MEMORIAL TRIBUTE

In Memory of Andrew J. Majda

Bjorn Engquist, Panagiotis Souganidis, Samuel N. Stechmann, and Vlad Vicol

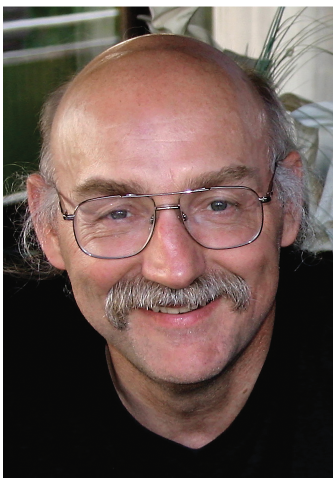

Figure 1. Andrew J. Majda 1949–2021.

Andrew Majda passed away on March 12, 2021, at the age of 72. He was hard working until the end even though he suffered from serious health issues for quite some time.

Andy was one of the great applied mathematicians of our time. He was passionate in his research and an applied mathematician in the best possible way, by using sophisticated mathematics to advance our understanding of science and also when needed to make fundamental contributions to mathematical analysis, in particular, to the theory of partial differential equations. He advocated a philosophy for applied mathematics research that involves the interaction of math theory, asymptotic modeling, numerical modeling, and observed and experimental data, as summarized in Figure 2 and often displayed by Andy in his research presentations. Andy's legacy lives on in the mathematical science he created, but also in the many students and postdocs he so enthusiastically taught and mentored.

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Communicated by Notices *Associate Editor Reza Malek-Madani.*

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DOI: https://doi.org/10.1090/noti2810

He was the son of Polish immigrants and grew up in a working-class environment in East Chicago, where he worked in the steel mills during summer breaks and played on his high school's football team. He got his bachelor of science degree from Purdue University as a mathematics major in 1970, and his PhD in mathematics from Stanford University in 1973. He wrote his thesis "Coercive Inequalities for Nonelliptic Symmetric Systems" with Ralph Philips as an advisor. Stanford was very important for Andy, not only because of his PhD, but above all that is where he met Gerta Keller. They married and stayed together for the rest of his life. She is a highly accomplished scientist herself and is now professor emeritus in the Geosciences Department at Princeton University. At Stanford, he also got to know the Ralph Phillips' collaborator Peter Lax, who became a mentor for Andy and encouraged him to join the Courant Institute as an instructor. This postdoc period exposed Andy to the Courant style of applied mathematics and nonlinear PDEs. After three years, he joined the faculty in the UCLA Mathematics Department in 1976 where he revitalized the applied and computational mathematics group. The period at UCLA was followed by five years at Berkeley, 1979–1984. During this productive time, he developed "Majda's model" for combustion in reactive flows [Maj81], and together with Tosio Kato and Tom Beale derived the so-called "Beale-Kato-Majda criterion," which characterizes a putative incompressible Euler singularity in terms of the accumulation of vorticity [BKM84]. From Berkeley, Andy moved to Princeton and stayed there from 1984–1995, where he briefly served as director for the Program in Applied and Computational Mathematics. During this time Andy continued his work in applied analysis, obtaining fundamental results concerning, e.g., finite-time singularities in ideal fluids [CLM85, CMT94], oscillations and concentrations in measure-valued solutions of the two-dimensional incompressible fluid equations together with Ron DiPerna [DM87, DM88], and turbulent diffusion with Marco Avellaneda [AM90, AM92a, AM92b], just to name a few. It was while working in Princeton that Andy started focusing on another area of applications: atmosphere and ocean science, which became his passion for the rest of his life and where his impact is felt most prominently. Andy returned to Courant in 1994 and founded the Center for Atmosphere Ocean Science within the Courant Institute of Mathematical Sciences.

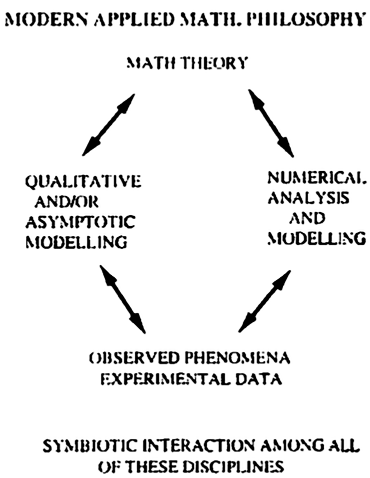

Figure 2. Andy Majda's modus operandi of modern applied mathematics, as a symbiotic relationship between (i) rigorous mathematical theory, (ii) numerical analysis and numerical modeling, (iii) observed phenomena and experimental data, and (iv) qualitative and/or asymptotic modeling. From [Maj00].

At Courant, Andy shifted his research efforts to cross-disciplinary research in modern applied mathematics with climate–atmosphere–ocean science. He created a variety of novel models of physical phenomena, such as his "multicloud model" with Boualem Khouider to explain the mechanisms of convectively coupled equatorial waves (CCEWs), his "skeleton model" with Sam Stechmann to explain the mechanisms of the Madden–Julian oscillation (MJO), his multi-scale asymptotic models for the tropics with Rupert Klein and Joe Biello, and many other models. He also designed novel computational methods for data assimilation, uncertainty quantification, stochastic climate modeling, ensemble prediction, and the fluctuation–dissipation theorem, among others. Common themes of Andy's work in these areas included a focus on intermittency and rare events, for instance in his novel data analysis technique with Dimitris Giannakis called Nonlinear Laplacian Spectral Analysis, and the use of information theory and entropy in many of his computational algorithms. He used full mathematical rigor to prove theorems and bring clarity to explanations of intriguing behavior such as catastrophic filter divergence in data assimilation. While Andy's favorite applications were always changing throughout his career, he maintained his modus operandi of modern applied mathematics (Figure 2) through it all. Andy stayed at Courant for the remainder of his life, even if it meant he had to commute weekly from his home in Princeton.

In recognition of Andy's achievements, he accumulated a grand collection of awards and honors. He was a member of the National Academy of Sciences and received numerous honors and awards including the National Academy of Science Prize in Applied Mathematics, the John von Neumann Prize of the Society of Industrial and Applied Mathematics (SIAM), the Gibbs Prize of the American Mathematical Society (AMS), the Wiener Prize of AMS/SIAM, the Lagrange Prize (awarded every four years by the International Council of Industrial and Applied Mathematics, ICIAM), and the Steele Prize of the AMS. He was also a member of the American Academy of Arts and Sciences, a Fellow of SIAM, and a Fellow of the AMS. He was awarded the Medal of the College de France, twice, and was a Fellow of the Japan Society for the Promotion of Science. He received an honorary doctorate from his undergraduate alma mater, Purdue University, as well as honorary degrees from Fudan University and China Northwestern University. He gave plenary one-hour lectures at both the International Congress of Mathematicians (ICM; Kyoto 1990) and the first ICIAM (Paris 1987).

In what follows, Andy's contributions in several research areas are described in more detail, and many of his friends and colleagues share their reminiscences of Andy.

1. Computation and Modelling

At Stanford, in 1976, Bjorn Engquist asked Ralph Phillips for some advice on pseudo differential operators. Phillips wisely suggested talking to Andy who was visiting for the summer. This turned out to be fortunate. The application was computational far-field boundary conditions which Bjorn had experienced in simulating wave propagation with applications in seismology. For infinite or very large domains, an artificial boundary is introduced to limit the computational domain where a wave equation is defined. If no boundary condition is added, which would be ideal, there is however no uniqueness. With the standard Dirichlet or Neumann boundary conditions the initial boundary value problem is well-posed but strong artificial reflections are generated at the new boundary distorting the solution in the interior. Ideally, the boundary conditions should support a well-posed problem and

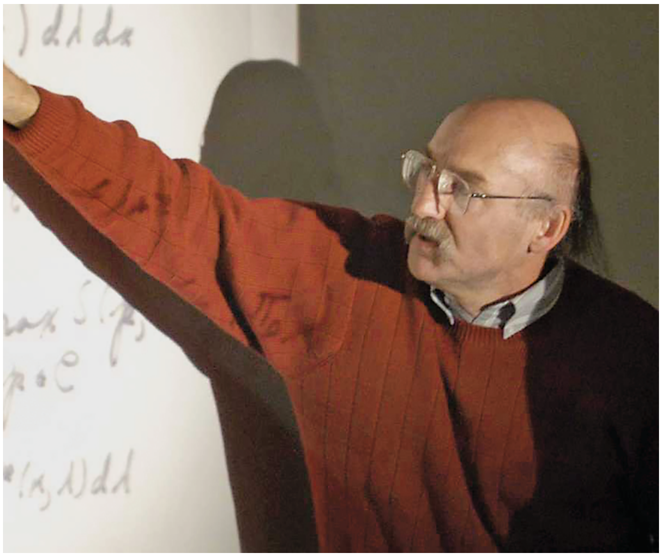

Figure 3. Andy giving a lecture.

"absorb" any waves coming from the interior. An excellent mathematical tool for generating and analyzing such absorbing boundary conditions is pseudodifferential operators. Andy's deep knowledge of microlocal analysis and his ability to rapidly grasp a new field was crucial. Pseudodifferential operators are however nonlocal and therefore not practical in computations. The next step is to approximate the nonlocal operators by local ones, which means differential operators. The obvious choice of using truncated Taylor expansions in this approximation turns out to produce ill-posed problems. By studying a wide class of local approximations in light of the recently developed normal mode analysis, it is possible to show that any order of approximations can be reached by using Padé approximations. This research was done during a few intensive weeks, and the first paper [EM77] came out in 1977. This remained one of Andy's most-cited papers throughout his career.

Andy made several significant contributions to numerical analysis, most often in connection with mathematical modeling of a particular physical process. In one important and purely numerical paper [CM80], Andy and Michael Crandall studied the challenging problem of numerical convergence in a nonlinear setting. This result together with a related paper by Kuznetsov and Volosin were game changers, which initiated a sequence of new results during the 1980s and 1990s. A difficulty is that the solution typically becomes discontinuous at a later time even if the initial value is smooth. A weaker notion of solution is needed for existence, and for uniqueness extra entropy conditions are needed and here the Kruzkov entropy is appropriate. Andy and Mike proved L^1 convergence of monotone, conservation form schemes to the correct entropy solution of scalar nonlinear conservation laws in the multidimensional case. A key tool is a lemma on L^1 contraction.

If Andy's work on absorbing boundary conditions [EM77] started his career in applied mathematics, another step was taken when he joined the faculty at Berkeley, 1979–1984. Using his talent to address open problems in analysis and applied mathematics, he focused on an application area, in this case reacting flows. He developed relevant mathematical models and analyzed their properties. The so-called, "Majda's model" plays the same role for combustion as Burgers' equation does for general compressible flow [Maj81]. This simple but ingenious model is phenomenological but explains the interplay between the advection, diffusion, and chemical reactions by just two quantities. It has been used extensively for developing and tuning numerical methods for the more general equations. Andy contributed to the area of reacting flows with a number of other results, but also continued with important works in applied analysis: the existence and stability of multidimensional shock fronts [Maj83, Maj84], and singular limits of quasilinear hyperbolic systems and in particular the zero Mach number limit for compressible flows [KM81], just to name a few.

2. Fluid Dynamics

Andy Majda's contributions to the modern development of mathematical fluid dynamics are immense, and it is not possible to provide a detailed account of his work on this subject in this short note. Instead, we will focus on two topics that have withstood the test of time, where the results of Majda and his collaborators are to date essentially the best available results: criteria characterizing putative finite time singularities in incompressible models, and the stability of multidimensional shock solutions to hyperbolic systems of conservation laws.

The quest for finite-time singularities in ideal fluids. The fundamental macroscopic model describing the motion of inviscid flows (these neglect viscous effects) are the Euler equations, written down by L. Euler in 1757. These PDEs encode the conservation of mass, momentum, and energy. When the change in density with pressure is negligible (which is the case for most liquids) the fluid flow is modeled as being incompressible. In spite of being around for more than 250 years, some of the fundamental properties of the resulting PDE model, namely the *incompressible Euler equations*, remain to date open. Chief among these is the question of finite time singularities: *Given an infinitely smooth initial distribution of velocity (and hence pressure), is it possible that in finite time the incompressible Euler dynamics creates nonsmooth solutions (meaning, with an unbounded velocity gradient)?* Majda appreciated the importance of this question for our understanding of fluids (e.g., the dynamical creation of small scales, cascades in inertial range turbulence), and his research program from the early 80s to the mid-1990s had a very large component investigating this problem.

Majda's most cited contribution to this subject (joint with Thomas Beale and Tosio Kato) is the so-called *Beale-Kato-Majda criterion* [BKM84], which states that the three-dimensional incompressible Euler equations form a finite time singularity at time $T < \infty$ if and only if $\int_{T_{\text{initial}}}^{T} \|\omega(t,\cdot)\|_{L^\infty} dt = +\infty$, where $\omega = \nabla \times u$ is the fluid vorticity (the curl of the fluid velocity). That is, finite time singularities from smooth initial data can occur if and only if the magnitude of the vorticity blows up in a nonintegrable way. To date, this is essentially the best available blowup criterion for incompressible three-dimensional Euler (up to log log corrections), and is widely used as a "test" in numerical simulations of putative Euler singularities. The paper [BKM84] foreshadowed a common theme in Majda's work on finite time singularities in ideal fluid models: the fundamental role played by vorticity dynamics and Lagrangian geometry. In hindsight, emphasizing the role of vorticity may not seem like that novel a perspective, especially since this was the leading point of view at the start of the 20th century; but one has to understand that mathematical fluid dynamics in the 70s and 80s was to a large extent influenced by a perspective through which fluid motion is an Eulerian dynamical system in a suitable Banach space, where the only important feature of the nonlinear term was that it obeyed certain cancellation properties, and satisfies certain bounds. The Lagrangian vorticity perspective championed by Andy Majda, Peter Constantin, and others was instrumental in the revival of mathematical fluid dynamics at the turn of the 21st century.

Subsequently, Majda approached the problem of finite-time singularities for the incompressible three-dimensional Euler equations through the analysis (both numerical and analytical) of a hierarchy of simpler toy models in which the potentially catastrophic effect of vortex stretching is in competition with the regularizing effect of advection. Perhaps the simplest such PDE model is the so-called *Constantin-Lax-Majda model* [CLM85]. Recall that the vorticity field ω in three-dimensional incompressible Euler solves is Lie-advected: $D_t\omega = \omega \cdot \nabla u$, where $D_t = \partial_t + u \cdot \nabla$ is the material derivative. The map $\omega \mapsto \nabla u$ may be represented as a matrix of Calderón-Zygmund operators (iterated Riesz transforms) applied to the vorticity vector, and thus the rough bounds for $\omega \cdot \nabla u$ (say in L^p spaces for $p \in (1,\infty)$) are the same as the bounds for $|\omega|^2$. This is the motivation behind the analogy with the Riccati ODE $\dot{x} = x^2$ in Lagrangian coordinates. The Constantin-Lax-Majda (CLM) model incorporates nonlocality in this analogy, by proposing the one-dimensional evolution equation $\partial_t\omega = \omega H\omega$, where H is the Hilbert transform (the canonical one-dimensional Calderón-Zygmund operator). In turns out that upon denoting $z = H\omega + i\omega$, the CLM evolution becomes the complex Riccati equation $\dot{z} = z^2/2$, from which finite time blowup can be directly deduced. This remarkably simple model has inspired a lot of deep subsequent works aimed at balancing the effect of nonlocal vorticity stretching and nonlocal advection. For instance, De Gregorio proposed a generalization of the CLM model in which the $\partial_t\omega$ term is replaced by the nonlocal advection operator $(\partial_t + u\partial_x)\omega$, with $\partial_x u = H\omega$. In certain regimes, this model is also known to blow up in finite time, but remarkably, a number of blowup questions remain open. Note that for a few decades the mathematical fluid dynamics community has criticized such simple toy models of the three-dimensional incompressible vortex dynamics, because they cannibalize the Euler equations to a seemingly absurd degree, and so drawing conclusions about finite time singularities seems not to be justified. We want to emphasize however that very recently Tarek Elgindi has proven the finite time breakdown of classical solutions of the three-dimensional incompressible equations from minimally smooth initial data ($C^{1,\alpha}$ for some $0 < \alpha \ll 1$). Remarkably, at the heart of Elgindi's construction lies an asymptotic (as $\alpha \to 0^+$) simplification of the three-dimensional Euler equations in self-similar variables, which has an exact solution obtained by solving (a version of) the CLM model!

Another toy model (which by now is considered to be "classical") of the three-dimensional incompressible Euler equations coansidered by Majda (jointly with Peter Constantin and Esteban Tabak) is the so-called *surface quasi-geostropic* (SQG) equation [CMT94]. The SQG equation has its origins in atmospheric dynamics (see the paper of Isaac Held and collaborators on this subject); at the same time, it has a very deep analytic and geometric connection with the three-dimensional Euler vorticity dynamics, while being an equation in two space dimensions (hence, making it more amenable to direct numerical simulations). The SQG equation may be written as the active scalar equation $D_t\theta = (\partial_t + u \cdot \nabla)\theta = 0$, where $u = \nabla^\perp \Delta^{-1/2}\theta$ is the nonlocal velocity. In particular, a quantity similar to vorticity is $\nabla^\perp\omega$, is also Lie-advected $D_t(\nabla^\perp\theta) = (\nabla^\perp\theta) \cdot \nabla u$, and the map $\nabla^\perp\theta \mapsto \nabla u$ is given by a matrix of Calderón-Zygmund operators. The numerical simulations reported in [CMT94] indicated a strong and potentially singular front formation in the shape of a hyperbolic saddle for the active scalar θ. Later numerical studies of Ohkitani and Yamada indicated that the direct numerical simulations were instead consistent with super-exponential growth for $\|\nabla^\perp\theta(\cdot,t)\|_{L^\infty}$ instead of finite time blowup, and this collapsing hyperbolic saddle scenario was ruled out rigorously as a path toward finite time blowup in the PhD thesis of Diego Córdoba. The work of Constantin, Majda, and Tabak [CMT94] has sparked an immense amount (over 700 citations) of beautiful mathematical works dedicated to the analysis of the SQG equation. It has led to a proof of finite time blowup in the case of patches, and in the presence of a solid boundary. Nonetheless, the question of whether the SQG equations posed on the plane (or on the two-dimensional periodic box) is capable of evolving smooth initial data into a finite time singularity, remains to date open; this is considered to be one of "the" outstanding mathematical challenges of inviscid incompressible fluid dynamics.

Coming back to the incompressible three-dimensional Euler equations themselves, Majda returned in the mid-1990s to the subject of blowup criteria. Motivated by the coherent structures observed to be ubiquitous in the inertial scales of fully developed hydrodynamic turbulence, Majda, Peter Constantin, and Charles Fefferman [CFM96] considered a geometric vorticity-based blowup criterium for three-dimensional Euler. The so-called *Constantin-Fefferman-Majda* (CFM) criterion states that certain regularity information on the direction of the vorticity vector $\frac{\omega(t,x)}{|\omega(t,x)|}$ gives information on the size of the vorticity vector. The mechanism is that of a hidden cancellation present in the vortex stretching term $\omega \cdot \nabla u$. The CFM criterion implies in particular that if a pair of potentially singular vortex tubes were to smoothly align during the stretching process, then the growth of vorticity is eventually subdued. This is yet another beautiful example of a leitmotif present in Majda's work: the Lagrangian geometry of the vorticity in three-dimensional incompressible Euler leads via delicate mathematical analysis, to bounds on the magnitude of the vorticity vector itself, which plays the key role in determining whether or not finite time singularities occur.

Multidimensional shocks. Systems of conservation laws are ubiquitous in the modeling of physical processes. In the case of scalar problems, these problems have already been quite well understood for a few decades, in part because several unreasonably nice things are available to the mathematical analyst: maximum principles, monotonicity formulas for the entropy, BV-contraction, etc. This was essentially done in the work of Kruzkov. The story is however very different in the case of systems and, in particular, systems in multiple space dimensions, which is the relevant case for continuum mechanics. For systems, very few rigorous results were known before Majda's fundamental contributions. There were only special solutions to piecewise constant initial values, so-called Riemann problems, and Glimm's existence proof for restricted problems in one dimension.

The prototypical example of a multidimensional conservation law is the compressible Euler equation of gas dynamics, in two or three space dimensions. Given a smooth initial condition for density, velocity, and energy, these equations have, at least for a small amount of time, a unique smooth solution. It is, however, well known that a C^1 smooth continuation of this solution cannot be defined for all times, due to the emergence of singularities. The prototypical stable singularity is a *multidimensional shock*: a codimension-one oriented hypersurface, such that on either side of this surface the unknowns (density, momentum, internal energy) are C^1 smooth functions and such that across the shock surface the density, normal velocity, and energy are discontinuous. The natural requirement that the globally defined fields define a weak solution to the compressible Euler system means that the jumps in these quantities across the shock surface are coupled to the evolution of the shock surface as a free boundary, through the classical Rankine-Hugoniot jump conditions. This naturally leads to the study of discontinuous weak solutions to systems of conservation laws.

Majda's monumental works [Maj83] and [Maj84] gave the first rigorous mathematical results for the *existence and stability of multidimensional shock fronts* for the compressible Euler equations; more generally, for systems of genuinely nonlinear hyperbolic conservation laws. Starting from a "shock-front" data, i.e., the shock front is given at the initial time together with compatible density, momentum, and internal energy fields, which are smooth on either side of the initial shock, Majda's results provide a rigorous short-time existence and structural stability of such shock fronts within the compressible Euler framework. Some of Majda's novel contributions to the mathematical analysis of this mixed hyperbolic free-boundary value problem include the notion of "uniform stability" for shock-front solutions based on normal mode analysis, the associated stability analysis for the linearization of a curved shock front, the nonlinear stability as an extremal form of linear stability, and the construction of the discontinuous solution via a nonlinear iteration scheme.

Majda's monographs [Maj83, Maj84] have essentially closed the subject for the local-in-time propagation of multidimensional shock fronts arising from discontinuous initial data for more than a decade. To date, these are essentially the best-known results in this generality. These works have been extremely influential in the hyperbolic community; for instance, they paved the way for the work of Gués, Métivier, Williams, and Zumbrun who proved the existence and stability of viscous multidimensional shocks. Modern research in this area is dedicated to connecting Majda's picture of the fully developed multidimensional shock, to the local smooth evolution arising from smooth initial data. How do multidimensional shocks emerge from smooth data in the first place? What is their geometry? How do they propagate? Is this propagation unique? Do other singularities emerge at the same time as the shock? This is an extremely active research area.

3. Turbulent Diffusion

Starting in 1990, Majda turned his attention to problems related to turbulent diffusion and combustion. Turbulent diffusion is associated with the problem of describing and understanding the transport of some physical entity, such as heat or particulate matter, which is immersed in a fluid flow. In most models, it is assumed that the fluid is undergoing some disordered or turbulent motion. If the transported quantity is passive, that is, it does not significantly influence the fluid motion, it is said to be passive, and its concentration density is termed a passive scalar field. Weak heat fluctuations in a fluid, dyes utilized in visualizing turbulent flow patterns, and chemical pollutants dispersing in the environment may all be reasonably modeled as passive scalar systems in which the immersed quantity is transported by ordinary molecular diffusion and passive advection by its fluid environment. Turbulent combustion is about the propagation of fronts in environments undergoing some disordered or turbulent motion.

The general and typical problem of describing turbulent diffusion of a passive quantity is the study of the quenched and or annealing properties of scaled versions of solutions of diffusion partial differential equations and solutions of stochastic differential equations, both transported by a velocity field which is assumed to be either periodic or random–typically a Gaussian random field of prescribed spectrum. The problem is really challenging and there are several important analytic and probabilistic contributions, which even to date do not give the complete answer.

In his joint program with Avellaneda which started in [AM90] and continued with several influential papers [AM92a, AM92b], Majda attacked a simplified version of the problem and obtained several exact results. This work represents another example of Majda's approach to Applied Mathematics—study a difficult problem by considering a simplified but yet challenging version which can be analyzed mathematically and obtain rigorous and new results which show the possible behavior of the underlying application.

Instead of looking at the most general problem, Avellaneda and Majda studied the asymptotic behavior of a simple model problem for a passive scalar with incompressible shear velocity fields admitting a statistical description and involving a continuous range of excited spatial and/or temporal scales, and developed a complete renormalization theory with full mathematical rigor. The analysis yielded explicit formulae for the anomalous time scaling in various regimes as well as the Green's function for the large-scale, long-time, ensemble averages, and the renormalized higher order statistics. The simple form of the model problem was deceptive. Indeed the renormalization theory in [AM90] provided a remarkable range of different phenomena as parameters in the velocity statistics vary. These included the existence of several distinct anomalous scaling regimes as the spectral parameter varies as well as explicit regimes where the effective equation for the ensemble average is not a simple diffusion equation but instead involves an explicit random nonlocal eddy diffusivity.

Turbulent diffusion combustion models are reaction diffusion equations perturbed by incompressible velocity fields. The aim is to understand the influence of the velocity field on the speed of evolving fronts resulting from the reaction. In a series of papers, Majda and Souganidis provided the first rigorous results for the effective speed of models with spatio-temporal periodic velocity fields, came up with explicit bounds for a model problem with a velocity field with incompressible shear velocity, and provided several important counterexamples to the general conjecture that the front velocity should be given by the so-called G equation.

4. Climate–Atmosphere–Ocean Science

In the 1990s, Andy made the decision to focus his research efforts on problems from climate–atmosphere–ocean science. In approaching this new area, Andy brought his modus operandi of modern applied mathematics, summarized in Figure 2, which is a synergy between rigorous mathematical theory, efficient computational methods, and physical insight. Furthermore, he aimed his sights at the biggest and most challenging open problems.

The tropics: Multiscale models, singular limits, moist convection, and dispersive waves. The tropics were seen as the biggest challenge, so they naturally grabbed Andy's attention. In fact, Andy would frequently recall being told that the tropics are too difficult, which to him only served as more motivation.

In jumping into the tropics, one of Andy's first and biggest impacts was the derivation of multiscale models. With Rupert Klein, Andy used the systematic asymptotic procedure of applied mathematics to derive different governing equations for the different scales of interest, including multiscale interactions between scales [MK03]. Later, with Alexandre Dutrifoy, Andy formulated rigorous proofs of some of these singular asymptotic limits.

One factor that drew Andy toward the tropics was that the tropics are home to a very interesting set of dispersive waves. They are called equatorial waves, since they are trapped near the equator and propagate around the globe with the equator acting as a waveguide, due to the vanishing of the Coriolis force at the equator. It was only in the 1960s that equatorial waves were discovered and their mathematical description was formulated (by Yanai, Matsuno, and others). Andy described his attraction to these waves by saying (perhaps overly simplistically) that he saw the dispersion curves for equatorial waves and knew he wanted to work on the tropics.

Another factor that drew Andy toward the tropics was a major open problem: the convective parameterization problem. The word "convective" refers to clouds and storms, which are essentially manifestations of convection within the atmosphere. They must be parameterized—i.e., represented as an unresolved, subgrid-scale process—in global climate models, since climate models must use large grid spacing to remain computationally feasible. The grid spacing is in the range of 10 to 100 km, which is too large to resolve the fluid dynamics of clouds and storms. The convective parameterization problem has persisted since around 1960 in the earliest days of global climate modeling or general circulation modeling.

These two factors—equatorial waves and the convective parameterization problem—come together in a phenomenon called convectively coupled equatorial waves (CCEWs). CCEWs are, as the name implies, equatorial waves that are coupled with clouds and storms: from an overly simplistic point of view, you can imagine that one phase of the wave is rainy, and the other phase of the wave is clear and sunny. The waves themselves have wavelengths of roughly 1,000 km or larger, and so in principle the waves could be resolved by a climate model with a grid spacing of 10 to 100 km. However, CCEWs are coupled with clouds and storms, which suffer from the convective parameterization problem, so climate models have struggled to adequately simulate CCEWs. Furthermore, many aspects of CCEWs were not well understood from a theoretical point of view when Andy began his research on the tropics in the 1990s and 2000s.

One of Andy's biggest contributions in atmospheric science is aimed at these two factors, and it is the multi-cloud model that he developed with Boualem Khouider (see Figure 4). It started as a nonlinear wave model to explain the mechanisms of CCEWs, and it grew into a sub-grid-scale model of clouds and convection for climate models. This work showed how the multicloud model could help to improve CCEWs and other tropical phenomena in global climate models (GCMs).

One mysterious phenomenon that doesn't fit in with equatorial wave theory is the Madden–Julian Oscillation (MJO). Despite many advances in understanding since its discovery in 1971, a generally accepted theory for the

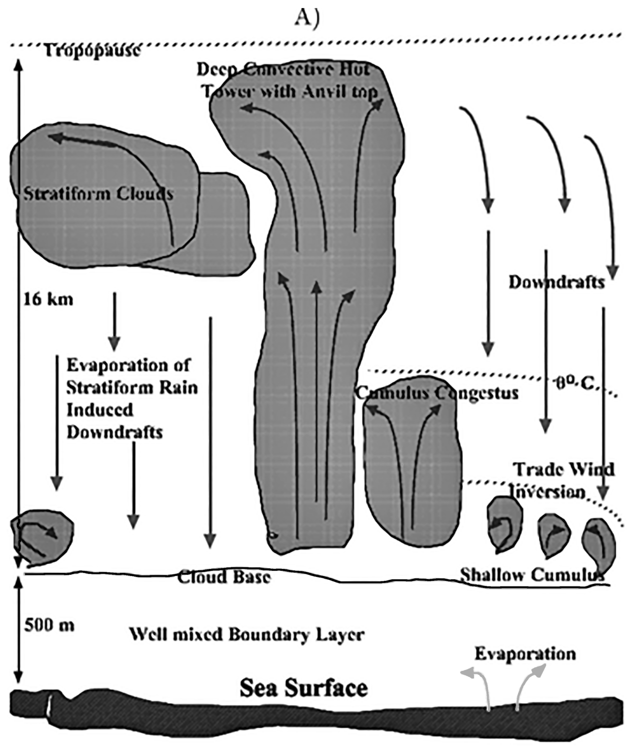

Figure 4. Schematic diagram of the physical processes in Andy Majda and Boualem Khouider's multicloud model. It illustrates the great amount of detail in the physical processes that Andy considered in his modeling efforts. From [KM08].

mechanisms of the MJO has remained elusive. Many theories have been proposed but none has been generally accepted. In light of this, the MJO has been called the holy grail of tropical meteorology. Of course it grabbed Andy's attention.

Andy brought the ideas of multi-scale asymptotics to try to explain the workings of the MJO. With Joseph Biello, he showed that CCEWs can impact the larger-scale MJO and help to shape the MJO's circulation structure. Later, with Sam Stechmann, he formulated an idea of how the MJO works as a multiscale phenomenon in a dynamical model, called the MJO Skeleton model [MS09]. It is a nonlinear PDE model, based on a nonlinear oscillation, and it isn't a multi-scale model per se but is based on multi-scale ideas. Andy wanted to name it the skeleton model because it captured only the most basic and large-scale features of the MJO (i.e., its skeleton), and not the smaller-scale details that would be part of a true multiscale description.

Computational methods: Data assimilation, ensemble forecasting, uncertainty quantification, and information theory. Another major theme of Andy's weather and climate research was computational methods and probabilistic forecasting, and what we would now refer to as data science. In these areas, data assimilation was one of Andy's favorite topics. Data assimilation is the process of creating the initial conditions for a forecast: a first guess at today's weather is provided by yesterday's forecast, and a better estimate is obtained by assimilating observational data into the model. Algorithms for data assimilation have undergone substantial advances in the decades since the advent of weather forecasting, and many of the most widely used algorithms are from the 1990s and 2000s with continuing advances today.

Part of the improvement in weather forecast skill over the past decades is due to improvements in data assimilation algorithms. Andy often said that "data assimilation" is a particularly unexciting name for such a vibrant and interesting research topic, but that is the terminology that is in common use. Other names for data assimilation are state estimation and filtering, and Andy became absorbed in this field enough that he wrote a book with John Harlim titled *Filtering Complex Turbulent Systems*. Two of Andy's results on data assimilation are described in the next paragraphs, as examples of his many novel ideas in these areas.

A first example began when Andy was attending a workshop at the National Center for Atmospheric Research (NCAR) in Boulder, Colorado. As Andy told the story, he was sitting in the audience during a presentation on data assimilation, and was surprised to see the results of a numerical calculation. He noticed that the numerical time step Δt was large and violated the Courant–Friedrichs–Lewy (CFL) stability condition, which should have caused the numerical solution to diverge toward an infinitely large value. However, the numerical solution didn't diverge. Andy noticed this on the spot and was intrigued. He suspected that the assimilation of observational data was providing a stabilizing effect. He soon went on to write a paper with Marcus Grote on this topic, and described how the traditional CFL stability criterion is modified in the presence of data assimilation. As in his modus operandi of modern applied mathematics, he developed rigorous mathematical theory and also computational examples.

A second example on data assimilation is Andy's work on "catastrophic filter divergence." Andy said that he coined the phrase himself to describe a type of divergence that might seem unexpected: a numerical solution with filtering/data assimilation can sometimes tend toward infinite values in finite time, even though the underlying stochastic dynamical system *without* filtering/data assimilation is dissipative and stable with the absorbing ball property. So in this case, the assimilation of observational data actually provides a *destabilizing* effect. Andy said he noticed this property for the first time while he was working on a project with John Harlim. They were developing computer codes for some data assimilation algorithms and their numerical simulations were diverging to infinite values. Since they were investigating examples that they thought were tame and well-behaved, they thought their computer code must have a bug. However, when they were unable to discover a bug, they began to suspect that the code was correct and searched for a possible explanation for the numerical divergence that they were seeing. In the end, Andy wrote several papers on this topic with collaborators including John Harlim, David Kelly, Georg Gottwald, and Xin Tong. Again following his modus operandi, Andy developed simple example models to illustrate the mechanisms of catastrophic filter divergence, investigated many numerical examples, and proved rigorous theorems about his ideas.

Three additional themes of Andy's work on computational methods were stochastic modeling/parameterization, uncertainty quantification, and information theory. He had many interesting results over the years, from some relatively early work on stochastic climate modeling with Ilya Timofeyev and Eric Vanden Eijnden [MTVE01], up to his last years with some of his last students, Nan Chen and Di Qi [MQ18]. Among his excellent results, he was particularly fond of his prediction, using empirical information theory in work with Bruce Turkington, of the location of the Great Red Spot of the planet Jupiter.

Anyone who worked with Andy or met him at a conference probably saw that Andy approached his work with great passion. He took his work very seriously and wanted it to have an impact on the world, and he expected others to approach their work in the same way. With such passion always turned on, Andy could be blunt in his criticisms, which Andy himself refered to as "tough love." A discussion with Andy might sometimes feel like the words of a tough sports coach. Perhaps this aspect of Andy's personality was shaped by his own experiences with sports coaches in his youth. He would often recall his time as a football player in high school and his later attraction to tennis. He was a sports fan and sports were always part of his life. He approached math and science with the type of intensity that an athlete might bring to a sports event. Many people who worked with Andy will miss the energy and enthusiasm that he always carried with him.

Andy was the founder and leader of two centers. The first was the Center for Atmosphere Ocean Science (CAOS) at New York University (NYU), a unit within the Courant Institute for Mathematical Sciences. From its founding in the 1990s to his last days there, CAOS grew to approximately 10 professors, and continues to foster a graduate program of approximately 20 current PhD students. The second center founded by Andy was the Center for Prototype Climate Modeling at NYU–Abu Dhabi. It provided a place where new ideas could be aided in the transition from an incipient research idea to a component of an operational global climate model. Among many successes of the center was the transition of Andy and Boualem Khouider's multi-cloud model into a convective parameterization in a GCM.

Andy had many colleagues and friends around the world, and perhaps two of his most substantial interactions were with colleagues in China and India. He received an honorary doctoral degree from Fudan University in Shanghai in 2008, and in a span of years he was a special visitor at Fudan University and cohosted several workshops there. Also, in India, he organized several workshops and formed collaborations with experts on the Indian summer monsoon.

Another leadership role for Andy was when he led a team of researchers (professors, postdocs, and students) as principal investigator (PI) of a Multidisciplinary University Research Initiative (MURI). The theme of the initiative was "Physics-Constrained Stochastic-Statistical Models for Extended Range Environmental Prediction" and it was funded by the Office of Naval Research with program managers Reza Malek–Madani and Scott Harper. The five years (2012–2017) of close interactions between Andy and the team were some of the most productive of Andy's career. He published approximately 20 papers per year in this period, and even 30 papers in 2015. He organized annual meetings for the team that were stimulating and enjoyable and were the source of many fond memories and some of the stories retold here.

5. Friends and Collaborators

Throughout his career, Andy worked with many people including many students. Part of his legacy is the mark he left not only on mathematics and science but also on the people who make up the mathematical and scientific communities. Below is a selection of memories of Andy from many of his friends, collaborators, and students.

Based on Andy's website and the Math Genealogy Project, Andy had 29 PhD students: Rafail Abramov, Robert Almgren, Miguel Artola, Andrea Bertozzi, Anne Bourlioux, Noah Brenowitz, Jonathan Callet, Dongho Chae, Nan Chen, Ryszard Dziuzynski, Pedro Embid, Jonathan Goodman, Roy Goodman, Margaret Holen, David Horntrop, Peter Kramer, Ding-Gwo Long, Richard McLaughlin, Robert Palais, Robert Pego, Di Qi, Victor Roytburd, Daniel Ruprecht, Steven Schochet, Sang-Yeun Shim, Sam Stechmann, David Stuart, Enrique Thomann, and Qiu Yang.

Included in the memories below are contributions from several of his PhD students, and many friends and colleagues.

Stan Osher

I met Andy in 1974, almost half a century ago. The reason these years were as good as they were for me was largely because of this meeting. I was at SUNY Stony Brook, spending a lot of time at Courant and somehow scored a 13th floor office there. I was told about this brilliant young postdoc, also interested in initial-boundary value problems. We met and wrote around seven papers together. But, more importantly, we became friends. Neither of us came from academic backgrounds, but rather from the streets of lower-class ethnic urban neighborhoods. I remember fondly a discussion we had with Mike Crandall where we tried to figure out whose background was more lower-class. Although Andy is seven years younger than I, I learned a lot from him. What I remember most is his belief that the best applied math research should also impact the larger scientific community and society in general.

I tried to recruit Andy for Stony Brook, but he wisely chose to go to UCLA. He then recruited me to UCLA Math, a department trying, with ambivalence, to build up in applied math. This turned out well and I am truly grateful. Eventually, Andy moved on from UCLA and we lost touch, but reconnected around twenty years ago. We attended and spoke at each other's 60th birthday conferences and enjoyed each other's company in the remaining years.

It was always a pleasure to hang out with Andy. He was a terrific applied mathematician, of course, but also an interesting person with a capacity for great compassion. His passing is a great loss to the community and a personal loss for me.

Peter Constantin

I met Andy in 1983–1984, introduced by Sergiu Klainerman. We started talking about blow-up in a simple model of Euler equations we were calling "the baby vorticity" model. This turned into our first paper, joint with Peter Lax. Andy liked to think in terms of a hierarchy of models. Next on his blow-up list was the contour dynamics equation for two-dimensional vortex patch boundaries, because, just as the baby vorticity model, it had a nonlinear structure resembling the quadratic nonlocal nonlinearity of vortex stretching in three-dimensional Euler equations. Vortex patch boundaries were proved later to remain

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smooth for all time, albeit at the price of super-exponential growth of norms measuring their smoothness.

Interested in incompressible turbulence, Andy was closely following the numerical and physics-theoretical approaches of the time. Numerical computations were showing certain Beltrami-like aspects of turbulent flows. This led us to our second joint paper, on the Beltrami spectrum. We continued to talk often, I would visit him in Princeton and he would visit me in Chicago, combining these trips with visits to his mother who lived close by, in East Chicago, across the Indiana border. I was thinking about a class of hydrodynamic models related to Euler equations which I was calling "active scalars" to contrast them with the "passive scalars" which were popular in the physical field-theoretical approaches to turbulence in those days. Andy told me he was talking with Isaac Held in Princeton about a quasi-geostrophic model, which was the same as one of the active scalars. We joined forces, and, together with Esteban Tabak we wrote a paper on the SQG model, with numerical predictions of blow-up. The scenario we were putting forward turned out differently, and numerical studies by Ohkitani-Yamada, and later Schorgoffer and Nie, Jiahong Wu and coauthors, and rigorous results of Diego Cordoba, showed that this scenario cannot sustain blow-up. In fact, an interesting geometric depletion of nonlinearity takes over the dynamics. We wrote a paper, joint with Charlie Fefferman on the subject of depletion of nonlinearity in three-dimensional Euler equations, due to the local alignment of vorticity direction.

Andy continued to be interested in incompressible fluids for a while. Our last joint paper was a 1997 paper with Ciprian Foias and Igor Kukavica on the density of the set of ancient solutions of two-dimensional Navier-Stokes equations. Around that time Andy decided to get involved in climate science. He immersed himself in the technical literature and then started to work in the area.

Although we have not collaborated since then, I continued to see him from time to time. Last time I saw him was in Victoria in 2019, at a celebration in his honor. Although gravely ill, he was in good spirits, and was challenging his friends to roast him more vigorously, without much success. Andy had a strong influence on many applied mathematical areas: multidimensional shock fronts in hyperbolic PDEs, detonation models in combustion, singular limits in fluid mechanics, incompressible formation of singularities, climate science, statistical modeling, and more. The mathematical sciences community lost in Andy Majda a major figure.

Victor Roytburd

Andy Majda was a great mathematician and an amazing man. I had the honor of being the first PhD student Andy graduated. I met him (and became his student) more or less incidentally.

Before coming to the US as a Jewish refugee in 1979, I had been a graduate student (aspirant) in Russia for three years (without getting a degree) and worked in industry for nine years on some optimization issues totally unrelated to my graduate research. After some unsuccessful and feeble attempts to find a job, friends of mine gave me wonderful advice to try getting into graduate school, no matter that I came in June and the application process to all graduate schools was long over.

I met Joe Keller through some Jewish contacts of my mother-in-law who lived in Palo Alto at the time. Joe very generously listened to me and told me of a star-quality mathematician doing a very important fantastic work in applied mathematics, Andy Majda, and suggested that I talk to him. Andy was a professor at Berkeley but usually spent summers at Stanford, and Keller connected me with Andy. Andy came to our first meeting quite sweaty after a tennis game with his tennis gear in tow (which in my eyes was a serious argument in his favor as a thesis advisor). After looking through my reprints, Andy said that he was not interested in the linear stuff and offered that I work under his guidance on combustion problems, which were one of his interests at the time. Needless to say, I agreed then and there and one of those combustion problems led to my dissertation.

Among other achievements, Andy is known for his early groundbreaking work on multidimensional shock waves. Once I asked him why he did not continue working in this or other areas of PDEs. His answer was that while there were several mathematicians who could do work of high quality (comparable to his) in analysis, he saw his principal and rather unique strength in extracting underlying mathematical structures from messy and unwieldy physical phenomena and analyzing those by all methods available, be it rigorous analysis, asymptotics, or numerical computation. From pure analysis he resolutely moved to more applied problems. Andy was second to none, a real artist, in designing simple conceptual models that were amenable to analysis while preserving salient features of underlying phenomena. One of his first models of this kind was the Majda's model of detonation. By the way, he used to say that scientists are divided into artists and art

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critics; I don't know what he meant by art critics but he definitely saw himself as the former, and an artist he was.

As an advisor, Andy was extremely generous and forgiving. He did not intervene much in what I was doing; however, he provided some crucial advice. Also, he made some attempts to teach me how to write mathematics. My Russian papers were very brief announcements of the Doklady type. Andy taught me to forget about this mathematical tendency to economize paper; "important ideas and notions deserve to be repeated." It was an amazing learning experience to write joint papers with him. Right on the early stages of the work he developed a detailed plan of the future paper that was headed with the title. He knew which scientific questions he wanted to resolve and apparently had no doubt that they would be resolved.

Andy was very passionate and brutally honest about science. I remember sitting next to him at a colloquium lecture by Vladimir Arnold on singularities theory that, according to Arnold, was developed mostly by him and his collaborators. Among other examples, he mentioned that shock waves dynamics could be given an elegant interpretation through his results. Andy was literally sitting on his hands not to jump up, telling me that it was complete nonsense (he used a stronger expression though). Finally, he asked the speaker how the scalar theory that Arnold was talking about could have anything to do with the shock waves which are inherently multidimensional. Arnold tried to talk his way out of this unpleasant situation but Andy did not let him off the hook. As a result, I believe the department chair either disinvited Andy from the colloquium party at somebody's house or elicited from him a promise not to get into scientific discussions with Arnold.

Andy secured a sabbatical position for me at Princeton from 1988 to 1989, and I worked with him and his very talented student Anne Bourlioux on detonations. Once, when visiting him at home, Andy stunned me by saying that he was preparing to move in the new direction, atmospheric science. He showed me a rather substantial pile of books and reprints he was ploughing through. Here again he was resolutely planning a turn in a new direction that he considered as an outstanding scientific direction for applied mathematics.

Andy was a multifaceted man. I think he played football in high school, and he was an excellent tennis player, although he never agreed to play matches with me; ("Victor, I am so much better than you that it does not make any sense for us to play games"). He surprised me once by admitting that he took ballet lessons while a postdoc at NYU. I asked him why in the world he would take ballet lessons. He answered, "Victor, you don't understand; that is where the girls are." He was a cinephile and we shared a love of movies by Bob Altman (even to some of those that were not universally accepted as the good ones). He was a loyal and supportive friend to me and others. I remember him regularly visiting Ron DiPerna during his terminal illness in Princeton. Andy Majda was a real *mensch*. He left us too early; he will be missed.

Robert Pego

I was one of Andy's first PhD students, thanks to a tip from Jerry Bona. It was a period of exciting progress in Andy's work on fluid mechanics, conservation laws, and numerics, with major works emerging on radiation boundary conditions, monotone difference schemes, vortex blob methods, multidimensional shock fronts, and instabilities of combustion waves. A lively circle of activity included visitors such as Mike Crandall, Ron DiPerna, Tai-Ping Liu, Tom Beale, Ruben Rosales, and Barbara Keyfitz.

Andy could switch topics with incredible swiftness and seemed to have all his knowledge instantly on tap. As a teacher, he gave clear and inspiring lectures, and as a mentor he was conscientious. He had played football in his youth and had an expansive and sometimes blunt sportsman's personality. He was famously not shy about expressing sharp opinions, but was extremely supportive of his students. He would explain to you enthusiastically what was interesting about the little lemma you proved and how it fit into the big picture. After my PhD studies, I felt for a while that math was in black-and-white, while it had been in technicolor with Andy.

In the 1980s, Andy's work diversified rapidly into fields like turbulence and stochastic modeling. At the end of that decade he made a major shift to focus on the mathematics of atmospheric flows and climate. Andy always articulated an inspiring vision of applied mathematics as part of the scientific enterprise in service of humanity. He esteemed the development of good models and better computational methods alongside rigorous analysis of fundamental ideas.

Although my work has been analytical for the most part, on more that one occasion I took heart from this heritage to address a problem by a formal approach that could benefit a broad scientific audience. It behooves us all to pick up the torch that Andy has been forced to lay down.

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Pierre-Louis Lions

On top of numerous seminars and lectures at Collège de France, Andrew Majda was invited twice to deliver a research course at Collège de France and thus twice received the Medal of Collège de France. Few mathematicians have received it once and Andrew Majda is the only mathematician who received it twice!

This should not be a surprise in view of Andrew Majda's depth and breadth. His lectures in Paris always attracted large audiences from many different scientific communities: applied mathematics, fluid mechanics, combustion theory, turbulence theory, meteorology, and climatology.... He also had real friends in Paris and I am proud to count myself among them.

These facts are a good illustration of the unique nature of Andrew Majda's work—in French, we would say "oeuvre"—and of his singular contributions to Mathematics and more generally to Science.

Charles Fefferman

When I knew Andy best, his interests were concentrated on classical problems of fluid mechanics. He had a vision of how that field should progress. He preached that neither numerical simulation, physical intuition, nor mathematical rigor alone could crack the tough, important problems. Rather, it was essential to combine them. In those days, Andy was one of very few people able to work that way: Andy could think like a fluid and also like a mathematician, an exceedingly rare combination of gifts.

It was a big mistake to try to put over any BS in Andy's presence. I repeatedly watched in a mixture of delight and awe as his sharp questions punctured pompous public pronouncements from eminent mathematicians and physicists in a matter of seconds. I wonder what Andy would say now about my preceding paragraph.

Dave McLaughlin

Andy was convinced that "modern applied mathematics" has a great deal to contribute to both the basic and the applied sciences—a conviction that Andy so often expressed and reiterated. To Andy, modern applied mathematics meant the union of today's computational science with both the philosophy and methodology of mathematics. He was convinced that the revolutionary advances that have occurred in computing power and data acquisition would be terribly under-utilized without the precision, perspectives and methodology of mathematics. As summarized in this article, Andy's work over the years has resulted in confirmation after confirmation of his conviction, with perhaps the best examples provided by the many results in theoretical atmosphere–ocean science that he obtained during the last 25 years of his career.

Russ Caflisch

Andy Majda was a powerhouse as a mathematician working on PDEs, fluid dynamics, atmospheric science, and scientific computing, but he was also much more. He had a large network of former students and postdocs, collaborators, and others who looked to him for advice and inspiration. At the Courant Institute he was the founder and longtime leader of the Center for Atmosphere/Ocean Science (CAOS). He also had great influence on the scientific direction of Courant and on faculty hiring through his wide-ranging knowledge and passion for math and science.

Eric Vanden Eijnden

I met Andy 25 years ago when I came to Courant fresh from my PhD. I was very much looking forward to talking to him about his work with Marco Avellaneda on turbulent diffusion, which was the topic of my thesis. The meeting did not go very well. True to form, Andy threw me out of his office after about a minute, declaring that I didn't know anything about anything. He was right of course, as Andy was often right. This first meeting was not our last, however: Andy invited me back and we ended up working together for many years afterward, first on turbulent diffusion (with Peter Kramer) then on stochastic parameterization (with Ilya Timofeyev, on what became known as the *MTV framework*, no pun intended). Working with Andy was intense, but always rewarding: even though he was tough, always telling it straight like it is, he was also very caring, both about the science and the people he was working with. Just as he had done for many others, Andy helped me a lot at the beginning of my career, with advice

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about what to do and what to learn to be able to do it—Andy had very good taste in problems and the ability to make a dent in them.

Andy was an extraordinary character in many ways. He was forward looking, always searching for new challenges with vision—even though he was well-established at a young age, he never rested on his laurels and kept going after new and timely topics throughout his career. Andy was also one of the most driven people I ever met. This was true all his life, but it became even more apparent after he had his stroke. He never gave up and the courage with which he faced his health challenges was truly inspirational. His attitude reminds me of a beautiful poem by Dylan Thomas whose first verse is:

"Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light."

That drive to push the limits is how I remember Andy—I miss him very much.

Weinan E

Andy was a master of using simple models to explain complex physical phenomenon. We have seen this again and again in his work on gas dynamics, combustion, singularity formation in fluid equations, and atmospheric flows. In this regard, he was the "Landau" in applied mathematics.

In somewhat of an opposite direction, Andy also defined a certain style of applied mathematics: getting to the bottom of things using a combination of rigorous analysis, asymptotic analysis, and numerical analysis. In it, we see the power of applied mathematics shown at its fullest. This style has greatly impacted a generation of applied mathematicians like myself. It is not just useful for fluid dynamics, but for many other problems across a wide range of fields. In fact, right now we are witnessing applications to the understanding of neural network models.

Andrea Bertozzi

I was introduced to Andy Majda when he first arrived at Princeton. I was starting my third year as an undergraduate and I was told to take his graduate course in Partial Differential Equations. This was a good transition from the rigorous pure math curriculum given to Princeton Math Majors and a research career in applied mathematics. He gave me a delightful problem to work on for my undergraduate thesis—something that became my first publication—in dynamical systems. Andy had incredible intuition, not only for good problems to work on, but about areas in which one would find future good problems. He steered me toward nonlinear PDE in the late 1980s during a time when desktop computing was just beginning to have an impact in science and when there were many open problems on the boundary between science, mathematics, and computing. His critical stance on other people's work gave me great perspective and helped to form my own taste in research problems. He had a great capacity to see well beyond "where we are now" into the realm of where we should be a decade or two down the road. This foresight has had an immense impact on both science and mathematics and also on the careers of many younger applied mathematicians.

Rupert Klein

According to Andy, I was his first postdoc. Our first encounter dates back to the Conference on Hyperbolic Systems in Bordeaux, France, in the summer of 1988. Forman Williams, an eminent combustion scientist and a good friend of both Andy and my PhD advisor Norbert Peters, had quizzed me earlier about what I was up to for my postdoc time. When I mentioned "combustion" and "turbulence," he asked whether I was interested in working with Andy Majda or Steve Orszag. "Sure," I said, "but I don't know either of them." He promised to ask them, and he did. The next thing I knew is that Andy had said I could come, that he would provide an office, but that I should bring my own salary. A grant proposal with the German Science foundation was successful, and everything was arranged without Andy and I ever having met or even talked to each other.

Then came the Bordeaux conference, which I attended for the chance of meeting my prospective host. Early during the week I heard all sorts of intimidating rumors about Andy, and these made me procrastinate a bit on approaching him. So, by Wednesday he had sensed me sneaking around in his vicinity, and over a coffee break—obviously quite annoyed—he stormed right at me booming "WHO ARE YOU??!!" Timidly I uttered my name, and in a split second Andy changed gears, opened his arms, beamed at me "you are Rupert Klein..." and then excitedly told the bystanders the whole story of the postdoc arrangement.

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Throughout the rest of the week we had tremendous fun discussing the weakly nonlinear asymptotics of acoustic waves, Andy and Miguel Artola's "kink-modes on a vortex sheet," the acoustic-chemistry-interactions I had been working on for my diploma, and the like.

Later in the year, I moved to Princeton, looking forward to working on my (self-funded) project in combustion theory and to learn all the pertinent math background from my host. But, alas, in the course of our first meeting in Andy's office up on the ninth floor of Fine Hall, he told me he was not working on combustion anymore, but that I would be welcome to discuss these issues with his PhD students Anne Bourlioux and Rob Almgren. Well, I did, and my exchange with the two of them was enjoyable and very interesting, for all involved, as I hoped. But there was no scientific exchange with Andy for the time being.

That is, until a few months into my stay, when Andy approached me over coffee and asked what I was doing with Lu Ting at the Courant Institute every second Thursday; and would I not be willing to give an Applied Math seminar talk on this one of these days. Andy had ventured into vortex dynamics and turbulence theory in the meantime and he was interested in finding a reduced analytically tractable model that would describe the self-stretching of vorticity. He had suspected for a while that Lu's pioneering matched asymptotic analyses of slender vortex filaments could provide a solid point of departure for the derivation of such a reduced model, but had thus far not found the time to go through these intricate derivations himself. Lu and I were indeed working on lecture notes that summarized that work, and so my seminar presentation was right on the money. Andy asked several pointed questions afterward with an eye to the self-stretching issue, and then he and I took off for our first adventure of close cooperation.

Two years earlier, my PhD advisor Norbert had not only suspected already that Lu's work should be of high interest for turbulence theory, but he also thought that Hidenori Hasimoto's paper on "A soliton on a vortex filament," which showed that the local induction approximation for slender vortices was equivalent to the focusing cubic nonlinear Schrödinger equation, could probably play an intriguing role in the context of Lu's work. So, he had me give lectures on both of these theories in his Turbulence Seminar at RWTH Aachen, and thus I was familiar with both. Hence, when I had gotten stuck with the geometric complexities of our asymptotic filament model and Andy suggested that I see whether Hasimoto's theory could help untangle the mess, I had the tools already and the door to the total of five papers on the self-stretching of vortex filaments with Andy was wide open.

I will never forget the enthusiasm and excitement Andy and I shared in the course of these developments, nor the incredible efficiency of communication that was possible with him. Yes, I was mathematically well-trained as far as mechanical engineers are concerned, but I was a "dyed-in-the-wool" fluid mechanicist. Andy could seamlessly shift from, e.g., the abstract math perspective on nonlinear Schrödinger to intuitive discussions of self-induced vortex motion and back, however, and this way we always found the right level of language very quickly and were able to move forward in big strides. This, in hindsight, is one of the most remarkable experiences in my life as a scientist. I have had this amazing and immediate feeling of mutual scientific understanding with no more than three colleagues over the past 30-odd years.

After my postdoc period I went back to Aachen and Andy and I lost touch for a while. Yet, in the late 1990s, when Andy had already moved on to the applied math of atmosphere–ocean science, I joined the Potsdam-Institute for Climate Impact Research in Potsdam, Germany. Shortly thereafter "it happened again," as Andy would later say: We met in Breckenridge for the 13th Atmospheric and Oceanic Fluid Dynamics Conference. He told me about flow regimes in the tropics and how he envisioned the construction of an entire scale-dependent model hierarchy, and two years later our next paper on "Systematic multi-scale Models for the Tropics" came out. This paper, he often jokingly told meteorologists in the audience of his various science lectures, was his revenge on the insiders of atmosphere–ocean science: Their field, he said, kept potential intruders from other disciplines from intruding by the impenetrable jungle of acronyms they had grown around it. So, here we went with MEWTG, SPEWTG, IPESD, QLELWE, and so on in the 2003 paper.

Later in 2006, we incorporated moist process models in our multiscale asymptotics framework, and studied the interaction of internal waves with arrays of deep convective towers. Then in 2009, we ventured into advanced time series analyses for atmospheric flow applications together with Christian Franzke and Illia Horenko.

Andy loved interdisciplinary advanced applied mathematics research, and he helped foster related workshops and conferences where ever there was a promising opportunity. Thus, in 2002, 2006, and 2010, he and I organized Oberwolfach workshops on atmosphere–ocean science together with Oliver Bühler and Bjorn Stevens, and today these have developed into a very productive multi-leaved workshop series at the institute. Our last major joint enterprise—again with Bjorn Stevens on board—was the stimulating long program on "Model and Data Hierarchies for Simulating and Understanding Climate" at the Institute for Pure and Applied Mathematics (IPAM), in Los Angeles in 2010. The spin-offs of this long program I think still reverberate quite positively in the community.

Andy has been a great supporter and a fabulous cooperating partner throughout my career, and this leaves me deeply grateful. As partnerships tend to have it, we have had our challenging moments, but these never rattled the foundations of our great mutual respect or the shared enthusiasm about our joint and mutual scientific achievements.

Leslie M. Smith

Andy was my close colleague, mentor, and friend for 30 years. His influence on pure and applied mathematics was extraordinary, as was everything about Andy. On the applied side, he made pioneering contributions to the theory of turbulence, combustion, and climate–atmosphere–ocean science. Andy's work never stopped evolving, and it was next to impossible to keep pace with him. One of his most important legacies was mentoring junior mathematicians and his peers, always ready to share his vision and insight. I was not Andy's student or postdoc and could not help but be surprised by his generosity, but there it was, genuine and with no strings attached.

Conversations with Andy changed my career at least twice, the first time as a beginning postdoc and later at midcareer. For my first postdoc at the Stanford Center for Turbulence Research, I was given the task of penetrating a new theory of turbulence based on renormalization that had just been introduced into the fluid dynamics community by Victor Yakhot and Steve Orszag. Two years later, after summing many loop diagrams, I went to work with Victor and Steve at Princeton, where Andy was separately working on a mathematically rigorous renormalization theory for advection-diffusion of a passive scalar. Thus, I came to know Andy in the middle of a heated rivalry. On request to meet with Andy, his assistant was instructed to tell me *"Professor Majda says that you should read his papers."* That I did, including the beautiful papers by Avellaneda and Majda on renormalization. After a bit of time and persistence, Andy and I became friends as well as colleagues. I admired him as both the mathematician and the person, insistent upon rigor, honest and straightforward to a fault, rough on the outside but big-hearted on the inside.

My friendship with Andy developed over decades and became one of the most important relationships in my professional life. Much later, during a shuttle trip from Banff to Calgary, I mentioned a desire to move beyond my current research projects, and Andy proceeded to give me ideas for tackling moist atmospheric dynamics that would be a foundation of my research for the next few years. I have a special memory of meeting Andy in Beijing when he received the Lagrange Prize at ICIAM 2014, and we had dinner with his spouse Gerta Keller, Boualem Khouider, Rupert Klein, and Sam Stechmann. The last time I saw Andy was in Victoria, at one of his 70th birthday parties, where Gerta gave a public lecture on her theory of the dinosaur extinction, and we shared another celebratory dinner. I was lucky to know Andy, and am extremely grateful for his brilliance, leadership, kindness, and companionship.

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Richard McLaughlin

Andy Majda was an applied math pioneer. He made a strong impact in so many different areas of science including compressible flow, combustion, turbulent transport, vortex dynamics, and climate modeling. In each of these fields, he brought powerful mathematics to bear upon real world phenomena, often utilizing mathematically exact solutions which captured many features of the system under study. He taught us to dive deep into the science whilst using the best mathematics possible to uncover new behavior and make new predictions about complex systems. His scientific impact and influence will be felt for a long time.

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Peter Kramer

The profound impact of Andy's mentoring while I was his PhD student would presumably echo largely what others are saying in more colorful ways so I'll share instead my encounter with Andy while I was an undergraduate physics major at Princeton. Not knowing what the subject really meant, I had never considered taking an "applied mathematics" class until Andy offered an undergraduate "Mathematical Fluid Mechanics" class my senior year. Beyond the usual introductory mathematical formalism and analysis of various fluid equations, Andy infused into this class his famous triangle of mathematics interacting with computing and the scientific discipline for insight and demonstrated this principle with concepts distilled from recent research by himself and his collaborators. His class was revelatory in illuminating the power of applied and computational mathematics toward understanding a richer spectrum of systems in the world than I was seeing in my physics classes. The following semester Andy directed a smaller undergraduate course in which we studied

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several key research papers in mathematical fluid mechanics. These classes with Andy, on their own, persuaded me to pursue graduate studies in applied mathematics rather than physics. Indeed, I had been having difficulty identifying an area of physics whose research prospects resonated with my interests, and Andy's uniquely inspirational undergraduate classes steered me to a career that I believe has been much more satisfying than it would otherwise have been. These, I believe, were the last undergraduate classes Andy ever taught (1992–1993), and I feel so privileged to have been introduced to Andy and his scientific world view by them.

Boualem Khouider

The passing of Professor Andy Majda on March 12th, 2021, sadly marks the end of a lifetime full of new mathematical and scientific discoveries and of breaking barriers between highly complex mathematics and very important applications including but not limited to climate, atmosphere and ocean sciences (CAOS). Majda's ingenuity, endless vision, contagious energy, and generosity in sharing and broadcasting his ideas and nonconservative approach to scientific research will be forever remembered. There is no doubt that his legacy will outlive his academic children and grandchildren. Professor Majda has made several groundbreaking advances in the theoretical description and understanding of many important atmosphere and ocean phenomena. His contributions to the theories of the Madden-Julian Oscillation (MJO), convectively coupled equatorial waves, and El Nino, are just a few examples.

By the time he decided to engage in climate change science, Andy Majda was one of the world's most famous applied mathematicians of his generation, known for his groundbreaking work in numerical analysis, shock waves, turbulence theory and combustion. Andy's "intrusion" to the CAOS community was received with a mixture of skepticism and awe. While a few people were saying that the problems he was tackling were too hard for a newcomer, many have admired his original and unconventional approach to these problems.

With his impeccable mathematical skills, his extensive experience in applying mathematics to describe and understand observed physical phenomena, his sharp vision to quickly grasp the most fundamental features from observations, and his tireless efforts in training young scientists, Professor Majda has made, unquestionably, some of the most exciting contributions in the area of atmospheric dynamics in the past two decades. His work has changed the way many people think about tropical atmospheric dynamics. Because of Professor Majda's contributions, the field of tropical atmospheric dynamics is now in a new era in which nonlinear scale interactions are no longer a speculation, a concept, or an observational perception. His mathematical models and computational strategies and algorithms for the multiscale interactions for atmosphere and ocean dynamics have paved new paths and solid grounds for CAOS research.

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Nan Chen

I was very fortunate to be Andy's PhD student as well as his postdoc. Andy taught me everything about research, offered many precious opportunities to improve my abilities, and provided enormous help to my career. I was extremely touched by how hard Andy worked to guide me in research and how generous he was in spending time helping me. In addition to being my research advisor, Andy was my close friend. He also often made me feel that he was a close family member of mine. I sincerely cherish all the time spent with Andy. Andy told me that the most important thing for someone who wants to do great scientific work is is that "he/she loves science." Andy himself was a person who really loves science. His spirit will continue to influence all of us. I keep in mind all that I learned from Andy and regard Andy as a model for my life and career. The last time I saw Andy was October 10, 2019, at his home. Andy brought me to the front door when I left. He had a slow pace, but a big smile. That was the scene that will stay in my memory forever.

Di Qi

Andy Majda was my PhD advisor and later Postdoctoral mentor. He was a great mathematician, sharp thinker, enthusiastic teacher, and most of all a true friend with an ardent heart. No doubt Andy has left an indelible mark on me in my scientific pursuit as well as in my perception of life.

During the years working with him, I was always amazed by the functioning of Andy's magical mind to quickly digest and employ new ideas with skill. He was always eager to spread his knowledge and share his thoughts with the people around him. It was well-known that Andy always showed up earliest in the morning in his office. He was able to continually discuss science on vastly different

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research topics tirelessly from morning till late afternoon. It is still a wonder to me how Andy could smoothly shift subjects and stay sharp in his many ongoing projects all at the same time.

Even when his health began to decline, Andy maintained his vitality to continue his everyday scientific routine. He never stopped the weekly scientific meetings with all of us. He forced himself to overcome great pain and showed little sign of his suffering. Besides the numerous scientific achievements, it is his shear strong-minded determination to keep on going that leaves the deepest impression for me.

Andy remains a persistent inspiration and gives me the courage to carry on. I was lucky to be guided by Andy, with his insight, energy, and intellectual power at the early stage of my career. I will forever miss the many exhilarating discussions around the long couch in his office, with warm sunlight pouring in through the wide window panes, where so many brilliant ideas emerged.

Reza Malek-Madani

I knew Andy Majda for over 35 years, first meeting him in 1975 as a graduate student at Brown when he gave a talk in the PDE Seminar. It was clear from that meeting that Andy not only was a great researcher, he was already a gifted teacher. His lectures throughout his career were organized as if meant to tell a story. We are so fortunate that many of his talks have been recorded and are available on YouTube and on other platforms.

I was of course aware of the remarkable contributions he was making during the 80s and 90s, but that was mostly as a distant observer. In the year 2000, I became the program officer at ONR, running the applied math program, and it was at that time that I got to interact with Andy regularly. Andy had just started his new research area in atmospheric and oceanic sciences, an area of critical importance to the Navy. In the early 2000s, each year Andy organized a Friday–Saturday workshop in early December and brought together applied mathematicians and oceanographers to discuss the many topics that our colleagues have already described here: Predictability for the atmosphere and ocean (2003), Large-Scale turbulence in the atmosphere and ocean (2004), Vortices and Waves in Geophysical Flows (2006), to mention just the first few. The brainstorming in these workshops were always stimulating, and on occasion heated and contentious—in those years, on the way home late on Saturday, my train would stop in Philadelphia. Many of my students who had just attended the Army-Navy game would get on board, telling stories about the battle they had just witnessed on the field, and I would wonder which of us had seen a more competitive event. The net result of watching Andy during those formative years, and learning from him how to learn, was to give me the tools and the courage to approach Scott Harper, my counterpart at ONR in charge of GFD research funding, to propose that we could come up with an interdisciplinary approach to our programming design to take advantage of what was being developed at Courant, at NCAR, at JPL, and elsewhere. Scott, a Princeton graduate, quickly agreed that we were observing something special. For the next two decades I would visit Andy regularly, and I was grateful for his enormous patience while I tried to keep up with him. On the Fridays of those workshops, Andy would take me aside at lunchtime, we would get a sandwich and then walk the streets of New York while he told me what was on his mind. My job for the next several months was to digest his thoughts and fill in the gaps in what I understood. I slowly began to understand what he meant by his modus operandi, which others have already alluded to. That concept impacted my career in two ways: Andy's framework for what modern applied mathematics should be ended up defining the underpinning for my program; and my attempt to understand Andy's approach became the focus of a course I taught annually at the Naval Academy for over 15 years.

What touched me the most about Andy was the courage he showed when he switched his research focus so dramatically to atmospheric sciences. ONR and the Navy benefitted enormously from this decision. Our community is so much richer because of Andy's brilliance, and the generous and unselfish manner in which he treated all of us.

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Gerta Keller

The first time I met Andy Majda was in the summer in Palo Alto, California. From the very beginning, I called Andy by his last name "Majda," which is an endearing name for the Swiss.

As the summer progressed, Majda and I became close friends. He often worked at the Stanford Coffee House and I met him for café lattes. At sunset, we ran across the Bay Area mud flats among thousands of birds and talked about all sorts of interests from science to nature, but I never was interested in Majda's favorite sport topics from football to tennis. And so began our lifelong friendship and love.

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Figure 5. Andy Majda and Gerta Keller in the Swiss Alps.

Majda and his younger twin brothers grew up in the middle of East Chicago's oil refineries, where they played among the slag heaps and roasted potatoes. Their father grew up as a Jesuit and learned many languages in Poland. When he left for the USA, he immediately enlisted in the US army as a radio technician during World War II. After the war, he worked in East Chicago's oil refineries. Andy Majda was a precocious youngster who quickly taught himself everything there was to learn in the encyclopedia, which was the only book in their home. He had a photographic memory and never forgot anything. Even during the first years of his encyclopedic memory, he was called "the little professor." His twin brothers followed the little professor's example. In a short time, Andy Majda became an extraordinary mathematics professor with full credentials by age 28. He followed the twins: George, also a mathematician, and John an oncologist. They were a remarkable trio of academics. Majda fell in love with the Swiss Alps. To introduce Majda to the beauty of the Swiss Alps, we organized a major hiking and climbing trip over three days. It was Majda's first trip into the high mountains. We steadily hiked upward toward the glacier, known as the Forno Glacier, and then carefully picked our way between ice, water, and rocks. After leaving the Forno glacier, we began a very hard steep switchback climb to the top of the mountain that ended the first day adventure at the Forno Hut, a Swiss Alp club house.

Near the end of Majda's life, I asked him: What was your favorite Alps trip? He smiled happily and said "Forno Hutte." We both laughed at our memory. "I will take you there" I said. He looked at me and replied, "G, I can't walk anymore." "But Majda, I will take you there," I repeated. He looked at me and smiled, realizing what I proposed and he was happy. On second thoughts he said, "G, you can't do it anymore, you're not strong enough." "I will do it," I replied. "G, it's better to take me to Val Rosegg, which is my second favorite Alps trip and has fantastic desserts. And then you can walk to Piz Bernina." And so, Majda planned his last trip to his favorite mountains and the best desserts in the Val Rosegg valley. It will happen in the summer of 2023.

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