EARLY CAREER

The Early Career Section offers information and suggestions for graduate students, job seekers, early career academics of all types, and those who mentor them. Krystal Taylor and Ben Jaye serve as the editors of this section. Next month's theme will be Black History Month.

What Does a Mathematician Do?

Does One Have to be a Genius to Do Maths?

Terence Tao

This article was initially written for my blog in May 2007, as part of my general advice to my graduate students, and was based on my experience interacting with many such students, postdocs, and colleagues as they went through the process of "learning the ropes" of how to be a research mathematician. It has been one of the more highly viewed and commented upon articles on my blog, perhaps in part due to the unintuitive conclusions drawn.

Better beware of notions like genius and inspiration; they are a sort of magic wand and should be used sparingly by anybody who wants to see things clearly. (José Ortega y Gasset,[1] "Notes on the novel")

Does one have to be a genius to do mathematics?

The answer is an emphatic **NO**. In order to make good and useful contributions to mathematics, one does need to work hard,[2] learn one's field well,[3] learn other fields[4] and tools,[5] ask questions,[6] talk to other mathematicians,[7] and think about the "big picture."[8] And yes, a reasonable amount of intelligence, patience,[9] and maturity[10] is also required. But one does **not** need some sort of magic "genius gene" that spontaneously generates *ex nihilo* deep insights,

Terence Tao is a professor of mathematics at UCLA. His email address is tao@math.ucla.edu.

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[1] *http://en.wikipedia.org/wiki/Jos%C3%A9_Ortega_y_Gasset*
[2] *https://terrytao.wordpress.com/career-advice/work-hard/*
[3] *https://terrytao.wordpress.com/career-advice/learn-and-relearn-your-field/*
[4] *https://terrytao.wordpress.com/career-advice/dont-be-afraid-to-learn-things-outside-your-field/*
[5] *https://terrytao.wordpress.com/career-advice/learn-the-power-of-other-mathematicians-tools/*
[6] *https://terrytao.wordpress.com/career-advice/ask-yourself-dumb-questions-and-answer-them/*
[7] *https://terrytao.wordpress.com/career-advice/attend-talks-and-conferences-even-those-not-directly-related-to-your-work/*
[8] *https://terrytao.wordpress.com/career-advice/think-ahead/*
[9] *https://terrytao.wordpress.com/career-advice/be-patient/*
[10] *https://terrytao.wordpress.com/career-advice/be-professional-in-your-work/*

unexpected solutions to problems, or other supernatural abilities.

The popular image of the lone (and possibly slightly mad) genius—who ignores the literature and other conventional wisdom and manages by some inexplicable inspiration (enhanced, perhaps, with a liberal dash of suffering) to come up with a breathtakingly original solution to a problem that confounded all the experts—is a charming and romantic image, but also a wildly inaccurate one, at least in the world of modern mathematics. We do have spectacular, deep, and remarkable results and insights in this subject, of course, but they are the hard-won and cumulative achievement of years, decades, or even centuries of steady work and progress of many good and great mathematicians; the advance from one stage of understanding to the next can be highly non-trivial, and sometimes rather unexpected, but still builds upon the foundation of earlier work rather than starting totally anew. (This is for instance the case with Wiles's[11] work on Fermat's last theorem,[12] or Perelman's[13] work on the Poincaré conjecture.[14])

Actually, I find the reality of mathematical research today—in which progress is obtained naturally and cumulatively as a consequence of hard work, directed by intuition, literature, and a bit of luck—to be far more satisfying than the romantic image that I had as a student of mathematics being advanced primarily by the mystic inspirations of some rare breed of "geniuses." This "cult of genius"[15] in fact causes a number of problems, since **nobody** is able to produce these (very rare) inspirations on anything approaching a regular basis, and with reliably consistent correctness. (If someone affects to do so, I advise you to be *very* sceptical of their claims.) The pressure to try to behave in this impossible manner can cause some to become overly obsessed with "big problems" or "big theories,"[16] others to lose any healthy scepticism in their own work[17] or in their tools,[18] and yet others still to become too discouraged[19] to continue working in mathematics. Also, attributing success to innate talent (which is beyond one's control) rather than effort, planning, and education (which are within one's control) can lead to some other problems as well.[20]

Of course, even if one dismisses the notion of genius, it is still the case that at any given point in time, some mathematicians are faster, more experienced, more knowledgeable, more efficient, more careful, or more creative than others. This does not imply, though, that only the "best" mathematicians should do mathematics; this is the common error of mistaking absolute advantage for comparative advantage.[21] The number of interesting mathematical research areas and problems to work on is vast—far more than can be covered in detail just by the "best" mathematicians—and sometimes the set of tools or ideas that you have will find something that other good mathematicians have overlooked, especially given that even the greatest mathematicians still have weaknesses in some aspects of mathematical research. As long as you have education, interest,[22] and a reasonable amount of talent, there will be some part of mathematics where you can make a solid and useful contribution. It might not be the most glamorous part of mathematics,[23] but actually this tends to be a healthy thing; in many cases the mundane nuts-and-bolts of a subject turn out to actually be more important than any fancy applications. Also, it is necessary to "cut one's teeth" on the nonglamorous parts of a field before one really has any chance at all to tackle the famous problems in the area; take a look at the early publications of any of today's great mathematicians to see what I mean by this.

In some cases, an abundance of raw talent may end up (somewhat perversely) to actually be *harmful* for one's long-term mathematical development; if solutions to problems come too easily, for instance, one may not put as much energy into working hard,[24] asking dumb questions,[25] or increasing one's range,[26] and thus may eventually cause one's skills to stagnate. Also, if one is accustomed to easy success, one may not develop the patience[27] necessary to deal with truly difficult problems (see also this talk by Peter Norvig[28] for an analogous phenomenon in software engineering, though see this clarification[29]). Talent is important, of course; but how one develops and nurtures it is even more so.

It's also good to remember that **professional mathematics is not a sport** (in sharp contrast to mathematics

[11] http://en.wikipedia.org/wiki/Andrew_Wiles
[12] http://en.wikipedia.org/wiki/Fermat%27s_last_theorem
[13] http://en.wikipedia.org/wiki/Grigori_Perelman
[14] http://en.wikipedia.org/wiki/Poincar%C3%A9_conjecture
[15] https://www.discovermagazine.com/mind/the-cult-of-genius
[16] https://terrytao.wordpress.com/career-advice/dont-prematurely-obsess-on-a-single-big-problem-or-big-theory/
[17] https://terrytao.wordpress.com/career-advice/be-sceptical-of-your-own-work/
[18] https://terrytao.wordpress.com/career-advice/learn-the-limitations-of-your-tools/
[19] https://terrytao.wordpress.com/career-advice/be-patient/
[20] http://nymag.com/news/features/27840/
[21] http://en.wikipedia.org/wiki/Comparative_advantage
[22] https://terrytao.wordpress.com/career-advice/enjoy-your-work/
[23] https://terrytao.wordpress.com/career-advice/don%E2%80%99t-base-career-decisions-on-glamour-or-fame/
[24] https://terrytao.wordpress.com/career-advice/work-hard/
[25] https://terrytao.wordpress.com/career-advice/ask-yourself-dumb-questions-and-answer-them/
[26] https://terrytao.wordpress.com/career-advice/continually-aim-just-beyond-your-current-range/
[27] https://terrytao.wordpress.com/career-advice/be-patient/
[28] https://www.youtube.com/watch?v=DdmyUZCl75s
[29] https://news.ycombinator.com/item?id=25426329

competitions[30]). The objective in mathematics is not to obtain the highest ranking, the highest "score," or the highest number of prizes and awards; instead, it is to increase understanding of mathematics (both for yourself, and for your colleagues and students), and to contribute to its development and applications. For these tasks, mathematics needs all the good people it can get.

Further reading:

- "How to be a genius,"[31] David Dobbs, New Scientist, 15 September 2006. [Thanks to Samir Chomsky for this link.]
- "The mundanity of excellence,"[32] Daniel Chambliss, Sociological Theory, Vol. 7, No. 1, (Spring, 1989), 70-86. [Thanks to John Baez for this link.]

Terence Tao

Credits

Photo of the author is courtesy of Reed Hutchinson/UCLA.

What Does a Mathematician Do?

William Thurston

The following question appeared on mathoverflow.net, and it was answered by the late William Thurston in October of 2010. We republish it here with the permission of Bill's widow, Julian Muriel Thurston.

Question: I find that mathematics is made by people like Gauss and Euler—while it may be possible to learn their work and understand it, nothing new is created by doing this. One can rewrite their books in modern language and notation or guide others to learn it too, but I never believed this was the significant part of a mathematicians work; which would be the creation of original mathematics. It seems entirely plausible that, with all the tremendously clever people working so hard on mathematics, there is nothing left for someone such as myself (who would be the first to admit they do not have any special talent in the field) to do. Perhaps my value would be to act more like cannon fodder? Since just sending in *enough* men in will surely break through some barrier.

Answer: It's not mathematics that you need to contribute to. It's deeper than that: how might you contribute to humanity, and even deeper, to the well-being of the world, by pursuing mathematics? Such a question is not possible to answer in a purely intellectual way, because the effects of our actions go far beyond our understanding. We are deeply social and deeply instinctual animals, so much that our well-being depends on many things we do that are hard to explain in an intellectual way. That is why you do well to follow your heart and your passion. Bare reason is likely to lead you astray. None of us are smart and wise enough to figure it out intellectually.

The product of mathematics is clarity and understanding. Not theorems, by themselves. Is there, for example any real reason that even such famous results as Fermat's Last Theorem, or the Poincaré conjecture, really matter? Their real importance is not in their specific statements, but their role in challenging our understanding, presenting challenges that led to mathematical developments that increased our understanding.

The world does not suffer from an oversupply of clarity and understanding (to put it mildly). How and whether specific mathematics might lead to improving the world (whatever that means) is usually impossible to tease out, but mathematics collectively is extremely important.

I think of mathematics as having a large component of psychology, because of its strong dependence on human minds. Dehumanized mathematics would be more like computer code, which is very different. Mathematical ideas, even simple ideas, are often hard to transplant from mind to mind. There are many ideas in mathematics that may be hard to get, but they are easy once you get them. Because of this, mathematical understanding does not expand in a monotone direction. Our understanding frequently deteriorates as well. There are several obvious mechanisms of decay. The experts in a subject retire and die, or simply move on to other subjects and forget. Mathematics is commonly explained and recorded in symbolic and concrete forms that are easy to communicate, rather than in conceptual forms that are easy to understand once communicated. Translation in the direction conceptual -> concrete and symbolic is much easier than translation in the reverse direction, and symbolic forms often replaces the conceptual forms of understanding. And mathematical conventions and taken-for-granted knowledge change, so older texts may become hard to understand.

[30] https://terrytao.wordpress.com/career-advice/advice-on-mathematics-competitions/

[31] http://talentdevelop.com/articles/HTBAG.html

[32] https://www.jstor.org/stable/202063

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