ADDENDUM: DERIVATIVES OF INFINITE ORDER

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We gave an incomplete proof[1] that, if $f(x)$ belongs to a quasi-analytic class $C\{M_n\}$ in $a<x<b$ and if $f^{(n)}(x_0)\to L$ for one x_0 in (a, b), then $f(x)$ is analytic in (a, b) (and consequently $f^{(n)}(x)\to Le^{x-x_0}$ in $a<x<b$); the proof was completed by S. Mandelbrojt.[2] We now wish to point out that in fact T. Bang[3] had already shown that if $f(x)$ belongs to a quasi-analytic class on $a<x<b$ and $g(x)$ is analytic, then $f^{(n)}(x_0)=g^{(n)}(x_0)$ for all n and $a<x_0<b$ implies $f(x)\equiv g(x)$, which is precisely the result which we needed for our proof.

Bang has also pointed out to us that a function constructed in his thesis[4] answers a question raised by us.[5] We asked whether it is possible to have $\lim_{n\to\infty} f^{(n)}(x)/\lambda_n=g(x)$, $a\leqq x\leqq b$, with lim inf $|\lambda_{n-1}/\lambda_n|=0$, and $g(x)\not\equiv 0$ in $a<x<b$. Bang constructed a function $f(x)$ analytic except for $x=0$, with $f^{(n)}(0)$ tending to ∞ arbitrarily rapidly; if we take $\lambda_n=|f^{(n)}(0)|$, we have $g(x)=0$ for $x\neq 0$, $g(0)=1$.

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[1] *Derivatives of infinite order*, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 523–526.

[2] R. P. Boas and K. Chandrasekharan, *Correction*: *Derivatives of infinite order*, Bull. Amer. Math. Soc. vol. 54 (1948) p. 1191.

[3] T. Bang, *Om quasi-analytiske Funktioner*, Copenhagen thesis, 1946, p. 84.

[4] Bang, loc. cit.

[5] Boas and Chandrasekharan, op. cit., p. 525.