Modeling America's Racial Wealth Disparities

Mathematical Models Help Chart Pathways for Closing Racial Wealth Gaps

Pedro Nascimento de Lima, Jonathan Lamb, Osonde Osoba, and Jonathan Welburn

Background

Will America always be a country marked by racial disparity? Will Black families always be systematically poorer than white families? Can the American society reach a stage where past disparities are mended, economic opportunity is expanded, and where everyone gets their fair share? Are such goals mathematically feasible, and if so, over what timeframe? What policies could achieve these goals?

For many, questions like these evoke a very strong response. Some may immediately object, arguing that America is and has always been a land of opportunity for all races and groups—a leading democracy that delivered prosperity through free markets. Others will say America has never been a land of opportunity for all—particularly for Black Americans. No matter where you lie along this spectrum, we invite you to dive into mathematical facts about racial wealth disparity in America and models that help shed light on America's wealth disparity struggle.

Building on our recently released report [1] describing the headwinds of a long and incessant racial wealth gap, we introduce a measure of racial wealth disparity and characterize the wealth distribution among Black and white US households. Next, we frame America's racial wealth disparity challenge within a computational optimization setting where the social objective is to close racial wealth gaps at a minimal cost. Finally, we discuss how models that describe long-term dynamics of wealth accumulation—specifically, overlapping generation (OLG) models—can be used to investigate whether the United States can ever become a country free of racial wealth disparities.

Measuring Racial Wealth Disparities

Consider a nation composed of N households. Because our analysis is focused on the Black-white wealth gap, we consider the set of races $R = \{b, w\}$, Black and white. Over the years, households are created, get older, and are dissolved; but at any point in time t, household i has wealth w_{it}. While wealth data is not collected for all American households i of race r, we can obtain information about the density function of wealth through the Survey of Consumer Finances (SCF), a triannual survey examining American households' wealth maintained by the Federal Reserve [2].

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Let $f_w(x)$ denote the density function of the wealth distribution of white households and $f_b(x)$ the density function of the wealth of Black households. In a society without racial wealth disparity, knowing a household's race would give no information about their wealth; $f_w(x)$ and $f_b(x)$ should be similar. To operationalize this idea, let $F_w(x)$ and $F_b(x)$ be the cumulative density functions of the white and Black household wealth, respectively. The inverse[1] of those functions $F_w^{-1}(p)$ and $F_b^{-1}(p)$ are quantile functions of both distributions and should be similar in a society free of racial wealth inequity. The existing literature commonly uses differences in the median wealth ($F_w^{-1}(0.5) - F_b^{-1}(0.5)$) to characterize wealth gaps, but a more complete representation of the wealth gap can be obtained if one considers multiple percentiles. We therefore propose the following disparity measure:

$$D = \frac{1}{n_p} \sum_{p=p_0}^{p_n} \frac{|F_w^{-1}(p) - F_b^{-1}(p)|}{F_w^{-1}(p)}, \tag{1}$$

where p are percentiles and D is the racial wealth disparity; index is computed using n_p percentiles of the wealth distribution of white and Black households. That is, D is constructed from pairwise differences between the wealth of white and Black households at each percentile,[2] p, which defines racial wealth disparity as a normalized average of differences between white and Black household wealth. Specifically, we normalize the difference at each percentile, p, by the level of white wealth $F_w^{-1}(p)$ to avoid weighting differences at the upper end of the distribution much more than those at the lower end.

This disparity index has some useful properties. First, it is a purely descriptive index and can be used to track racial wealth inequity gaps using available SCF data. It also has a clear interpretation—if there is no disparity and Black households are as wealthy as white households, then $D = 0$. In a society where Black households hold no wealth, $D = 1$. If Black households hold about 60 percent of the wealth of white households at each percentile in the wealth distribution, $D = 0.4$. Moreover, policies or factors that increase wealth disparity increase D even if they increase wealth overall, and policies that decrease racial wealth disparities decrease D. Finally, this disparity metric simplifies to a difference between median wealth if one only considers the 50th percentile.

Nevertheless, this disparity measure should not be seen as the only approach for measuring racial wealth disparities, nor as a metric that fully accounts for the *accumulated* injustices committed against Black Americans. To clarify that difference, it is useful to distinguish between *disparity*—a mathematical difference between groups—and *inequity*—an unjust difference between groups. Researchers in this field (notably, Darity, Jr. and Mullen [3]) have used several methods to calculate the amount that would have to be repaid in reparations to atone for injustices committed against Black Americans throughout US history. In doing so, this literature seeks to provide an account of the monetary value that would compensate for past racial *inequity*. Rather than being a metric meant to fully capture racial wealth *inequities*, index D is simply a disparity metric that sheds light on how racially segregated wealth is in any society at *a given time*.

Note that a society free of racial wealth disparities may still be highly unequal. Just as we differentiate between inequity and disparity, we may also differentiate between inequity and *inequality*—which refers to a difference in wealth within a group rather than between groups. Eq. (1) imposes no constraints on what the overall wealth distribution is in this society. Wealth could be highly concentrated with high inequality yet with little disparity. Therefore, D in eq. (1) is not a wealth inequality metric.

America's Racial Wealth Gap is a Persistent Phenomenon

Figure 1 shows the racial wealth inequity D metric from 1987–2019—the full range of available time-series data from the SCF. After decreasing from 1989 to 2002, disparities increased from 2002 to 2013 and since then have decreased slightly, from 0.91 to 0.87—meaning that Black households hold only about 13% of the wealth of white households. The data confirms a disappointing trend; wealth disparities have not substantially improved over the past 30 years.

The easiest way to understand why wealth disparity will not go to zero on its own is to realize that white households are positioned to gain wealth at a higher rate than Black households. Let the wealth of household i at time t be represented by random variable X_{it}. Wealth grows over time following $X_i(t) = X_i(t-1) + x_{it}$ where $x_{it} \sim F(\mu, \sigma)$ are stochastic gains. Now let the wealth Black households be represented by $Y_j(t) = Y_j(t-1) + y_{jt}$ where $y_{it} \sim F(M, \Sigma)$. Take $\mu <$ as given, then as time advances, $E[X_i(t)]$ only diverges from $E[Y_i(t)]$, and the probability that Black households will catch up with Black households in their wealth is $P(E[X_i(t)] < E[Y_i(t)]) = 0$. Thus, as long as $\mu < M$, Black households will *never* catch up with white households in wealth. Therefore, a difference in starting conditions in wealth and a difference in the savings is sufficient to cause an indefinite disparity in wealth.

[1] *Note that $F(x)$ is nondecreasing, so $F^{-1}(p)$ can be defined as a generalized inverse distribution function as $F^{-1}(p) = inf\{x \in R : F(x) \geq p\}$.*

[2] *In our report, we use $W_{w,p}$ as shorthand for $F^{-1}(p)$ for the wealth distribution of white households and $W_{b,p}$ for $F^{-1}(p)$ of the wealth distribution of Black households.*

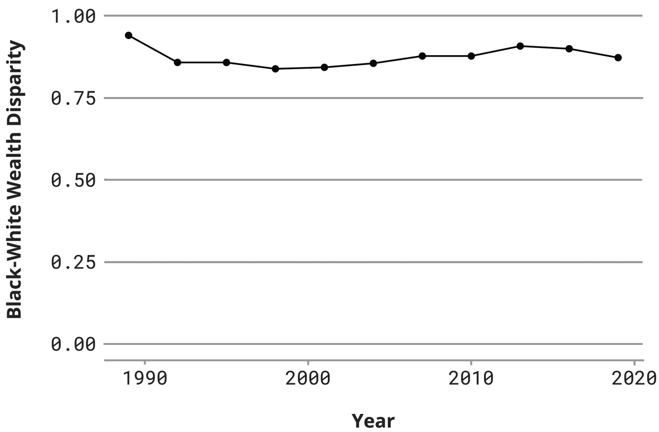

Figure 1. America's racial wealth gap has been persistently high. Note: This figure plots the measure of Black-white wealth disparity introduced in Equation 1, considering only median disparity, from 1989 to 2019. The plot includes the entire possible range of this measure to highlight that the trends in Black and white wealth have been minimal toward reducing disparities relative to a line of no racial disparity. A disparity of 0 implies no disparity. A disparity value of 0.87 implies that median Black households hold 13 percent of the wealth of a median white household. Source: Author's analysis of 2019 SCF time-series summary data (Board of Governors of the Federal Reserve Board, 2022).

One might hope that this simplified model does not capture important features of the real world and that the wealth and income of white and Black households are on track to converge. Perhaps the incomes of white and Black households would converge as education opportunities are leveled, or perhaps white households would spend their savings, naturally closing the wealth gaps. Unfortunately, this optimistic view is neither supported by decades of data nor by models that estimate the long-run dynamics of wealth (discussed later).

The long-run difference in $E[X_i(t)] < E[Y_i(t)]$ shows that disparities can never close unless Black households are able to increase their wealth at a faster rate than white households. Yet, the legacy of past disparity is transferred to younger generations through several mechanisms [4], including lower levels of intergenerational transfer (i.e., inheritance) for Black households relative to white households and Black households earn less income even for the same educational level of their white counterparts. The combined effect results in the strong and enduring headwinds shown in Figure 1.

Closing the Racial Wealth Gap as an Optimization Problem

Society has many competing priorities, so what is the minimum amount of wealth that would have to be created or allocated to close the racial wealth gap? Our recent research explores these questions through a sequence of optimization exercises. We start by framing the wealth inequity problem as an optimization problem, where wealth is transferred to eligible households within a wealth transfer policy. We abstract away from the policy mechanisms that create such wealth, and, for the purposes of this exercise, the policy costs approximately the same amount of wealth it creates. For example, debt forgiveness is arguably one of such policies that result in an immediate wealth effect. Other policies, such as tax incentives to home ownership, combined with easy monetary policy (i.e., low interest rates) also affect the distribution of household wealth through home values appreciation. For instance, a policy structure that results in house appreciation well above inflation results in a wealth transfer from young to old households.

In this analysis, we consider two eligibility criteria. The first is the race of the eligible households. Reparations policies exclusively target Black households (r = Black) while so-called race-blind policies, such as baby bonds,[3] allocate wealth to all households (r = all races/ethnicities). The second is the wealth of the eligible household; we consider a threshold maximum value of wealth $\widetilde{W}$ below which households would be eligible for the wealth-allocation policy. Finally, the total monetary value allocated to all eligible households A and the number of eligible households n_e determine all households' immediate post-allocation wealth $W'_{i,r}$ of household i of race r as follows:

$$W'_{i,r} = \begin{cases} W_{i,r} + \frac{A}{n_e}, & \text{if } W_{i,r} \leq \widetilde{W} \text{ and } r \in \{\text{Black, all races}\} \\ W_{i,r}, & \text{otherwise.} \end{cases} \tag{2}$$

Equation 2 assumes that each eligible household would receive the equal resulting allocation A/n_e[4] It also assumes that eligibility would always be determined by current wealth, such that the least wealthy households (in terms of n_e) would receive an equal wealth allocation.

Given this setup, we use a many-objective evolutionary algorithm (NSGA-II) to find the sets of policies (pairs of $\widetilde{W}$ *and* A) that minimize racial wealth disparities as measured by D at a minimum cost. We run the optimization exercise both making all households eligible (targeted allocations to all households) and only making Black households eligible (targeted allocations to Black households). We also

[3] *Baby bonds are a proposed policy for endowing every US-born child with a government-financed trust account, created at birth and accessible at age 18. Although baby bonds could be given to all children at birth, they are often discussed as a way to endow children from lower-income families with disparity-reducing allocations. Hamilton and Darity [5] propose this progressive implementation as a solution for eliminating the racial wealth gap over time.*

[4] *A policy wherein households receive an equal allocation is only one amongst many potential policy structures. For simplicity, this is the only policy structure we consider in this paper. Future work may relax this assumption and explore other types of policies.*

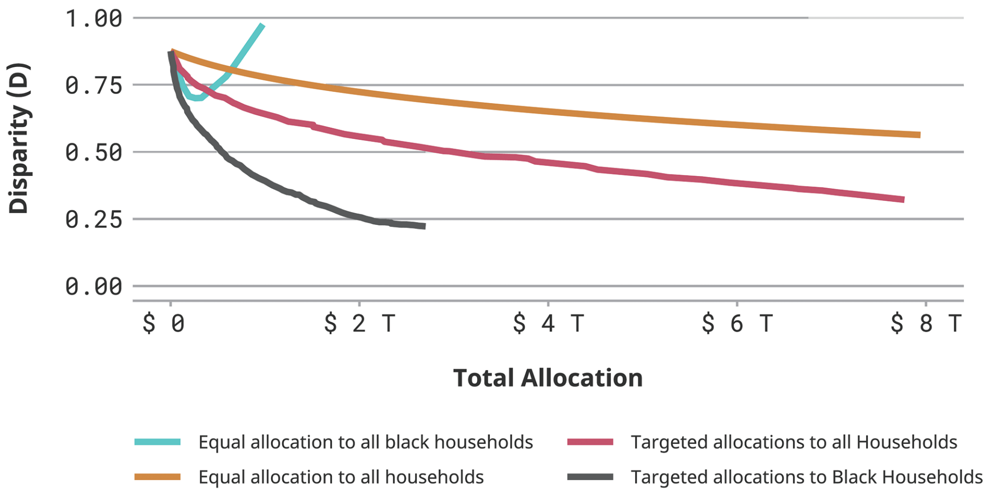

Figure 2. First-order effects of disparity-minimizing policies. Note: This figure presents a set of policies that minimize disparity measure D, as discussed in our report [1]. Our report also presents the effect of policies considering alternative measures of wealth disparity, including only the effect on median and average disparity. Each curve is composed by a set of policies. The blue and orange curves are obtained by simulating a one-time transfer to either all Black households (blue) or all households (orange). The black and red lines present a pareto-efficiency frontier composed of policy solutions found by our many-objective optimization. They represent a "feasibility frontier"—i.e., it is not possible to reduce disparities with a smaller total allocation, using equal allocations to all households.

evaluate policies that resemble a reparations policy (equal allocations to all Black households) and policies that allocate wealth to all households irrespective of race (equal allocation to all households). Figure 2 presents the first-order, immediate effect of these policies on the wealth disparity measure D.

Figure 2 sheds light on some of the dilemmas policymakers would face when choosing a type of policy to close wealth gaps. First, policies that create an equal amount of wealth to all households (orange line) will reduce wealth disparities, but only at a high cost. Second, a reparations policy (blue line) can reduce disparities, but also may imply a short-term increase in disparities (as measured by disparity measure D) across the wealth distribution. This arises because with uniform transfers, selecting a value of A sufficient to eliminate disparities in one quantile will invert any disparity in quantiles with a smaller absolute gap, resulting in a U-shaped function for D. Models describing long-term wealth dynamics (discussed later) project these effects to wash away over time and disparities to reverse again. Another way to reduce wealth disparities without any short-term increase in disparity measure D would be to use *targeted* policies—i.e., allocations targeted to the least-affluent households, with allocation amounts calibrated to decrease wealth disparities at the minimum cost (red and black lines). The black curve demonstrates how much racial wealth disparities could decrease immediately if such policies were race-neutral (targeted allocations to all households) and if they were targeted to Black households (targeted allocations to Black households).

Such a model of the impact of wealth-changing policies on racial wealth disparities can help inform policy decisions that would otherwise be made in the absence of this information. Policymakers interested in understanding the immediate effect of policies that shape wealth outcomes can use this optimization approach to better calibrate the magnitude of policies such that interventions reduce, rather than add to racial wealth inequalities. The analysis presented earlier only shows the immediate effect of policies, but policymakers should look over a longer time-horizon when making policy choices that affect wealth disparities. Looking beyond one time-period however requires both more data and more modeling effort. Hence, we turn to models used in macroeconomics to shed light on long-term dynamics of disparity and the potential impacts of interventions.

The Long-term Dynamics of Racial Wealth Disparities

Overlapping generation (OLG) models are used in macroeconomics to study the long-run dynamics of fiscal policies and the accumulation and distribution of wealth. OLG models have been used to clarify the need for policies that are foundational to society, such as social security. Recently, OLG models have been used to study long-term consequences of policies to reduce racial wealth disparities [6], [7]. Where other common approaches derived from Solow or Ramsey make a simplifying assumption that economic agents are identical and infinitely lived [8], OLG models incorporate multiple subpopulations at different stages of a finite lifecycle to capture more realistic aggregate economic measures and the evolution of variables like wage levels and savings rates. The simplest OLG formulation [9] follows two generations: "young" wage-earning households, and "old" retired households. Each period, the "young" earn income and save to fund consumption in retirement, while the "old" generation spends remaining savings plus any earned interest. In the next period, the young household transitions to become the new "old," and a new "young" is born. Wages are determined by a production sector using labor of the "young" and capital supplied by the "old" savings. This simplistic scenario can be extended with additional generations, the introduction of taxation and social safety nets, bequests between generations, investments in productivity, and so on.

Consider the following model, described in more detail in [10]. At time t, the young generation, g, consumes $c_{g,t}$, chooses whether to attend post-secondary education $e_{g,t} = \{0 \text{ if no}, 1 \text{ if yes}\}$ at cost $\theta_{g,t}$, pays housing rent $h_{g,t}$ saves $s_{g,t}$ and pays a net intergenerational transfer (i.e., the difference $R_{gj,t} - R_{jg,t}$). The young generation also earns income from an interest earning endowment received in the prior period, $\omega_{g,t-1}$, and pre-tax labor income $y_{g,t}$ where $\tau(y_{g,t})$ is a tax rate function leading to the following budget constraint

$$c_{g,t} + e_{g,t}\theta_{g,t} + \widetilde{h}_{g,t} + s_{g,t} + \sum_{j\neq g} (R_{gj,t} - R_{jg,t}) \leq \omega_{g,t} + \tilde{y}_{g,t}\left(1 - \tau(y_{g,t})\right). \quad (3)$$

The middle generation, $g-1$, consumes $c_{g-1,t}$, chooses whether to buy a house $I_{g-1,t} = \{0 \text{ if no}, 1 \text{ if yes}\}$, with down payment $\eta_{g-1,t}$, pays housing rent $h_{g,t}$ if they do not own a home (defined by the binary indicator $H_{g-1,t} = \{0 \text{ if no}, 1 \text{ if yes}\}$), saves $s_{g-1,t}$ and pays a net intergenerational transfer. This generation also earns income from interest earning savings from the prior period, $s_{g-1,t-1}$, and pre-tax labor income $y_{g-1,t}$ as follows:

$$\begin{aligned} c_{g-1,t} \quad & + I_{g-1,t}\eta_{g-1,t} + h_{g-1,t} + s_{g-1,t} \\ & + \sum_{j\neq g-1} (R_{g-1j,t} - R_{jg-1,t}) \\ \leq & \ (1+r)\, s_{g-1,t-1} \\ & + y_{g-1,t}(e)\left(1 - \tau(y_{g-1,t})\right), \end{aligned} \quad (4)$$

where the down payment $\eta_{g,t} = \psi_i V_t$ is an exogenously imposed percentage ψ_i of home value V_t calculated as the discounted present value of future rents:

$$V_t = \kappa \sum_{t=t}^{\infty} \delta^t h_t,\ \delta = \frac{1}{1+r}; t = \infty \Rightarrow V_t = \kappa \frac{h_t}{r}. \quad (5)$$

Finally, at time t, the old generation, $g-2$, consumes $c_{g-2,t}$, chooses whether to buy a house $I_{g-2,t} = \{0 \text{ if no}, 1 \text{ if yes}\}$, pays housing rent $h_{g,t}$ if they do not own, saves $s_{g-2,t}$ and pays a net intergenerational transfer. Old earns income from interest earning savings from the prior period, $s_{g-2,t-1}$, and pre-tax labor income $y_{g-2,t}$ (i.e., retirement income) as follows:

$$\begin{aligned} & c_{g-2,t} + (1 - H_{g-2,t})\, h_{g-2,t} + \sum_{j\neq g-2} (R_{g-2j,t} - R_{jg-2,t}) \\ & \leq y_{g-2,t}(e)\left(1 - \tau(y_{g-2,t})\right) + (1+r)\, s_{g-2,t-1} + H_{g-2,t} V_t. \end{aligned} \quad (6)$$

In each period for each generation, households receive a stochastic income stream as a function of the median income for their generation at period t, a set of demographic characteristics including race, and an income shock conditional on education and demographics:

$$y_{g,t} = F_y\left(\mu_{\mathrm{gt}}, \sigma_{\mathrm{gt}} | e_g, D_g\right). \quad (7)$$

Net wealth in period t is given by $W_{g,t} = s_{g,t} + H_{g,t}V_t$ and generations gain utility with constant relative risk aversion

$$u_{g,t}(c_{g,t}) = \beta c_{g,t}^{1-\rho}. \quad (8)$$

We express the value function iteration, known as the Bellman equation, as the Q-function:

$$Q^*(s_t, a_t) = u(s_t, a_t) + \gamma \cdot Q^*(s_{t+1}, a). \quad (9)$$

After specifying a model that describes how wealth evolves over time, and how households make decisions, one can set initial conditions for this model—including race heterogeneities using the SCF data described previously to understand how wealth might evolve over the next several decades or even centuries.

While models like these are scarce, they recently clarified potential pathways to reduce racial wealth disparities in the long run [6], [7]. First, no model-based analysis predicts that racial wealth disparities can close absent any policy intervention. One paper [6] finds that absent any other changes, even a reparations policy that would immediately close average wealth gaps would not necessarily lead to convergence of wealth in the long run. In their model, different beliefs about risk returns would prevent Black households from realizing the benefits of reparation transfers and would cause the D disparity measure to increase back to around 0.5 after being reduced to 0 under the reparations policy. Their analysis finds that an alternative policy, investment subsidies (i.e., low-interest loans) targeted to Black households *can* close wealth gaps in the long run. In their model, such policies would eventually increase the expected returns from investments among Black households and help catalyze a virtuous cycle of Black wealth creation. Another paper based on an OLG model emphasizes the importance of initial conditions, in particular firm ownership [7]. In this model, an initial gap in firm ownership is sufficient to prevent Black households to catch up in wealth. This result holds *even if there was no racism* post Jim-Crow era. Those analyses highlight that the US government must carefully consider the potential effects of policy interventions in the long run if it aims to close wealth gap over a meaningful time-horizon.

Conclusion

The extent to which model assumptions would hold in the real world is a subject of debate. For instance, existing OLG models may not fully account for present-day racism that could hinder wealth accumulation amongst Black households. Yet mathematical models agree that racial wealth disparities will be perpetuated absent policy interventions and help chart potential pathways to reduce racial wealth disparities in the United States over the next few centuries.

While mathematical models do not replace fundamental value judgements about the merits of different policies, they can inform the debate about how to close racial wealth disparities by allowing researchers to clearly state model assumptions, anticipate predictable consequences of policy interventions, and ultimately help steer the United States towards a more equitable society.

Data and Code Availability

All the data used by this paper is publicly available. All figures and tables in this paper and our report can be reproduced with free software, and we made our code publicly available at `https://github.com/RANDCorporation/racial-wealth-gap`.

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