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A NOTE ON GENERALIZED TAUBERIAN THEOREMS. ADDENDUM

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Lemma 9 of my paper mentioned above, in Proceedings of the American Mathematical Society vol. 2 (1951) pp. 335-349, may be stated thus.

LEMMA. *If there exists a sequence $\{\lambda_p\}$ such that $1 < \lambda_p \rightarrow 1$ as $p \rightarrow \infty$ and*

$$(1) \quad \liminf_{u \rightarrow \infty} \text{lower bound } \{A(u') - A(u)\}_{u < u' < \lambda_p u} = \begin{cases} \text{either } o_L(\lambda_p - 1), \\ \text{or } o_L(\log \lambda_p), \end{cases} \quad \text{as } p \rightarrow \infty,$$

then

$$(2) \quad \liminf_{u \rightarrow \infty} A(u) = \liminf_{u \rightarrow \infty} \frac{1}{u} \int_0^u A(x) dx,$$

$$(3) \quad \limsup_{u \rightarrow \infty} A(u) = \limsup_{u \rightarrow \infty} \frac{1}{u} \int_0^u A(x) dx.$$

Dr. T. Vijayaraghavan has kindly pointed out to me that the proof of (3) which I have merely indicated might require some clarification as follows.

Taking one of the alternatives of (1), say the first, we can show that it implies, for any $\lambda > 1$,

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$$(1') \quad \text{lower bound } \{A(u') - A(u)\}_{u < u' < \lambda u} = o_L(1)(\lambda - 1) \quad \text{as } u \rightarrow \infty.$$

For, the first alternative of (1) shows that we can find $\lambda_p < \lambda$ such that, for all large u ,

$$(4) \quad A(u') - A(u) > -\frac{\epsilon}{2}(\lambda_p - 1), \quad u < u' < \lambda_p u.$$

Plainly there is a positive integer $r \geq 2$ such that $\lambda_p^{r-1} < \lambda \leq \lambda_p^r$ and so (4) gives, for all large u and $u < u' < \lambda u$,

$$\begin{aligned} A(u') - A(u) &= \{A(u') - A(\lambda_p^{r-1} u)\} + \{A(\lambda_p^{r-1} u) - A(\lambda_p^{r-2} u)\} + \dots \\ &\quad + \{A(\lambda_p u) - A(u)\} \\ &> -\frac{\epsilon}{2} r(\lambda_p - 1) > -\frac{\epsilon}{2} \frac{r}{r-1} (\lambda_p^{r-1} - 1) \\ &> -\epsilon(\lambda - 1). \end{aligned}$$

The last inequality leads at once to (1').

If we are given the first alternative of (1), we write it in the form (1') and use (1') in connection with familiar arguments such as those of Karamata referred to in my paper [cf. S. Minakshisundaram, *On generalised Tauberian theorems*, Math. Zeit. vol. 45 (1939) pp. 495-506, §5.1]. The second alternative of (1) can be used to prove (3) in the same way.

I take this opportunity to correct a mistake which appears in the step preceding (4.10), p. 345, of my paper. The step in question is

$$\liminf_{u \rightarrow \infty} \text{lower bound } [\sigma_1(u') - \sigma_1(u)]_{u < u' < \lambda u} = o_L(1) \log \lambda, \quad u \rightarrow \infty,$$

and it should be read without $\liminf_{u \rightarrow \infty}$ on the left side.