Knots, Links and Their Invariants: An Elementary Course in Contemporary Knot Theory
By A. B. Sossinsky.

This new addition to the Student Mathematical Library is an excellent concise introduction to knot theory and some of its associated algebra and topology. It is written clearly and simply, and well illustrated with many figures. The book is quite short, though not without demands on the reader. It is based on a course given online to participants in the Math in Moscow program and, because of the format of the course, the book contains rather frequent exercises for the reader, some of which are significant steps in proofs. However, a reader prepared to put in the work will be well rewarded. The exercises are interesting and not too difficult for undergraduates, and they lead the reader to some of the most important and useful knot invariants via surprisingly elementary methods—certainly more elementary than those used when the invariants were originally discovered.

As the book amply illustrates, knot theory is a topic that is easy to approach through elementary, easily visualized, combinatorial arguments. Yet it is also able to grow in many directions due to its connections with algebra, geometry, topology, and even physics. There can be no better example of “seeing the (mathematical) world in a grain of sand.”

Lecture 1 begins with some examples of knots, defined as equivalence classes of simple polygons in $\mathbb{R}^3$, and introduces the three Reidemeister moves, which underlie most of the invariance proofs in the lectures that follow. Lecture 2 introduces the Conway polynomial, a variant of the classical Alexander polynomial which is well-suited to illustrate the inductive construction of invariants via skein relations (unlike Alexander’s construction via covering spaces and homology). Skein relations are also the key to the treatment of the Jones polynomial and the Vassiliev invariants, which come up next.

The proof of the invariance of the Conway polynomial is only sketched, with some parts left as exercises, but proofs are more complete in Lectures 5 and 6, where we come to the more powerful Jones polynomial. The even more powerful Vassiliev invariants are the subject of Lectures 9, 10, and 11. These too are treated in a mostly elementary manner, and their nonelementary aspect—invoking Kontsevich integrals—is nicely explained in Lecture 11. To my knowledge, the Vassiliev material was previously unavailable in a book for undergraduates, so Sossinsky’s book is unique for this reason alone.

Most of the “meat” of this book is in the lectures on the knot polynomials and Vassiliev invariants; however, these are interspersed with very attractive informal sketches of related material on knots, braids, and other topics in topology and algebra. Also included is Lecture 13 on the history of knot theory, which mentions all the main contributors to the subject.

Altogether, this book is ideal for self-study by undergraduates with an interest in topology. It is proof that “old school” methods of topology—concrete, visual, and elementary—are still more than capable of providing insight into classical problems.

John Stillwell is a professor emeritus at the University of San Francisco. His email address is stillwell@usfca.edu.

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