CHOICE FUNCTIONS AND TYCHONOFF’S THEOREM

W. H. GOTTSCALK

The purpose of this note is to point out that the following set-theoretic theorem [R. Rado, Axiomatic treatment of rank in infinite sets, Canadian Journal of Mathematics vol. 1 (1949) pp. 337–343] is an easy consequence of Tychonoff’s theorem that the cartesian product of a family of compact spaces is compact.

**Theorem.** Let \( (X_\alpha | \alpha \in I) \) be a family of finite sets, let \( \mathcal{A} \) be the class of all finite subsets of \( I \), and for each \( \Lambda \in \mathcal{A} \) let \( \phi_\Lambda \) be a choice function of \( (X_\alpha | \alpha \in \Lambda) \). Then there exists a choice function \( \phi \) of \( (X_\alpha | \alpha \in I) \) such that \( \Lambda \in \mathcal{A} \) implies the existence of \( B \in \mathcal{A} \) such that \( B \supseteq \Lambda \) and \( \alpha \phi = \alpha \phi_B \) \( (\alpha \in \Lambda) \).

**Proof.** For \( \Lambda \in \mathcal{A} \) let \( E_\Lambda \) be the set of all \( \phi \in X = \bigtimes_{\alpha \in I} X_\alpha \) such that \( \alpha \phi = \alpha \phi_B \) \( (\alpha \in \Lambda) \) for some \( B \in \mathcal{A} \) with \( B \supseteq \Lambda \). Provide each \( X_\alpha \) \( (\alpha \in I) \) with its discrete topology. Since \( X \) is compact and \( \{ E_\Lambda | \Lambda \in \mathcal{A} \} \) is a class of nonvacuous closed subsets of \( X \) with the finite intersection property, there exists \( \phi \in \bigcap_{\Lambda \in \mathcal{A}} E_\Lambda \). The proof is completed.

**Corollary.** A family of finite sets has a one-to-one choice function if and only if each of its finite subfamilies has a one-to-one choice function. [Cf. C. J. Everett and G. Whaples, Representations of sequences of sets, Amer. J. Math. vol. 71 (1949) pp. 287–293.]

University of Pennsylvania

Presented to the Society, April 22, 1950; received by the editors January 16, 1950.