be equal by the Cauchy integral theorem (by [1] and a result in the author's dissertation not yet published).

Theorem 1 follows directly from Theorem 3.

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# REMARK ON A FORMULA FOR THE BERNOULLI NUMBERS

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Some years ago Garabedian [1] proved the following formula:

$$B_{k+1} = \frac{(-1)^{k+1}(k+1)}{2^{k+1}-1} \sum_{r=0}^{k} (-1)^r \frac{\Delta^r 1^k}{2^{r+1}} \qquad (k \geqq 0), \tag{1}$$

where the even suffix notation is employed for the Bernoulli numbers. The proof of (1) made use of the sum of a certain divergent series.

We wish to point out that (1) is not new. It can be found (in somewhat different notation) in [3, p. 224, formula (68)].

It may be of interest to give a short proof of (1). We use the formula [2, p. 28]

$$C_k = 2^{k+1}(1 - 2^{k+1}) \frac{B_{k+1}}{k+1}, \tag{2}$$

where the $C_k$ are the coefficients in the Euler polynomial:

$$E_k(x) = \left(x + \frac{C}{2}\right)^k = \sum_{s=0}^{k} \binom{k}{s} 2^{-s} C_s x^{k-s}. \tag{3}$$

Then in view of

$$E_k(x+1) + E_k(x) = 2x^k, \tag{4}$$

we have

$$E_k(x) = \left(1 + \frac{1}{2}\Delta\right)^{-1} x^k = \sum_{s=0}^{k} (-1)^s 2^{-s} \Delta^s x^k. \tag{5}$$

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Received by the editors September 5, 1952.

If we take $x=1$ in (5) and use (3) and (4), we get

$$C_k = -2^k E_k(1) = -\sum_{s=0}^{k} (-1)^s 2^{k-s} \Delta^s 1^k.$$

Substitution in (2) leads at once to (1).

In a similar way we can prove

$$B_{k+1} = \frac{(-1)^{k+1}(k+1)}{2^{k+1}-1} \sum_{r=0}^{k} (-1)^r 2^{-r-1} \Delta^r 0^k. \tag{6}$$

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