CHOICE FUNCTIONS AND TYCHONOFF'S THEOREM

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The purpose of this note is to point out that the following set-theoretic theorem [R. Rado, *Axiomatic treatment of rank in infinite sets*, Canadian Journal of Mathematics vol. 1 (1949) pp. 337–343] is an easy consequence of Tychonoff's theorem that the cartesian product of a family of compact spaces is compact.

THEOREM. *Let* $(X_\alpha | \alpha \in I)$ *be a family of finite sets, let* $\mathcal{A}$ *be the class of all finite subsets of* I, *and for each* $A \in \mathcal{A}$ *let* ϕ_A *be a choice function of* $(X_\alpha | \alpha \in A)$. *Then there exists a choice function* ϕ *of* $(X_\alpha | \alpha \in I)$ *such that* $A \in \mathcal{A}$ *implies the existence of* $B \in \mathcal{A}$ *such that* $B \supset A$ *and* $\alpha\phi = \alpha\phi_B$ $(\alpha \in A)$.

PROOF. For $A \in \mathcal{A}$ let E_A be the set of all $\phi \in X = X_{\alpha \in I} X_\alpha$ such that $\alpha\phi = \alpha\phi_B$ $(\alpha \in A)$ for some $B \in \mathcal{A}$ with $B \supset A$. Provide each X_α $(\alpha \in I)$ with its discrete topology. Since X is compact and $[E_A | A \in \mathcal{A}]$ is a class of nonvacuous closed subsets of X with the finite intersection property, there exists $\phi \in \bigcap_{A \in \mathcal{A}} E_A$. The proof is completed.

COROLLARY. *A family of finite sets has a one-to-one choice function if and only if each of its finite subfamilies has a one-to-one choice function.* [Cf. C. J. Everett and G. Whaples, *Representations of sequences of sets*, Amer. J. Math. vol. 71 (1949) pp. 287–293.]

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