements, Subscribe to Open, and other models such as Diamond OA journals seen in the new AMS flagship journal, *Communications of the American Mathematical Society.*

Glossary of Terms

APC – Article Processing Charge.

arXiv – A repository of electronic preprints (known as e-prints) approved for posting after moderation, but not full peer review. In many fields of mathematics and physics, almost all scientific papers are self-archived on the arXiv repository.

Big Deal – Pricing deal offered to libraries for subscriptions to a large set of journals, typically from large commercial publishers.

bioRxiv – A free submission, distribution and archive service for unpublished preprints in the life sciences.

chemRxiv – A free submission, distribution and archive service for unpublished preprints in chemistry and related areas.

CHORUS – Clearing House for the Open Research of the United States (it is also international now).

cOAlition S/Plan S – cOAlition S funders have proposed that "By 2020 scientific publications that result from research funded by public grants provided by participating national and European research councils and funding

[23]https://scholarlykitchen.sspnet.org/

[24]https://www.all4os.org/

bodies, must be published in compliant Open Access Journals or on compliant Open Access Platforms." Whether or not this will become a formal mandate is yet to be determined.

Content Repository – A content repository or content store is a database of digital content with an associated set of data management, search and access methods allowing application-independent access to the content, rather like a digital library, but with the ability to store and modify content in addition to searching and retrieving. Examples include Funder repositories (e.g., NSF-PAR, and institutional repositories such as Deep Blue at the University of Michigan, and DSpace at MIT).

COPE – Committee on Publication Ethics

Creative Commons Copyright Licenses

CC BY – this license lets others distribute, remix, tweak, and build upon your work, even commercially, as long as they credit you for the original creation.

CC BY-NC – this license lets others remix, tweak, and build upon your work non-commercially, and although their new works must also acknowledge you and be non-commercial, they don't have to license their derivative works on the same terms.

CC BY-NC-ND – this license is the most restrictive of the six main licenses, only allowing others to download your works and share them with others as long as they credit you, but they cannot change them or use them commercially.

DOE – United States Department of Energy.

DOI – A unique and persistent string of characters used to identify a journal article, website, or other item of intellectual property, typically one in digital form.

Embargo Period – The amount of time articles are kept behind a paywall prior to becoming open access.

Hybrid Journal – a subscription journal where select articles are made openly available through an APC.

Mirror Journal – A "mirror" journal shares the same editorial board as a traditional subscription model journal but is itself a separate, Gold OA publication.

NSF – National Science Foundation.

NSF-PAR – NSF/DOE – central article/data repository.

OA – Open Access

Gold OA articles – (versions of record) freely available to the reader because the author has paid an article processing charge (APC).

Green OA – articles freely available at no cost to the author or reader. This can happen when the author deposits the article (most often the accepted manuscript) in an external repository or when the article (version of record) becomes open in the online version of the journal after a time delay known as an embargo period.

Diamond/Platinum OA – articles (version of record) available freely at no cost to the author or reader, with no time delay. These journals tend to be small ones with private grant funding or financial support from their host institution or library.

OSTP – Office of Science and Technology Policy, an office in the White House.

PAR – Publish and Read agreement. With PAR, a consortium pays a pre-agreed amount for papers published by affiliated authors, and everyone in the library/consortium gets access to the subscription content for no extra cost. The recently announced agreement between Wiley and Projekt DEAL in Germany is an example.

Paywall – an arrangement whereby access is restricted to users who have paid to subscribe to the site.

Preprint – a version of a scholarly or scientific paper that precedes formal peer review and publication in a peer-reviewed scholarly or scientific journal. The preprint may be available, often as a non-typeset version available free, before and/or after a paper is published in a journal.

RAP – Read and Publish agreement. The amount of money currently paid to the publisher (for subscriptions and sometimes also for APCs where there has been additional funding for OA publishing) is guaranteed, and in exchange authors can publish OA without paying an additional APC. In some instances—for example where a country publishes many articles with a publisher or there is strong article submission growth to the publisher from authors in the country—additional money is made available. Consortia sometimes cap the total number of articles for which they will pay in order to control costs.

Subscribe to Open – a new OA publishing model from the non-profit publisher, Annual Reviews. It is designed to motivate collective action by libraries who are asked to continue to subscribe even though the content will be published OA.

Version of Record – the final typeset and edited version of the journal article that has been made available by the publisher by formally and exclusively declaring the article "published."

AMS Updates

Whitaker to Give Einstein Public Lecture in Mathematics

"From Segregation to Research Mathematician": Nathaniel Whitaker of the University of Massachusetts, Amherst will give the Einstein Public Lecture in Mathematics at the AMS Spring Central Sectional meeting, April 15–16, 2023, University of Cincinnati.

"In this presentation, I will describe my research in fluid flow and math biology in the context of my personal journey into academia," Whitaker writes. "I'll describe my work with modeling interfaces in a Hele-Shaw cell and two-dimensional turbulence as well as my work in modeling tumor angiogenesis and blood flow in the kidney.

"I will weave into the narrative my journey of growing up in the segregated south to becoming a research mathematician. In this narration, I hope to communicate how acceptance and belonging to a community are essential for success."

Learn more by visiting `https://www.ams.org/meetings/sectional/2308_program.html`.

—AMS Communications Department

"Mathematical Foundations for Democratic Processes" is the First in a New AMS Online Workshop Series

In April, the AMS will offer an online workshop, "Mathematical Foundations for Democratic Processes," the first in a new Engaged Pedagogy Series of online workshops for faculty.

In the span of three afternoon sessions, slated to run April 23–25, 2023 (Sunday–Tuesday), the organizers—Stanley Chang, Andrew Schultz, and Ismar Volić of Wellesley College's Institute for Mathematics and Democracy—will engage participants in topics that include:

- voting and social choice theory;
- apportionment and the Electoral College;
- gerrymandering; and
- quantification of power.

Participants in this online workshop will learn the tools and resources to create courses or incorporate modules on mathematics and politics into their own teaching. We aim to create a collaborative, creative, and active learning environment. All levels of experience with teaching these topics are welcome!

Through the new Engaged Pedagogy Series, the AMS will provide mathematical sciences faculty with experiences and resources to strengthen them as professionals and empower their engagement with students learning mathematics. Offered in the spring and fall, these online workshops will be inspiring opportunities to connect with others in the mathematical community who teach; to engage in current applications of mathematics; to innovate with curriculum; and to learn how to use online and software tools.

Register for "Mathematical Foundations for Democratic Processes" by April 7, 2023. Space is limited. The cost is $200 for AMS members; $300 for nonmembers. Watch for updates at `www.ams.org/engaged-pedagogy-series` and in *Headlines & Deadlines*, the biweekly AMS member publication.

—AMS Programs Department

Ten Students Receive 2022 AMS Undergraduate Opportunity Awards

Ten students have received 2022 AMS Undergraduate Opportunity Awards, designed to help them pursue careers in mathematics at a college or university that is an institutional member of the AMS. These funds support students

DOI: https://doi.org/10.1090/noti2655

who lack adequate financial resources and who may be in danger of not completing their degree program in mathematics for financial reasons. Each year the AMS selects a number of geographically distributed schools, which in turn make one-time awards to undergraduate mathematical students. The amount of each scholarship is currently $3,000, and the number of scholarships awarded each year varies.

Nine of this year's awardees received Waldemar J. Trjitzinsky Memorial Awards, named in honor of the mathematician. Since 1991, more than 200 promising math students have received Trjitzinsky Awards to ensure that financial hardship does not stand in the way of completing their degree programs.

In 2022, the following nine undergraduates received Trjitzinsky Awards:

Michael Brito, Middle Tennessee State University;
Nevin Etter, Washington and Lee University;
Necef Kavrut, California Institute of Technology;
Aniela E. Mendez, University of Texas Rio Grande Valley;
Kayleigh Page, University of Central Oklahoma;
Mahdi Rahman, University of Buffalo–SUNY;
Leslye Rodriguez, DePaul University;
Aaron Thomas, University of Louisville;
Heather Vogler, Northern Arizona University.

In addition, the Edmund Landau Award of $3,000, named in honor of the mathematician and instituted in 2021, was presented to **Kaia De Vries,** University of Maine, Orono.

—AMS Communications Department

Apply for an AMS-Simons Research Enhancement Grant for Primarily Undergraduate Institution (PUI) Faculty

The new AMS-Simons Research Enhancement Grants Program for PUI Faculty fosters and supports research collaboration by mathematicians employed full-time at institutions that do not award doctoral degrees in mathematics. In support of AMS's continuing efforts to promote equity, diversity, and inclusion in the mathematical research enterprise, we strongly encourage and welcome applicants from diverse backgrounds and experiences. The AMS is committed to selecting a diverse group of researchers from PUIs across the country, from small liberal arts colleges to large public universities with master's (but not doctoral) mathematics degree programs.

This funding will provide support essential to growing and sustaining the investigator's research. Each year for three years, awardees will receive $3,000 to support research-related activities, with some additional modest support for the awardees' department and institution. With generous funding from the Simons Foundation supplemented by the AMS and individual donors, at least 40 awards will be made. Funds for the first round of grants will be disbursed in July 2023.

Tenured and tenure-track mathematicians with an active research program at a PUI, who have earned a PhD at least five years before the start of the grant, and who do not have more than $3,000 in external research support are eligible to apply. Applications will be accepted on MathPrograms.org through March 20, 2023. To see additional information and restrictions, please visit the link to the application: `https://www.ams.org/ams-simons-pui-research`.

—AMS Programs Department

Deaths of AMS Members

Thomas R. Butts, of Dallas, Texas, died on January 26, 2019. Born on July 5, 1943, he was a member of the Society for 50 years.

Richard H. Dale, of Las Cruces, New Mexico, died on February 1, 2020. Born on June 8, 1936, he was a member of the Society for 60 years.

Martin David Davis, of Berkeley, California, died on January 1, 2023. Born on March 8, 1928, he was a member of the Society for 76 years.

Jacob Eli Goodman, of San Rafael, California, died on October 10, 2021. Born on November 15, 1933, he was a member of the Society for 66 years.

Yuly Makovoz, of Johns Creek, Georgia, died on November 9, 2022. Born on May 19, 1937, he was a member of the Society for 39 years.

R. Bradford Murphy, of Basking Ridge, New Jersey, died on August 28, 2017. Born on June 7, 1922, he was a member of the Society for 65 years.

M. Wachman, of Storrs, Connecticut, died on September 29, 2021. Born on February 1, 1931, he was a member of the Society for 52 years.

NEWS

Mathematics People

Kakde Awarded Infosys Prize 2022 in Mathematical Sciences

The Infosys Prize 2022 in Mathematical Sciences was awarded to Mahesh Kakde, professor of mathematics, Indian Institute of Science, Bengaluru for his outstanding contributions to algebraic number theory. "Prof. Kakde's deep work on the non-commutative Iwasawa main conjecture, his work on the Gross-Stark conjecture (with Samit Dasgupta and Kevin Ventullo), and his work on the Brumer-Stark conjecture (with Samit Dasgupta), resolves outstanding conjectures at the heart of modern number theory," according to the citation.

The Infosys Prize celebrates the achievements of the recipients and awards them for their contributions to science and research impacting India. The Infosys Science Foundation (ISF) awarded the Infosys Prize 2022 in six categories: Engineering and Computer Science, Humanities, Life Sciences, Mathematical Sciences, Physical Sciences, and Social Sciences. The prize for each category is comprised of a gold medal, a citation, and a prize of US$100,000 (or its equivalent in rupees). "The Infosys Prize counts among its laureates two Nobel Prize winners, two Fields Medal winners, a MacArthur fellow, and others who have gone on to hold high posts in government and academia," according to a press release.

2022 Clay Research Awards Presented

The 2022 Clay Research Awards were presented by the Clay Mathematics Institute (CMI) to John Pardon, Søren Galatius, and Oscar Randal-Williams.

Pardon, of Princeton University, was recognized for his wide-ranging and transformative work in geometry and topology, particularly his achievements in symplectic topology.

DOI: https://doi.org/10.1090/noti2654

"Pardon's work displays a remarkable clarity of vision, sustained through extensive projects in which he develops conceptually sophisticated theories that cast long-standing problems into settings where they become tractable," according to the citation. "His novel treatment of the theory of moduli spaces of pseudo-holomorphic curves, leading to an elegant construction of virtual fundamental cycles and chains, provides a compelling example."

Galatius (University of Copenhagen) and Randal-Williams (University of Cambridge) were honored for their profound contributions to the understanding of high-dimensional manifolds and their diffeomorphism groups. "They have transformed and reinvigorated the subject," according to the prize citation.

"In a celebrated trilogy of papers on moduli spaces of high-dimensional manifolds, they established homology stability for diffeomorphism groups of manifolds in even dimensions $2n > 4$, where the stabilization is performed by taking connected sums with an increasing number of copies of the product of two n-dimensional spheres," according to the citation. "They explained how to compute the limiting homology in terms of a specific spectrum, and beyond their explicit results they developed an array of ideas that have stimulated waves of subsequent advances in the field."

Karagulyan Awarded 2023 Emil Artin Junior Prize in Mathematics

Davit Karagulyan of KTH Royal Institute of Technology, Stockholm, has been awarded the 2023 Emil Artin Junior Prize in Mathematics for his paper "On certain aspects of the Möbius randomness principle," *Colloquium Mathematicum* 157 (2019), no. 2, 231–250.

Established in 2001, the Emil Artin Junior Prize in Mathematics is awarded under the auspices of the Armenian Mathematical Union and carries a cash prize of US$1,400. It is presented usually every year to a student or former student of an Armenian educational institution who is under the age of thirty-five, for outstanding contributions to algebra, geometry, topology, and number theory: the fields in which Artin made major contributions.

2022 AMSI Awards Announced

The Australia Mathematical Society Inc. (AMSI) announced its 2022 awardees.

The 2022 AustMS Medal was awarded to Guoyin Li (University of New South Wales), recognizing his outstanding contributions to optimization, variational analysis, and multilinear algebra. The medal is awarded for distinguished research in the mathematical sciences to a member of the Society within 15 years of the award of their PhD.

The 2022 George Szekeres Medal—AustMS's most prestigious medal recognizing research achievement and outstanding support of the mathematical sciences—was awarded to Igor Shparlinski (UNSW). "Professor Shparlinski has made notable contributions in many areas of mathematics with specialisms in number theory and its applications, and his influence on mathematics is reflected in his incredible citation rate," according to a press release.

The 2022 Gavin Brown Prize for best paper was awarded to Mark Holmes (University of Melbourne) and Edwin Perkins (University of British Columbia) for their paper "On the range of lattice models in high dimensions," *Probability Theory and Related Fields* 176 (2020), no. 3, 941–1009.

The 2022 Mahony-Neumann-Room Prize for outstanding contributions to AustMS's research publications was awarded to Satish K. Pandey (University of Waterloo) and Vern I. Paulsen (University of Waterloo), for their paper "A spectral characterization of AN operators," *J. Aust. Math. Soc.* 102 (2017), no. 3, 369–391.

Journal of Complexity Names Best Paper Award 2022

Thomas Kühn and Martin Petersen are winners of the 2022 Best Paper Award. According to the Award Committee, the paper "Approximation in periodic Gevrey spaces" by Kühn and Petersen, published in Volume 73, December 2022, Article 101665, exhibits exceptional merits. The winners will divide a US$4,000 prize and each author will receive a plaque.

US Rhodes Scholarship Winners Announced

Two US students who study math were among those elected as 2023 Rhodes Scholars.

Sophie A. Bryant is a senior at Columbia University, where she is completing dual majors in East Asian Languages and Cultures and Mathematics. Isaac A. Robinson is a senior at Harvard College, where he majors in Computer Science and Mathematics.

G. de B. Robinson Award to Coulembier, Chen

Kevin Coulembier and Chih-Whi Chen received the G. de B. Robinson Award, which recognizes the publication of excellent papers in the *Canadian Journal of Mathematics* (CJM) and the *Canadian Mathematical Bulletin* (CMB), and encourages the submission of the highest quality papers to these journals.

Classified Advertising

Employment Opportunities

CHINA

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at the Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

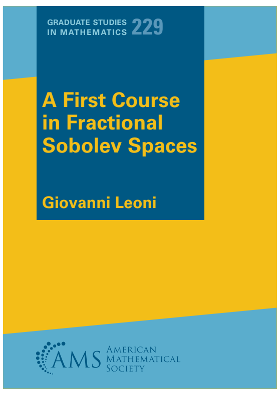
Please send your resume to `mathjobs@tju.edu.cn`.

For more information, please visit `cam.tju.edu.cn` or contact Mr. Albert Liu at `mathjobs@tju.edu.cn`, telephone: 86-22-2740-6039.

2

New Books Offered by the AMS

Analysis



A First Course in Fractional Sobolev Spaces

Giovanni Leoni, *Carnegie Mellon University, Pittsburgh, PA*

This book provides a gentle introduction to fractional Sobolev spaces which play a central role in the calculus of variations, partial differential equations, and harmonic analysis. The first part deals with fractional Sobolev spaces of one variable. It covers the definition, standard properties, extensions, embeddings, Hardy inequalities, and interpolation inequalities. The second part deals with fractional Sobolev spaces of several variables. The author studies completeness, density, homogeneous fractional Sobolev spaces, embeddings, necessary and sufficient conditions for extensions, Gagliardo-Nirenberg type interpolation inequalities, and trace theory. The third part explores some applications: interior regularity for the Poisson problem with the right-hand side in a fractional Sobolev space and some basic properties of the fractional Laplacian.

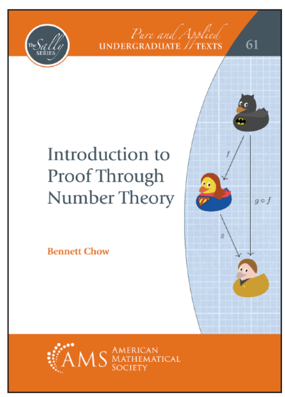
The first part of the book is accessible to advanced undergraduates with a strong background in integration theory; the second part, to graduate students having familiarity with measure and integration and some functional analysis. Basic knowledge of Sobolev spaces would help, but is not necessary. The book can also serve as a reference for mathematicians working in the calculus of variations and partial differential equations as well as for researchers in other disciplines with a solid mathematics background. It contains several exercises and is self-contained.

Graduate Studies in Mathematics, Volume 229
May 2023, approximately 579 pages, LC 202204094, 2010 *Mathematics Subject Classification*: 46E35; 26A33; 26A46; 30H25; 35R11

Hardcover, ISBN: 978-1-4704-6898-9, List US$135, **AMS members US$108**, **MAA members US$121.50**, Order code GSM/229
`bookstore.ams.org/gsm-229`

Softcover, ISBN: 978-1-4704-7253-5, List US$89, **AMS members US$71.20**, **MAA members US$80.10**, Order code GSM/229.S
`bookstore.ams.org/gsm-229-s`

Discrete Mathematics and Combinatorics



Introduction to Proof Through Number Theory

Bennett Chow, *University of California, San Diego, La Jolla, CA*

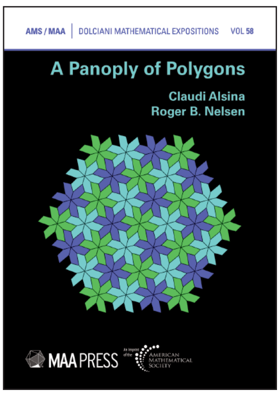
Lighten up about mathematics! Have fun. If you read this book, you will have to endure bad math puns and jokes and out-of-date pop culture references. You'll learn some really cool mathematics to boot. In the process, you will immerse yourself in living, thinking, and breathing logical reasoning. We like to call this *proofs*, which to some is a bogey word, but to us it is a boogie word. You will learn how to solve problems, real and imagined. After all, math is a game where, although the rules are pretty much set, we are left to our imaginations to create. Think of this book as blueprints, but you are the architect of what structures you want to build. Make sure you lay a good foundation, for otherwise your buildings might fall down. To help you through this, we guide you to think and plan carefully. Our playground consists of basic math, with a loving emphasis on number theory. We will encounter the known and the unknown. Ancient and modern inquirers left us with elementary-sounding mathematical puzzles that are unsolved to this day. You will learn induction, logic, set theory, arithmetic, and algebra, and you may one day solve one of these puzzles.

This item will also be of interest to those working in logic and foundations, general interest, and number theory.

Pure and Applied Undergraduate Texts, Volume 61
May 2023, approximately 456 pages, Softcover, ISBN: 978-1-4704-7027-2, LC 2022041609, 2010 *Mathematics Subject Classification*: 00–XX, 03–XX, 05–XX, 11–XX, 97–XX; 68–XX, List US$89, **AMS members US$71.20**, **MAA members US$80.10**, Order code AMSTEXT/61

bookstore.ams.org/amstext-61

Geometry and Topology



A Panoply of Polygons

Claudi Alsina, *Universitat Politècnica de Catalunya, Barcelona, Spain* and **Roger B. Nelsen**, *Lewis & Clark College, Portland, OR*

A Panoply of Polygons presents and organizes hundreds of beautiful, surprising and intriguing results about polygons with more than four sides. (*A Cornucopia of Quadrilaterals*, a previous volume by the same authors, thoroughly explored the properties of four-sided polygons.) This panoply consists of eight chapters, one dedicated to polygonal basics, the next ones dedicated to pentagons, hexagons, heptagons, octagons and many-sided polygons. Then miscellaneous classes of polygons are explored (e.g., lattice, rectilinear, zonogons, cyclic, tangential) and the final chapter presents polygonal numbers (figurate numbers based on polygons). Applications, real-life examples, and uses in art and architecture complement the presentation where many proofs with a visual nature are included.

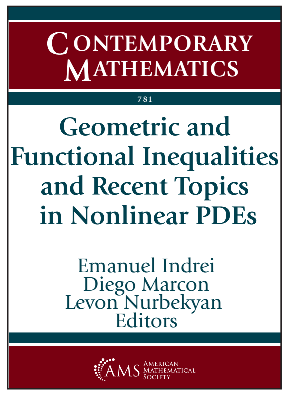
A Panoply of Polygons can be used as a supplement to a high school or college geometry course. It can also be used as a source for group projects or extra-credit assignments. It will appeal, and be accessible to, anyone with an interest in plane geometry. Claudi Alsina and Roger Nelsen are, jointly and individually, the authors of thirteen previous MAA/AMS books. Those books, and this one, celebrate and illuminate the power of visualization in learning, teaching, and creating mathematics.

Dolciani Mathematical Expositions, Volume 58
April 2023, approximately 274 pages, Softcover, ISBN: 978-1-4704-7184-2, 2010 *Mathematics Subject Classification*: 51M05, 51M15, 97G40, List US$65, **AMS Individual member US$48.75**, AMS Institutional member US$52, **MAA members US$48.75**, Order code DOL/58

bookstore.ams.org/dol-58

New in Contemporary Mathematics

Differential Equations



Geometric and Functional Inequalities and Recent Topics in Nonlinear PDEs

Emanuel Indrei, *Purdue University, West Lafayette, IN*, **Diego Marcon**, *Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil*, and **Levon Nurbekyan**, *University of California, Los Angeles, CA*, Editors

This volume contains the proceedings of the virtual conference on Geometric and Functional Inequalities and Recent Topics in Nonlinear PDEs, held from February 28–March 1, 2021, and hosted by Purdue University, West Lafayette, IN.

The mathematical content of this volume is at the intersection of viscosity theory, Fourier analysis, mass transport theory, fractional elliptic theory, and geometric analysis. The reader will encounter, among others, the following topics: the principal-agent problem; Maxwell's equations; Liouville-type theorems for fully nonlinear elliptic equations; a doubly monotone flow for constant width bodies; and the edge dislocations problem for crystals that describes the equilibrium configurations by a nonlocal fractional Laplacian equation.

This item will also be of interest to those working in analysis.

Contemporary Mathematics, Volume 781
February 2023, 135 pages, Softcover, ISBN: 978-1-4704-6652-7, LC 2022032626, 2010 *Mathematics Subject Classification*: 35–XX, 49–XX, 34–XX, 31–XX, 53–XX, 45–XX, 42–XX, 51–XX, 52–XX, List US$130, **AMS members US$104**, **MAA members US$117**, Order code CONM/781

bookstore.ams.org/conm-781

New in Memoirs of the AMS

Algebra and Algebraic Geometry

Higher Ramanujan Equations and Periods of Abelian Varieties

Tiago J. Fonseca, *IMECC-UNICAMP, Campinas-SP, Brazil*

Memoirs of the American Mathematical Society, Volume 281, Number 1391
February 2023, 125 pages, Softcover, ISBN: 978-1-4704-6019-8, 2010 *Mathematics Subject Classification*: 14D23, 14K99, 37F75, 11J81, 11G18, List US$85, **AMS members US$68**, **MAA members US$76.50**, Order code MEMO/281/1391

bookstore.ams.org/memo-281-1391

Affine Hecke Algebras and Quantum Symmetric Pairs

Zhaobing Fan, *Harbin Engineering University, China*, **Chun-Ju Lai**, *Institute of Mathematics, Academia Sinica, Taipei, Taiwan*, **Yiqiang Li**, *SUNY at Buffalo, New York*, **Li Luo**, *East China Normal University, Shanghai, China*, and **Weiqiang Wang**, *University of Virginia, Charlottesville, Virginia*

Memoirs of the American Mathematical Society, Volume 281, Number 1386
February 2023, 92 pages, Softcover, ISBN: 978-1-4704-5626-9, 2010 Mathematics Subject Classification: 17B37; 20C08, List US$85, **AMS members US$68**, **MAA members US$76.50**, Order code MEMO/281/1386

bookstore.ams.org/memo-281-1386

Analysis

Symbolic Extensions of Amenable Group Actions and the Comparison Property

Tomasz Downarowicz, *Wroclaw University of Science and Technology, Poland* and **Guohua Zhang**, *Fudan Unversity, Shanghai, China*

Memoirs of the American Mathematical Society, Volume 281, Number 1390
February 2023, 95 pages, Softcover, ISBN: 978-1-4704-5587-3, 2010 *Mathematics Subject Classification*: 37B05, 37C85, 37B10; 43A07, 20E07, List US$85, **AMS members US$68**, **MAA members US$76.50**, Order code MEMO/281/1390

bookstore.ams.org/memo-281-1390

Multi-Parameter Hardy Spaces Theory and Endpoint Estimates for Multi-Parameter Singular Integrals

Guozhen Lu, *University of Connecticut, Storrs, CT*, **Jiawei Shen**, *Wayne State University, Detroit, MI*, and **Lu Zhang**, *Shaanxi Normal University, Xian, China, and Binghamton University, New York*

Memoirs of the American Mathematical Society, Volume 281, Number 1388
February 2023, 87 pages, Softcover, ISBN: 978-1-4704-5537-8, 2010 *Mathematics Subject Classification*: 42B20, 42B25, 42B30, List US$85, **AMS members US$68**, **MAA members US$76.50**, Order code MEMO/281/1388

bookstore.ams.org/memo-281-1388

Geometry and Topology

The Regularity of the Linear Drift in Negatively Curved Spaces

François Ledrappier, *Sorbonne Université, Paris, France* and **Lin Shu**, *Peking University, Beijing, China*

Memoirs of the American Mathematical Society, Volume 281, Number 1387
February 2023, 151 pages, Softcover, ISBN: 978-1-4704-5542-2, 2010 *Mathematics Subject Classification*: 37D40, 58J65, List US$85, **AMS members US$68**, **MAA members US$76.50**, Order code MEMO/281/1387

bookstore.ams.org/memo-281-1387

Mathematical Physics

The $\mathscr{P}(\varphi)_2$ Model on de Sitter Space

João C. A. Barata, *Universidade de São Paulo, Brasil*, **Christian D. Jäkel**, *Universidade de São Paulo, Brasil*, and **Jens Mund**, *Universidade de Juiz de Fora, Brasil*

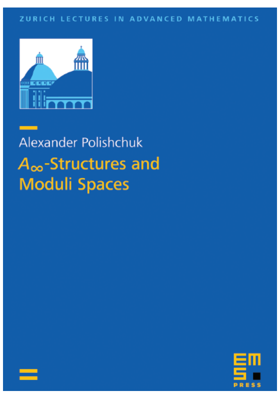
Memoirs of the American Mathematical Society, Volume 281, Number 1389

February 2023, 261 pages, Softcover, ISBN: 978-1-4704-5548-4, 2010 *Mathematics Subject Classification*: 81T05, 81T08, 81T20, 22E43, 47L90, List US$85, **AMS members US$68**, **MAA members US$76.50**, Order code MEMO/281/1389

bookstore.ams.org/memo-281-1389

New AMS-Distributed Publications

Algebra and Algebraic Geometry



A_∞-Structures and Moduli Spaces

Alexander Polishchuk, *University of Oregon, Eugene, OR*

This book discusses certain moduli problems related to A_∞-structures. These structures can be viewed as a way of recording extra information on cohomology algebras. They are useful in describing derived categories appearing in geometry, and, as such, they play an important role in homological mirror symmetry.

The author presents some general results on the classification of A_∞-structures. For example, he gives a sufficient criterion for the existence of a finite-type moduli scheme of A_∞-structures extending a given associative algebra. He also considers two concrete moduli problems for A_∞-structures. The first is related to the moduli spaces of curves, while the second is related to the classification of solutions of an associative version of the Yang–Baxter equation.

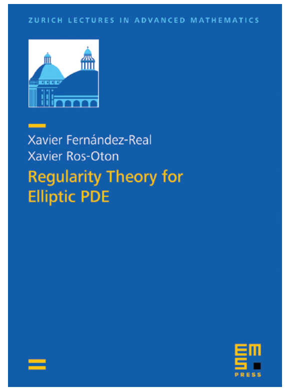
The book will be of interest to graduate students and researchers working in homological algebra, algebraic geometry, and related areas.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Zurich Lectures in Advanced Mathematics, Volume 29
December 2022, 178 pages, Softcover, ISBN: 978-3-98547-026-6, 2010 *Mathematics Subject Classification*: 18–02, 18G70; 14A22, 14F05, 14H10, 16E35, 16E45, List US$45, **AMS members US$36**, Order code EMSZLEC/29

bookstore.ams.org/emszlec-29

Differential Equations



Regularity Theory for Elliptic PDE

Xavier Fernández-Real, *École Polytechnique Fédérale de Lausanne, Switzerland* and **Xavier Ros-Oton**, *ICREA, Universitat de Barcelona and Centre de Recerca Matemàtica, Spain*

One of the most basic mathematical questions in PDE is that of regularity. A classical example is Hilbert's XIXth problem, stated in 1900, which was solved by De Giorgi and Nash in the 1950s. The question of regularity has been a central line of research in elliptic PDE during the second half of the 20th century and has influenced many areas of mathematics linked one way or another with PDE. This text aims to provide a self-contained introduction to the regularity theory for elliptic PDE, focusing on the main ideas rather than proving all results in their greatest generality. It can be seen as a bridge between an elementary PDE course and more advanced books.

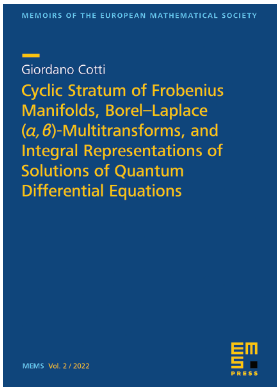
The book starts with a short review of the Laplace operator and harmonic functions. The theory of Schauder estimates is developed next but presented with various proofs of the results. Nonlinear elliptic PDE are covered in the following, both in the variational and non-variational setting, and, finally, the obstacle problem is studied in detail, establishing the regularity of solutions and free boundaries.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Zurich Lectures in Advanced Mathematics, Volume 28
December 2022, 236 pages, Softcover, ISBN: 978-3-98547-028-0, 2010 *Mathematics Subject Classification*: 35J15; 35B65, 35J05, 35J20, 35J60, 35R35, List US$55, **AMS members US$44**, Order code EMSZLEC/28

bookstore.ams.org/emszlec-28

NEW BOOKS



Cyclic Stratum of Frobenius Manifolds, Borel–Laplace (α,β)-Multitransforms, and Integral Representations of Solutions of Quantum Differential Equations

Giordano Cotti, *Universidade de Lisboa, Portugal*

In the first part of this book, the author introduces the notion of cyclic stratum of a Frobenius manifold *M*. This is the set of points of the extended manifold $\mathbb{C}^* \times M$, at which the unit vector field is a cyclic vector for the isomonodromic system defined by the flatness condition of the extended deformed connection. The study of the geometry of the complement of the cyclic stratum is addressed. The author shows that at points of the cyclic stratum, the isomonodromic system attached to *M* can be reduced to a scalar differential equation, called the *master differential equation* of *M*. In the case of Frobenius manifolds coming from Gromov–Witten theory, namely quantum cohomologies of smooth projective varieties, such a construction reproduces the notion of quantum differential equation.

In the second part of this book, the author introduces two multilinear transforms, called *Borel–Laplace* (α,β)*-multitransforms*, on spaces of Ribenboim formal power series with exponents and coefficients in an arbitrary finite-dimensional $\mathbb{C}$-algebra *A*.

In the third and final part of the book, as an application, the author shows how to use the new analytic tools, introduced in the previous parts, in order to study the quantum differential equations of Hirzebruch surfaces.

This item will also be of interest to those working in algebra and algebraic geometry.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Memoirs of the European Mathematical Society, Volume 2 December 2022, 134 pages, Softcover, ISBN: 978-3-98547-023-5, 2010 *Mathematics Subject Classification*: 53D45; 14N35, 18G80, List US$75, **AMS members US$60**, Order code EMSMEM/2

bookstore.ams.org/emsmem-2

Interactions between Elasticity and Fluid Mechanics

Maurizio Garrione, *Politecnico di Milano, Italy* and **Filippo Gazzola**, *Politecnico di Milano, Italy*, Editors

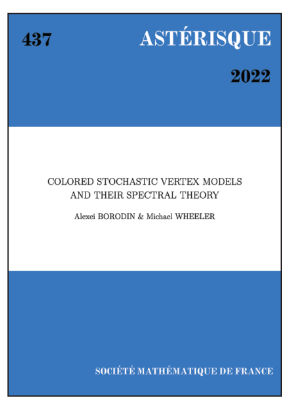
Partial differential equations arise naturally in mathematical physics and have numerous applications in real life. This book mainly focuses on fluid mechanics, elasticity, and their interactions. As a typical model of such phenomena, one may consider the fluid-structure interactions between the wind and a suspension bridge. Not much is known about the mechanisms generating instabilities (in a broad sense) and many problems are still open, while an interdisciplinary approach is necessary for a better understanding of all the involved phenomena.

This book collects different points of view on these phenomena and is addressed both to junior researchers entering the field as well as to experienced professionals aiming to expand their scientific knowledge to closely related disciplines. The book also aims to bring the mathematical and engineering communities closer to create a common language and to encourage future collaborations.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Industrial and Applied Mathematics, Volume 3 December 2022, 248 pages, Hardcover, ISBN: 978-3-98547-027-3, 2010 *Mathematics Subject Classification*: 35–02; 76D05, 74B20, List US$75, **AMS members US$60**, Order code EMSIAM/3

bookstore.ams.org/emsiam-3



Colored Stochastic Vertex Models and Their Spectral Theory

Alexei Borodin, *Massachsetts Institute of Technology, Cambridge, and Institute for Information Transmission Problems, Moscow, Russia* and **Michael Wheeler**, *The University of Melbourne, Parkville, Victorria, Australia*

This work is dedicated to $\mathfrak{sl}_{n+1}$-related integrable stochastic vertex models; these models are called colored. The authors prove several results about these models, which include the following:

- The authors construct the basis of (rational) eigenfunctions of the colored transfer-matrices as partition functions of their lattice models with certain boundary conditions. Similarly, they construct a dual basis and prove the corresponding orthogonality relations and Plancherel formulae.
- The authors derive a variety of combinatorial properties of those eigenfunctions, such as branching rules, exchange relations under Hecke divided-difference operators, (skew) Cauchy identities of different types, and monomial expansions.
- The authors show that their eigenfunctions are certain (non-obvious) reductions of the nested Bethe Ansatz eigenfunctions.
- For models in a quadrant with domain-wall (or half-Bernoulli) boundary conditions, the authors prove a matching relation that identifies the distribution of the colored height function at a point with the distribution of the height function along a line in an associated color-blind ($\mathfrak{sl}_2$-related) stochastic vertex model. Thanks to a variety of known results about asymptotics of height functions of the color-blind models, this implies a similar variety of limit theorems for the colored height function of their models.
- The authors demonstrate how the colored/uncolored match degenerates to the colored (or multi-species) versions of the ASEP, q-PushTASEP, and the q-boson model.
- The authors show how their eigenfunctions relate to non-symmetric Cherednik-Macdonald theory, and they make use of this connection to prove a probabilistic matching result by applying Cherednik-Dunkl operators to the corresponding non-symmetric Cauchy identity.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the US, Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 437
December 2022, 225 pages, Softcover, ISBN: 978-2-85629-963-0, 2010 *Mathematics Subject Classification*: 05E05, 05E10, 82B23, List US$74, **AMS members US$59.20**, Order code AST/437

bookstore.ams.org/ast-437

CELEBRATING WOMEN MATHEMATICIANS

RESEARCHERS & ROLE MODELS

Rosemary Guzman

Topology and its applications in other areas of mathematics

"Form your networks with intent, understanding that the formation of different kinds of networks—research, mentoring, and teaching—serve distinct purposes."

Fern Y. Hunt

Discrete approximation of dynamical systems and applications of Markov chains

"Attend as many research conferences as time and money allow, make use of online resources such as the arXiv and MathSciNet, and organize a special session at an annual conference."

Emily Riehl

Category theory, particularly as related to homotopy theory

"Ultimately, how you want to spend your time engaging with mathematical ideas is up to you. Give talks and take on extracurricular writing or teaching projects if this makes you happier, even if this means less time for research and other things."

Éva Tardos

Algorithms and algorithmic game theory

"Be open to exciting new opportunities in research."

Amie Wilkinson

Dynamical systems

"Enjoy the process of discovery, there's an infinite zoo of possibilities to explore and that's the joy of mathematics."

Melody Chan

Tropical geometry, combinatorialalgebraic geometry and combinatorics

"Do examples! Try to do as much mathematics as you can standing at the board, writing things down, and explaining them to people."

Tara S. Holm

Symplectic geometry and its applications in other areas of mathematics

"Find collaborators, especially ones who are close enough to your field so that you can talk but far enough that they will teach you some new mathematics."

Andrea Nahmod

Nonlinear Fourier and harmonic analysis and partial differential equations

"Have a broad mathematical culture, follow your intuition, keep a long view about research, and love what you do."

Gigliola Staffilani

Partial differential equations that model nonlinear wave phenomena

"Work with other mathematicians. The model of the lonely researcher in an ivory tower does not match with most of the mathematicians I know. This is a myth that definitely needs to be busted: it is dangerous and not encouraging."

Chelsea Walton

Noncommutative algebra and noncommutative algebraic geometry

"Find, value, and support your network of people, in math or not, who can selflessly give you words of encouragement, because the happier you are, the more math you will do!"

Read about the mathematical research, inspiring stories, and advice of these AMS members and more women researchers and role models at

www.ams.org/women-18

AMS American Mathematical Society

Meetings & Conferences of the AMS

March Table of Contents

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. *Paid meeting registration is required to submit an abstract to a sectional meeting.*

Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at `www.ams.org/meetings`.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to `https://www.ams.org/meetings/meetings-general` for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in LaTeX. Visit `www.ams.org/cgi-bin/abstracts/abstract.pl`. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Associate Secretaries of the AMS

Central Section: Betsy Stovall, University of Wisconsin–Madison, 480 Lincoln Drive, Madison, WI 53706; email: `stovall@math.wisc.edu`; telephone: (608) 262-2933.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: `steve.weintraub@lehigh.edu`; telephone: (610) 758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403; email: `brian@math.uga.edu`; telephone: (706) 542-2547.

Western Section: Michelle Manes, University of Hawaii, Department of Mathematics, 2565 McCarthy Mall, Keller 401A, Honolulu, HI 96822; email: `mamanes@hawaii.edu`; telephone: (808) 956-4679.

Meetings in this Issue

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. Please see our full Policy on a Welcoming Environment at `https://www.ams.org/welcoming-environment-policy`.

Meetings & Conferences of the AMS

IMPORTANT information regarding meetings programs: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `https://www.ams.org/meetings`.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL.

New: Sectional Meetings Require Registration to Submit Abstracts. In an effort to spread the cost of the sectional meetings more equitably among all who attend and hence help keep registration fees low, starting with the 2020 fall sectional meetings, you must be registered for a sectional meeting in order to submit an abstract for that meeting. You will be prompted to register on the Abstracts Submission Page. In the event that your abstract is not accepted or you have to cancel your participation in the program due to unforeseen circumstances, your registration fee will be reimbursed.

Atlanta, Georgia

Georgia Institute of Technology

March 18–19, 2023

Saturday – Sunday

Meeting #1184

Southeastern Section
Associate Secretary for the AMS: Brian D. Boe

Program first available on AMS website: To be announced
Issue of *Abstracts*: Volume 44, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Betsy Stovall, University of Wisconsin-Madison, *Reverse Inequalities in Harmonic Analysis*.

Blair Dowling Sullivan, University of Utah, *Title to be announced*.

Yusu Wang, University of California San Diego, *Comparing Graphs with Attributes: Weisfeiler-Lehman Distances and Connection to Graph Neural Networks*.

Amie Wilkinson, University of Chicago, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `https://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advanced Topics in Graph Theory and Combinatorics, **Songling Shan**, Illinois State University, and **Guangming Jing**, Augusta University.

Advances in Applied Dynamical Systems and Mathematical Biology, **Chunhua Shan**, The University of Toledo, and **Guihong Fan**, Columbus State University.

Advances in Mathematical Finance and Optimization, **Ibrahim Ekren**, **Arash Fahim**, and **Lingjiong Zhu**, Florida State University.

Algebraic Methods in Algorithms, **Kevin Shu** and **Mehrdad Ghadiri**, Georgia Institute of Technology.

Combinatorial Matrix Theory, **Zhongshan Li**, **Marina Arav**, and **Hein Van der Holst**, Georgia State University.

Combinatorics, Probability and Computation in Molecular Biology, **Christine Heitsch** and **Brandon Jerome Legried**, Georgia Institute of Technology.

Commutative Algebra and its Interactions with Algebraic Geometry, **Michael Brown** and **Henry K. Schenck**, Auburn University.

Contact and Symplectic Topology in Dimensions 3 and 4, **Akram Alishahi**, **Peter Lambert-Cole**, and **Gordana Matic**, University of Georgia.

Discrete Analysis, **Giorgis Petridis**, **Neil Lyall**, and **Akos Magyar**, University of Georgia.

Disordered and Periodic Quantum Systems, **Rodrigo Bezerra de Matos** and **Wencai Liu**, Texas A&M University, and **Xiaowen Zhu**, University of Washington.

Diversity in Mathematical Biology, **Daniel Alejandro Cruz**, University of Florida, and **Margherita Maria Ferrari**, University of Manitoba.

Dynamics of Partial Differential Equations, **Gong Chen**, Georgia Institute of Technology, **Hao Jia**, University of Minnesota, and **Dallas Albritton**, Princeton University.

Fractal Geometry and Dynamical Systems, **Mrinal Kanti Roychowdhury**, School of Mathematical and Statistical Sciences, University of Texas Rio Grande Valley, and **Scott Kaschner**, Butler University.

Geometric and Combinatorial Aspects of Lie Theory, **William Graham**, University of Georgia, **Amber Russell**, Butler University, and **Scott Larson**, University of Georgia.

Geometric Group Theory, **Ryan Dickmann**, Georgia Institute of Technology, **Sahana H. Balasubramanya**, University of Münster, and **Abdoul Karim Sane** and **Dan Margalit**, Georgia Institute of Technology.

Harmonic Analysis, **Betsy Stovall**, University of Wisconsin-Madison, **Benjamin Jaye**, Georgia Tech, and **Manasa Vempati**.

High-dimensional Convexity and Probability, **Galyna Livshyts** and **Orli Herscovici**, Georgia Institute of Technology, and **Dan Mikulincer**, MIT.

Knots, Skein Modules and Categorification, **Marithania Silvero-Casanova**, Universidad De Sevilla, **Rhea Palak Bakshi**, The George Washington University, **Jozef Henryk Przytycki**, George Washington University, and **Radmila Sazdanovic**, North Carolina State University.

Logic, Combinatorics, and their Interactions, **Anton Bernshteyn**, Georgia Institute of Technology, and **Robin Tucker-Drob**, University of Florida.

Macdonald Theory at the Intersection of Combinatorics, Algebra, and Geometry, **Olya Mandelshtam**, University of Waterloo, **Sean Griffin**, UC Davis, and **Andy Wilson**, Kennesaw State University.

Mathematical Modeling and Simulation Techniques in Fluid Structure Interaction Problems, **Pejman Sanaei**, Georgia State University.

Mathematical Modeling of Populations and Diseases Transmissions, **Yang LI**, Georgia State University, **Jia Li**, University of Alabama in Huntsville, and **Xiang-Sheng Wang**, University of Louisiana at Lafayette.

Multiscale Approaches to Modeling Ecological and Evolutionary Dynamics, **Daniel Brendan Cooney**, University of Pennsylvania, **Denis Daniel Patterson**, Princeton University, **Olivia Chu**, Dartmouth College, and **Chadi M Saad-Roy**, University of California, Berkeley.

Qualitative Aspects of Nonlinear PDEs: Well-posedness and Asymptotics, **Atanas G. Stefanov**, University of Alabama Birmingham, **Fazel Hadaifard**, University of California - Riverside, and **Jiahong Wu**, Oklahoma State University.

Quasi-periodic Schrödinger Operators and Quantum Graphs, **Fan Yang**, Louisiana State University, **Matthew Powell**, UCI, and **Burak Hatinoglu**, UC Santa Cruz.

Recent Advances and Applications in Imaging Sciences, **Carmeliza Luna Navasca**, University of Alabama at Birmingham, **Fatou Sanogo**, Bates College, and **Elizabeth Newman**, Emory University.

Recent Development in Advanced Numerical Methods for Partial Differential Equations, **Seulip Lee** and **Lin Mu**, University of Georgia.

Recent Developments in Commutative Algebra, **Thomas Polstra**, University of Alabama, and **Florian Enescu**, Georgia State University.

Recent Developments in Graph Theory, **Guantao Chen**, Georgia State University, **Zhiyu Wang**, Georgia Institute of Technology, and **Xingxing Yu**, Georgia Tech.

Recent Developments in Mathematical Aspects of Inverse Problems and Imaging, **Yimin Zhong** and **Junshan Lin**, Auburn University.

Recent Developments on Analysis and Computation for Inverse Problems for PDEs, **Dinh-Liem Nguyen**, Kansas State University, **Nguyen Hoang Loc**, UNC Charlotte, and **Khoa Vo**, Florida A&M University.

Recent Trends in Structural and Extremal Graph Theory, **Joseph Guy Briggs** and **Jessica McDonald**, Auburn University.

Representation Theory of Algebraic Groups and Quantum Groups: A Tribute to the Work of Cline, Parshall and Scott (CPS), **Daniel K Nakano**, University of Georgia, **Chun-Ju Lai**, Institute of Mathematics, Academia Sinica, Taipei 10617 Taiwan, and **Weiqiang Wang**, University of Virginia.

Singer-Hopf Conjecture in Geometry and Topology, **Luca Di Cerbo**, University of Florida, and **Laurentiu George Maxim**, University of Wisconsin-Madison.

Spectral Theory, **Rudi Weikard**, University of Alabama at Birmingham, and **Stephen P. Shipman**, Louisiana State University.

Stochastic Analysis and its Applications, **Parisa Fatheddin**, Ohio State University, Marion, and **Kazuo Yamazaki**, Texas Tech University.

Stochastic Processes and Related Topics, **Ngartelbaye Guerngar**, University of North Alabama, and **Le Chen**, **Erkan Nane**, and **Jerzy Szulga**, Auburn University.

Topological Persistence: Theory, Algorithms, and Applications, **Luis Scoccola**, Northeastern University, **Hitesh Gakhar**, University of Oklahoma, and **Ling Zhou**, The Ohio State University.

Topology and Geometry of 3- and 4-Manifolds, **Miriam Kuzbary**, Georgia Institute of Technology, **David T Gay**, University of Georgia, **Jon Simone**, Georgia Institute of Technology, and **Nur Saglam**, Georgia Tech.

Undergraduate Mathematics and Statistics Research, **Leslie Julianna Meadows**, Georgia State University, **Tsz Ho Chan** and **Asma Azizi**, Kennesaw State University, and **Mark Grinshpon**, Georgia State University.

Spring Eastern Virtual Sectional Meeting

Meeting virtually, hosted by the American Mathematical Society

April 1–2, 2023

Saturday – Sunday

Meeting #1185

Eastern Section
Associate Secretary for the AMS: Steven H. Weintraub

Program first available on AMS website: To be announced
Issue of *Abstracts*: Volume 44, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Kirsten Eisentraeger, Pennsylvania State University, *Title to be announced*.

Jason Manning, Cornell University, *Title to be announced*.

Jennifer L Mueller, Colorado State University, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `https://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Analysis and Differential Equations at Undergraduate Institutions, **William R. Green**, Rose-Hulman Institute of Technology, and **Katharine A. Ott**, Bates College.

Analysis of Markov, Gaussian and Stationary Stochastic Processes, **Alan C. Krinik**, California State Polytechnic University, Pomona, and **Randall J. Swift**, Cal Poly Pomona.

Cybersecurity and Cryptography, **Lubjana Beshaj**, Army Cyber Institute, **Shekeba Monshref**, IBM, and **Angela Robinson**, NIST.

Fractal Geometry and Dynamical Systems, **Mrinal Kanti Roychowdhury**, School of Mathematical and Statistical Sciences, University of Texas Rio Grande Valley, **Sangita Jha**, Department of Mathematics, National Institute of Technology Rourkela, India, and **Saurabh Verma**, Indian Institute of Information Technology Allahabad.

Gauge Theory, Geometric Analysis, and Low-Dimensional Topology, **Paul M. N. Feehan**, Rutgers University, New Brunswick, and **Thomas Gibbs Leness**, Florida International University.

Hypergeometric Functions, q-series and Generalizations, **Howard Saul Cohl**, National Institute of Standards and Technology, **Robert Maier**, University of Arizona, and **Roberto Costas-Santos**, Universidad Loyola de Andalucía.

Modeling, Analysis, and Control of Populations Impacted by Disease and Invasion, **Rachel Natalie Leander** and **Wandi Ding**, Middle Tennessee State University.

Quasiconformal Analysis and Geometry on Metric Spaces, **Dimitrios Ntalampekos**, Stony Brook University, and **Hrant Hakobyan**, Kansas State University.

Recent Advances in Differential Geometry, **Bogdan D. Suceava**, California State University Fullerton, **Adara M. Blaga**, West University of Timişoara, Romania, **Cezar Oniciuc**, "Al.I.Cuza" University of Iaşi, Romania, **Marian Ioan Munteanu**, "Al.I.Cuza" University of Iaşi, Iaşi, Romania, **Shoo Seto**, California State University, Fullerton, and **Lihan Wang**, California State University, Long Beach.

Recent Advances in Infinite-Dimensional Stochastic Analysis, **Vincent R. Martinez**, Hunter College (CUNY), **Hung Nguyen**, UCLA, and **Nathan E. Glatt-Holtz**, Tulane University.

Recent Advances in Ion Channel Models and Poisson-Nernst-Planck Systems, **Zilong Song**, Utah State University, and **Xiang-Sheng Wang**, University of Louisiana at Lafayette.

Recent Progress in Chromatic Graph Theory, **Hemanshu Kaul** and **Samantha Dahlberg**, Illinois Institute of Technology.

Cincinnati, Ohio

University of Cincinnati

April 15–16, 2023

Saturday – Sunday

Meeting #1186

Central Section
Associate Secretary for the AMS: Betsy Stovall

Program first available on AMS website: To be announced
Issue of *Abstracts*: Volume 44, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 14, 2023

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Johnny Guzman, Brown University, *Title to be announced*.

Lisa Piccirillo, MIT, *Exotic phenomena in 4-dimensional topology*.

Krystal Taylor, The Ohio State University, *Title to be announced*.

Nathaniel Whitaker, University of Massachusetts, *From Segregation to Research Mathematician* (Einstein Public Lecture in Mathematics).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `https://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Dispersive Partial Differential Equations I (Code: SS 13A), **William R. Green**, Rose-Hulman Institute of Technology, **Mehmet Burak Erdogan**, University of Illinois at Urbana Champaign, and **Michael J. Goldberg**, University of Cincinnati.

Advances in Radial Basis Functions for Numerical Simulation I (Code: SS 7A), **Jonah A. Reeger**, Air Force Institute of Technology, and **Cecile Piret**, Michigan Technological University.

Algorithms, Number Theory, and Cryptography I (Code: SS 8A), **Jonathan P. Sorenson** and **Jonathan Webster**, Butler University.

Arithmetic Statistics I (Code: SS 19A), **Brandon Alberts**, Eastern Michigan University, and **Soumya Sankar**, Ohio State University.

Brauer Groups in Algebraic Geometry and Arithmetic I (Code: SS 25A), **Jack Petok** and **Sarah Frei**, Dartmouth College.

Cluster Algebras, Positivity and Related Topics I (Code: SS 28A), **Eric Bucher**, Xavier University, **John Machacek**, The University of Oregon, and **Nicholas Ovenhouse**, Yale University.

Combinatorial and Geometric Knot Theory I (Code: SS 11A), **Micah Chrisman**, The Ohio State University, **Sujoy Mukherjee**, University of Denver, and **Robert G. Todd**, Mount Mercy University.

Commutative Algebra with Connections to Combinatorics and Geometry I (Code: SS 24A), **Aleksandra C. Sobieska**, University of Wisconsin - Madison, and **Jay Yang**, Washington University in St. Louis.

Ends and Boundaries of Groups: On the Occasion of Mike Mihalik's 70th Birthday I (Code: SS 2A), **Craig R. Guilbault**, University of Wisconsin-Milwaukee, and **Kim E. Ruane**, Tufts University.

Extremal Graph Theory I (Code: SS 18A), **Neal Bushaw**, Virginia Commonwealth University, **Puck Rombach** and **Calum Buchanan**, University of Vermont, and **Vic Bednar**, Virginia Commonwealth University.

Geometric and Analytic Methods in PDE I (Code: SS 20A), **Dennis Kriventsov**, Rutgers University, **Mariana Smit Vega Garcia**, Western Washington University, and **Mark Allen**, Brigham Young University.

Growth Models, Random Media, and Limit Theorems I (Code: SS 6A), **Magda Peligrad**, **Wlodek Bryc**, and **Xiaoqin Guo**, University of Cincinnati.

Harmonic Analysis and its Applications to Signals and Information I (Code: SS 12A), **Dustin G. Mixon**, The Ohio State University, and **Matthew Fickus**, Air Force Institute of Technology.

Homological Methods in Commutative Algebra I (Code: SS 27A), **Michael DeBellevue**, Syracuse University, and **Josh Pollitz**, University of Utah.

Inequalities in Harmonic Analysis I (Code: SS 26A), **Ryan Gibara**, University of Cincinnati, **Kabe Moen**, University of Alabama, and **Leonid Slavin**, University of Cincinnati.

Interactions between Analysis, PDE, and Probability in Non-smooth Spaces I (Code: SS 1A), **Nageswari Shanmugalingam**, University of Cincinnati, **Luca Capogna**, Smith College, and **Jeremy T. Tyson**, National Science Foundation.

Interactions between Noncommutative Ring Theory and Algebraic Geometry I (Code: SS 3A), **Jason Gaddis**, Miami University, and **Robert Won**, George Washington University.

Mathematical Modeling in Biosciences I (Code: SS 29A), **Sookkyung Lim**, University of Cincinnati, **Jeungeun Park**, SUNY at New Paltz, **Yanyu Xiao**, University of Cincinnati, **Hem R. Joshi**, Xavier University, Cincinnati, and **David Gerberry**, Xavier University.

Modern Trends in Numerical PDEs I (Code: SS 4A), **Johnny Guzman**, Brown University, and **Michael Neilan**, University of Pittsburgh.

Nonlinear Partial Differential Equations from Variational Problems and Fluid Dynamics I (Code: SS 16A), **Tao Huang**, Wayne State University, **Hengrong Du**, Vanderbilt University, and **Changyou Wang**, Purdue University.

Probabilistic and Extremal Combinatorics I (Code: SS 14A), **Jozsef Balogh**, University of Illinois at Urbana-Champaign, and **Tao Jiang**, Miami University.

Quantitative Aspects of Symplectic Topology I (Code: SS 15A), **Jun Li**, University of Dayton, **Olguta Buse**, IUPUI, and **Richard Keith Hind**, University of Notre Dame.

Recent Advances in Finite Element Methods: Theory and Applications I (Code: SS 21A), **Tamas L. Horvath**, Oakland University, and **Giselle Sosa Jones**, University of Houston.

Recent Developments in the Study of Fluid Flows, Turbulence, and its Applications I (Code: SS 30A), **Vincent Martinez**, CUNY Hunter College & Graduate Center, and **Samuel Punshon-Smith**, Tulane University.

Recent Trends in Graph Theory I (Code: SS 23A), **Adam Blumenthal**, Westminster College, and **Katherine Perry**, Soka University of America.

Recent Trends in Integrable Systems and Applications I (Code: SS 5A), **Deniz Bilman** and **Robert J. Buckingham**, University of Cincinnati.

Representation Theory, Geometry and Mathematical Physics I (Code: SS 22A), **Daniele Rosso**, Indiana University Northwest, and **Jonas T. Hartwig**, Iowa State University.

Stochastic Analysis and its Applications I (Code: SS 9A), **Po-Han Hsu**, University of Cincinnati, **Tai-Ho Wang**, Baruch College, CUNY, and **Ju-Yi Yen**, University Of Cincinnati.

The Interface of Geometric Measure Theory and Harmonic Analysis I (Code: SS 10A), **Eyvindur Ari Palsson**, Virginia Tech, and **Krystal Taylor**, The Ohio State University.

Topological and Geometric Methods in Combinatorics I (Code: SS 17A), **Zoe Wellner** and **R. Amzi Jeffs**, Carnegie Mellon University.

Fresno, California

California State University, Fresno

May 6–7, 2023

Saturday – Sunday

Meeting #1187

Western Section
Associate Secretary for the AMS: Michelle Ann Manes

Program first available on AMS website: Not applicable
Issue of *Abstracts*: Volume 44, Issue 3

Deadlines

For organizers: Expired
For abstracts: March 7, 2023

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Sami H. Assaf, University of Southern California, *A Littlewood–Richardson rule for Grassmannian Schubert varieties*.
Natalia Komarova, UCI, *To be announced*.
Joseph Teran, University of California, Los Angeles, *To be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `https://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances by CSU Student Scholars, **Jessica De Silva**, California State University, Stanislaus, **Andrea Arauza Rivera**, California State University, East Bay, and **Mario Banuelos**, California State University, Fresno.

Advances in Functional Analysis and Operator Theory, **Igor Nikolaev**, St. John's University, **Marat V. Markin**, California State University, Fresno, and **Michel L. Lapidus**, University of California Riverside.

Algebraic Structures in Knot Theory, **Carmen L. Caprau**, California State University, Fresno, **Sam Nelson**, Claremont McKenna College, and **Neslihan Gügümcu**, Izmir Institute of Technology in Turkey.

Algorithms in the Study of Hyperbolic 3-manifolds, **Maria Trnkova**, University of California In Davis, and **Robert C Haraway**, University of California, Davis.

Analysis of Fractional Differential and Difference Equations with its Application, **Bhuvaneswari Sambandham**, Dixie State University, and **Aghalaya Vatsala**, University of Louisiana at Lafayette.

Artin-Schelter Regular Algebras and Related Topics, **Ellen E Kirkman**, Wake Forest University, and **James J. Zhang**, University of Washington.

Combinatorics and Representation Theory (associated with the Invited Address by Sami Assaf), **Nicolle Gonzalez**, University of California, Berkeley, and **Sami H. Assaf**, University of Southern California.

Complexity in Low-Dimensional Topology, **Jennifer Schultens**, University of California Davis, and **Eric Sedgwick**, DePaul University.

Data Analysis and Predictive Modeling, **Earvin Balderama**, California State University, Fresno, and **Adriano Zambom**, California State University, Northridge.

Inverse Problems, **Robert M. Owczarek**, University of New Mexico, and **Hanna E. Makaruk**, Los Alamos National Laboratory, Los Alamos, NM.

Math Circle Games and Puzzles that Teach Deep Mathematics, **Maria Nogin**, **Agnes Tuska**, **Yaomingxin Lu**, and **Gábor Molnár-Sáska**, California State University, Fresno.

Mathematical Biology: Confronting Models with Data, **Erica Marie Rutter**, University of California, Merced.

Mathematical Methods in Evolution and Medicine (associated with the Invited Address by Natalia Komarova), **Jesse Kreger**, University of California, Irvine, and **Natalia Komarova**, UCI.

Mathematics in Data Science, **Elena S. Dimitrova**, California Polytechnic State University, San Luis Obispo, and **Ruriko Yoshida**, Naval Postgraduate School.

Methods in Non-Semisimple Representation Categories, **Paul Sobaje**, Georgia Southern University, **Eric Friedlander**, University of Southern California, Los Angeles, and **Julia Pevtsova**, University of Washington.

Modeling and Analysis of Cellular Processes in Biomedical Problems, **Joyce Lin** and **Warren Roche**, Cal Poly State University.

Nonlinear PDEs in Fluid Dynamics, **Juhi Jang**, **Igor Kukavica**, and **Linfeng Li**, University of Southern California.

Recent Developments in Mathematical Biology, **Lihong Zhao**, University of California, Merced, and **Christina Edholm**, Scripps College.

Research in Mathematics by Early Career Graduate Students, **Marat V. Markin**, **Doreen De Leon**, and **Khang Tran**, California State University, Fresno.

Scientific Computing, **Changho Kim**, University of California, Merced, and **Roummel F. Marcia**, Universtiy of California, Merced.

The Use of Computational Tools and New Augmented Methods in Networked Collective Problem Solving, **Agnes Tuska** and **Mario Banuelos**, California State University, Fresno, and **Andras Benedek**, Research Centre for the Humanities, Institute of Philosophy, Hungary.

Women in Mathematics, **Doreen De Leon**, **Katherine Kelm**, and **Oscar Vega**, California State University, Fresno.

Zero Distribution of Entire Functions, **Khang Tran** and **Tamás Forgács**, California State University, Fresno.

Buffalo, New York

University at Buffalo (SUNY)

September 9–10, 2023

Saturday – Sunday

Meeting #1188

Eastern Section
Associate Secretary for the AMS: Steven H. Weintraub

Program first available on AMS website: July 27, 2023
Issue of *Abstracts*: To be announced

Deadlines

For organizers: February 9, 2023
For abstracts: July 18, 2023

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Jennifer Balakrishnan, Boston University, *Title to be announced*.

Sigal Gottlieb, University of Massachusetts, Dartmouth, *Title to be announced*.

Samuel Payne, University of Texas, *Title to be announced*.

Omaha, Nebraska

Creighton University

October 7–8, 2023

Saturday – Sunday

Meeting #1189

Central Section
Associate Secretary for the AMS: Betsy Stovall, University of Wisconsin-Madison

Program first available on AMS website: August 17, 2023
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 7, 2023
For abstracts: August 8, 2023

Mobile, Alabama

University of South Alabama

October 13–15, 2023

Friday – Sunday

Meeting #1190

Southeastern Section
Associate Secretary for the AMS: Brian D. Boe

Program first available on AMS website: August 24, 2023
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 13, 2023
For abstracts: August 15, 2023

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Theresa Anderson, Carnegie Mellon University, *Title to be announced*.
Laura Miller, University of Arizona, *Title to be announced*.
Cornelius Pillen, University of South Alabama, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `https://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Mathematical Modeling of Problems in Biological Fluid Dynamics (Code: SS 1A), **Laura Miller**, University of Arizona, and **Nick Battista**, The College of New Jersey.

Albuquerque, New Mexico

University of New Mexico

October 21–22, 2023

Saturday – Sunday

Meeting #1191

Western Section
Associate Secretary for the AMS: Michelle Ann Manes
Program first available on AMS website: August 31, 2023
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: March 21, 2023
For abstracts: August 22, 2023

San Francisco, California

Moscone West Convention Center

January 3–6, 2024

Wednesday – Saturday
Associate Secretary for the AMS: Michelle Ann Manes
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Submit Your Proposals for AMS Special Sessions at the 2024 Joint Mathematics Meetings

All members of the mathematics community are invited to submit proposals for American Mathematical Society (AMS) Special Sessions at the 2024 Joint Mathematics Meetings (JMM). If you have a topic that you would like to explore in a special session, now is the time to put your great idea into motion.

The 2024 JMM will be held January 3–6, 2024 in San Francisco, CA. On behalf of the American Mathematical Society, Prof. Michelle Manes (`mamanes@hawaii.edu`), the AMS Associate Secretary responsible for the AMS program at this meeting, solicits proposals for AMS Special Sessions for this meeting. Proposals that reflect the full spectrum of interests of the mathematical community are welcome.

A special session is a collection of talks devoted to a single area of mathematics or a single topic. Special sessions can be proposed by teams of organizers.

Please go to the submission form at `https://meetings.ams.org/math/jmm2024/cfs.cgi` and provide the following information:

1. the title of the session
2. the name, affiliation, and email address of each organizer, with one organizer designated as the contact person for all communication about the session
3. a brief description of the topic of the proposed special session

4. a *sample* list of speakers whom the organizers plan to invite. (It is not necessary to have received confirmed commitments from these potential speakers, but the sample list should demonstrate sufficient potential interest in such a session.)
5. either the primary two-digit MSC (Mathematics Subject Classification) number that most closely matches the topic (see `http://www.ams.org/mathscinet/msc/msc2020.html`) or one of the following new code numbers adopted for topics (see `http://www.ams.org/journals/notices/202010/rnoti-p1602.pdf`):
   - 101: Teaching and learning
   - 102: Recreational mathematics
   - 103: Professional development and professional concerns
   - 104: Wider issues

The deadline for submission of proposals is **April 14, 2023**. Late proposals will not be considered. No decisions will be made on proposals until after the submission deadline has passed. For questions about using the submission form, contact `meet@ams.org`.

Organizers are encouraged to read the *AMS Manual for Special Session Organizers* at `https://www.ams.org/meetings/meet-specialsessionmanual` in its entirety.

Some key information:

Special sessions will in general be allotted between 5 and 10 hours in which to schedule speakers. To enable maximum movement of participants between sessions, organizers must schedule each session speaker for either (a) a 20-minute talk with 5-minute discussion and 5-minute break or (b) a 45-minute talk with 5-minute discussion and 10-minute break. A special session may include any combination of 20-minute and 45-minute talks that fits within the time allotted to the session, but all talks must begin and end at the scheduled time.

The number of special sessions in the AMS program at the JMM is limited, and because of the large number of high-quality proposals, not all can be accepted. Please be sure to submit as detailed a proposal as possible for review by the Committee on Special Sessions and Contributed Paper Sessions. Decisions will be made on acceptance and scheduling of sessions by early June 2023. At that time, contact organizers will be notified whether their proposal has been accepted. If so, they will be informed of their session's schedule and will be sent additional information about organizational details.

We look forward to reviewing your proposals.

Call for Proposals for Professional Enhancement Programs (PEPs) at the Joint Mathematics Meetings in San Francisco, CA, January 3–6, 2024

Send by May 9, 2023 to `https://meetings.ams.org/math/jmm2024/cfs.cgi`.

The AMS solicits proposals for Professional Enhancement Programs (PEPs) at the 2024 Joint Mathematics Meetings (JMM) to be held January 3–6, 2024 in San Francisco, California. Proposers of a PEP should keep in mind the aim of PEPs is to improve mathematical education at all levels, contribute to building a more mathematically literate society, make mathematics and statistics more attractive as a discipline and career pathway, and enhance the careers of mathematical scientists.

Information:

Mathematical and statistical instruction has grown in importance due to the expanding connections between mathematical/statistical ideas and computing, interdisciplinary foundational connections, as well as the ever-growing impact of technology on the social, economic and cultural fabric of our society. In educating a skilled and diverse workforce at both the undergraduate and graduate levels, departments must be able to provide a broad overview as well as learning opportunities for advanced topics in mathematics and statistics. In doing so, departments face a wide range of recent and historical issues and challenges, including the interface between online teaching, active learning and inclusive pedagogy, the development of effective foundational courses that address the importance of data analysis and the need for modeling, effective outreach to regional and national communities, collaboration with broader scholarly communities, and increasing diversity and reducing implicit bias in recruitment, hiring and promotion practices. A major part of this challenge is to broaden access to mathematical and statistical education at all levels, which has important implications for addressing society's most pressing and complex problems and for erasing long-standing socio-economic inequities in our society.

JMM brings together the entire mathematical sciences community. With the aim of PEPs in mind, members of the community are invited to propose a PEP that will provide professional development opportunities for participants and serve to strengthen departments' efforts to meet their responsibilities and challenges.

- PEPs are expected to be hands-on and participants should leave with a tangible product
- PEPs are scheduled for two, 2-hour sessions

- AMS will provide a total of $1000 to the designated organizer of approved PEPs
- Participants will pay a program fee

Suggested topics:

- Effective pedagogical and assessment practices for on-line instruction in the mathematical and statistical sciences (especially given lessons learned from the pandemic)
- Inclusive pedagogy in the mathematical and statistical sciences and curricular practices that promote diversity, equity and inclusiveness in undergraduate and graduate programs
- Pedagogical practices that promote active learning in specific mathematical and/or statistical courses
- Novel methods and applications for incorporating data science into established undergraduate and/or graduate courses and programs or into established faculty research agendas
- Innovative implementation of software
- Interdisciplinary research collaborations within the mathematical community or new connections between mathematics and/or statistics and other disciplines
- Any mathematical or statistical topic that enhances the professional development for faculty and is suited for the PEP format

Proposals should include:

1. Title of the event
2. Name, affiliation, email address of each organizer, with one organizer designated as the contact person for all communication about the event, and their website(s), if available
3. Give a description of your experiences and articles that support the efficacy of your PEP (character limit: 1500)
4. Some sessions associate their event with an organization; list the organization(s) agreeing to be associated with your event
5. A detailed (character limit: 3000) description of the proposed event to include participant takeaways
6. Time slots to avoid (where possible)
7. Other JMM events to avoid (where possible)
8. A list of other events proposed by the organizer
9. If applicable, a two-digit MSC (Mathematics Subject Classification) number (`https://mathscinet.ams.org/mathscinet/msc/msc2020.html`) matching the topic of the event or a three-digit code from the New Expanded Classification System (`https://www.ams.org/journals/notices/202010/rnoti-p1602.pdf`) adopted for topics included previously in an MAA or other JMM session

Proposals for PEPs should be submitted to the JMM Program Committee (`https://meetings.ams.org/math/jmm2024/cfs.cgi`) by May 9, 2023.

We look forward to receiving your submissions.

Call for Proposals for Panels, Workshops, and Other Events at the Joint Mathematics Meetings in San Francisco, CA, January 3–6, 2024

Send by April 13, 2023 to `https://meetings.ams.org/math/jmm2024/cfs.cgi`.

The AMS solicits proposals for panels, workshops and other events at the 2024 Joint Mathematics Meetings (JMM) to be held January 3–6, 2024 in San Francisco, California.

We invite proposals that reflect the full spectrum of interests of the mathematical sciences community.

The topics of proposals could be in areas in the mathematical sciences, but there are others, such as: creating an inclusive atmosphere in the mathematical sciences community; teaching and learning; professional development and professional concerns; recreational mathematics; and wider issues. Events that connect communities are welcome, such as interdisciplinary workshops or collaborations between mathematical sciences researchers and mathematics education researchers.

Proposals should include:

1. Type of event (panel, workshop, other (specify))
2. Title of the event
3. Name, affiliation, email address of each organizer, with one organizer designated as the contact person for all communication about the event, and their website(s), if available
4. Some sessions associate their event with an organization; list the organization(s) agreeing to be associated with your event
5. A detailed (character limit: 3000) description of the proposed event, including the intended audience

6. A proposed list of panelists/moderators/speakers and their institutional affiliation whom the organizers plan to invite. Have they agreed to participate?
7. Duration of time requested for the event (1 hour, 1.5 hours, 2 hours, 4 hours)
8. Time slots to avoid (where possible)
9. Other JMM events to avoid (where possible)
10. If applicable, a two-digit MSC (Mathematics Subject Classification) number (`https://mathscinet.ams.org/mathscinet/msc/msc2020.html`) that matches the topic of the event or a code from the New Expanded Classification System (`https://www.ams.org/journals/notices/202010/rnoti-p1602.pdf`) adopted for topics included previously in an MAA or other JMM session

Proposals for panels, workshops, and other events should be submitted to the JMM Program Committee (`https://meetings.ams.org/math/jmm2024/cfs.cgi`) by April 13, 2023.

Further information:

Panels, workshops, and other events approved by the Program Committee will normally be allotted a 60–90-minute slot, with the exception of a Professional Enhancement Program (PEP) that could run for 4 hours. Decisions will be made on proposals after the submission deadline has passed.

The number of panels, workshops, and other events in the AMS program at the Joint Mathematics Meetings is limited, and because of the large number of high-quality proposals anticipated, not all can be accepted, nor can all scheduling requests be honored.

Please submit as detailed a proposal for your event as soon as possible for review by the JMM 2024 Program Committee. Organizers of proposals will be notified whether their proposal has been accepted by **May 25, 2023**. Additional instructions and the event schedule will be sent to the contact organizer of each accepted proposal shortly after that deadline.

We look forward to receiving your submissions.

Tallahassee, Florida

Florida State University in Tallahassee

March 23–24, 2024

Saturday – Sunday

Southeastern Section

Associate Secretary for the AMS: Brian D. Boe, University of Georgia

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Washington, District of Columbia

Howard University

April 6–7, 2024

Saturday – Sunday

Eastern Section

Associate Secretary for the AMS: Steven H. Weintraub

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

San Francisco, California

San Francisco State University

May 4–5, 2024

Saturday – Sunday
Western Section
Associate Secretary for the AMS: Michelle Ann Manes
Program first available on AMS website: Not applicable

Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `https://www.ams.org/amsmtgs/sectional.html`.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `https://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Recent Advances in Differential Geometry, **Zhiqin Lu**, University of California, **Shoo Seto** and **Bogdan Suceavă**, California State University, Fullerton, and **Lihan Wang**, California State University, Long Beach.

Palermo, Italy

July 23–26, 2024

Tuesday – Friday
Associate Secretary for the AMS: Brian D. Boe
Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Savannah, Georgia

Georgia Southern University, Savannah

October 5–6, 2024

Saturday – Sunday
Southeastern Section
Associate Secretary for the AMS: Brian D. Boe, University of Georgia
Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Riverside, California

University of California, Riverside

October 26–27, 2024

Saturday – Sunday
Western Section
Associate Secretary for the AMS: Michelle Ann Manes
Program first available on AMS website: Not applicable

Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 8–11, 2025

Wednesday – Saturday

Associate Secretary for the AMS: Steven H. Weintraub

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Auckland, New Zealand

December 4–8, 2025

Monday – Friday

Associate Secretary for the AMS: Steven H. Weintraub

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center and Marriott Marquis Washington DC

January 4–7, 2026

Sunday – Wednesday

Associate Secretary for the AMS: Betsy Stovall

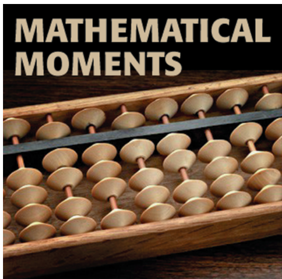
Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced



Using Math to Support Cancer Research

Cancer research is a crucial job, but a difficult one. Tumors growing inside the human body are affected by all kinds of things: oxygen delivered by blood, energy delivered in the form of glucose, not to mention the myriad other cells and chemicals in the surrounding environment. These conditions are difficult (if not impossible) to recreate in the lab, and using real patients as subjects can be painful and invasive. Mathematical models give cancer researchers the ability to run experiments virtually, testing the effects of any number of factors on tumor growth and other processes — using far less money and time than an experiment on human subjects or in the lab would use.

But these models don't work in isolation. They rely on real-world data, and their predictions must be carefully tested for accuracy using experiments and clinical trials.

Example: In one exciting potential advance, laboratory scientists grew tumors surrounded by a new type of cell. Mathematical modelers then used data gathered from studying them to simulate what would happen under various conditions. The results suggest that the tumor's growth might be halted with a drug that's already been invented — speeding up the timeline for patients to benefit from this research. Now, clinical trials must be done to ensure the model's results translate to the real world.

In a different study, Colin Cess and Stacey Finley used a computational model to predict what happens when a tumor is treated with nothing (left), with chemotherapy (center), or with a drug that prevents blood vessels from reaching the tumor (right). Dead cells are shown in black; live cells in gray; yellow cells migrate more freely.

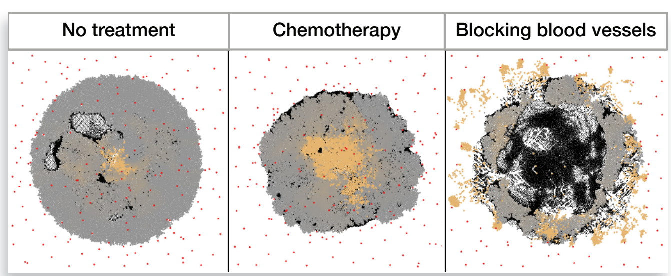


Image taken from: Cess, C. G., and Finley, S. D., Multiscale modeling of tumor adaption and invasion following anti-angiogenic therapy. *Comp. Sys. Oncol.* **2** (2022), e1032. https://doi.org/10.1002/cso2.1032. Licensed under CC BY 4.0.

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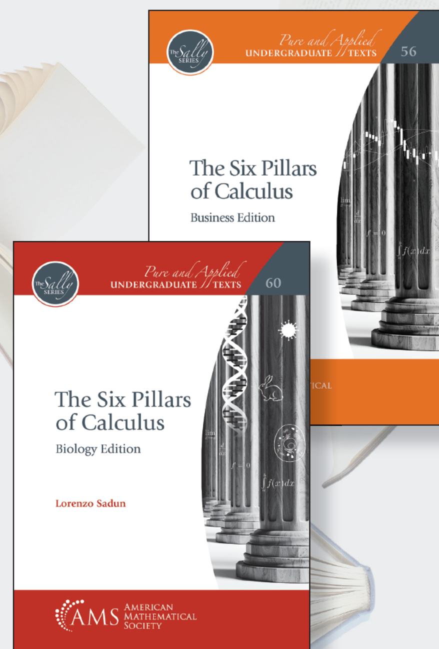

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