3. O. D. Kellogg, *Foundations of potential theory*, Berlin, 1929.

4. O. Perron, *Eine neue Behandlung der ersten Randwertaufgabe für* $\Delta u=0$, Math. Zeit. vol. 18 (1923).

5. H. Weyl, *The method of orthogonal projection in potential theory*, Duke Math. J. vol. 7 (1940).

6. O. Frostman, *Potential d'équilibre et capacité des ensembles avec quelques applications à la théorie des fonctions*, Meddel. Lunds Univ. Mat. Sem. vol. 3 (1935).

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A NOTE ON AREA[1]

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If $\bar{x}$ is continuous on a square Q into a metric space D, then it is known that there exists a continuous function x on Q into m, the space of bounded sequences, which is isometric to $\bar{x}$[1].[2] By an *area* let us understand a functional on the set of continuous functions on Q into m which has the property that if $\bar{x}$ maps Q into E_3, if $\bar{x}$ is sufficiently smooth, and if x on Q into m is isometric with $\bar{x}$, then the area of x agrees with the generally accepted area of $\bar{x}$.

If x and y map Q into m and if $\|x(p)-x(q)\| \leqq \|y(p)-y(q)\|$ for all p and q in Q, then let us write $x \prec y$.

Let μ be a measure on m such that if $G \subset m$, G is isometric to H, $H \subset E_3$, then the μ-measure of G is equal to the 3-dim Lebesgue measure of H.

Suppose that it is considered reasonable to impose upon an area the following (Kolmogoroff's) principle: if $x \prec y$, then area $x \leqq$ area y. We shall see, then, that an area must suffer from one, at least, of the following two maladies:

(i) There exists a function whose area is finite and whose range has positive μ-measure.

(ii) There exists a function whose area is infinite and whose range is a (simply covered) simple arc.

It is sufficient to show that an area α which satisfies Kolmogoroff's principle and is not subject to (i) must necessarily satisfy (ii).

Let x be continuous on Q into m. It is not difficult to construct a monotone function x' with $x \prec x'$ such that x' is the monotone factor

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[2] Numbers in brackets refer to the bibliography at the end of the paper.

in a monotone-light factorization of x. (In [2] a construction is given such that the Lebesgue area of x' is equal to the Lebesgue area of x.)

Let I be the closed interval $[0, 1]$, $Q=I\times I$, $J=I\times\{0\}$, and R be a cube. Since R is a Peano space, there exists a continuous function f from J onto R and we may take f to be light. Define $\bar{x}$ on Q by $\bar{x}(u, v)=f(u)$, and let x on Q into m be isometric to $\bar{x}$. Since the μ-measure of range x is positive, $\alpha(x)=+\infty$, and so, by Kolmogoroff's principle, $\alpha(x')=+\infty$. Let X be x' restricted to J. Then X is the monotone factor in a monotone-light factorization of f. Hence X is monotone and light, and thus topological, on J. Since the continua of constancy of x' are identical with those of x, we have range x' $=$range X and so range x' is a simple arc. Finally, the inverse image of each point of range x' is a continuum.

Bibliography

1. E. Silverman, *Definitions of Lebesgue area for surfaces in metric spaces*, *Rivista* di Matematica della Università di Parma vol. 2 (1951) pp. 47–76.

2. ———, *An intrinsic property of Lebesgue area*, Rivista di Matematica della Università di Parma vol. 2 (1951) pp. 195–201.

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