

NOTE ON SOME FORMULAS OF RODEJA F.

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Generalizing some identities proved by A. W. Goodman [1], Rodeja F. [3] proved the formula

$$(1) \quad \sum_{m=k}^n \frac{G_n \prod_{\alpha=1}^{m-1} (F_n - F_\alpha)}{G_k \prod_{\alpha=1}^{k-1} (F_k - F_\alpha) \prod_{\alpha=k+1}^m (F_k - F_\alpha)} = \delta_n^k \quad (n \geq k \geq 1),$$

where as usual a vacuous product is defined equal to 1; it is remarked that the factor G_n/G_k may be omitted without any loss in generality. It is easily verified that (1) is equivalent to

$$(2) \quad \sum_{m=k}^n \prod_{\alpha=1}^{m-1} (F_n - F_\alpha) \prod_{\alpha=m+1}^n (F_k - F_\alpha) = \delta_n^k \prod_{\alpha=1, \alpha \neq k}^n (F_k - F_\alpha).$$

The author also states the somewhat more general formula

$$(3) \quad \sum_{m=k}^s \prod_{\alpha=k}^{m-1} (F_n - F_\alpha) \prod_{\alpha=m+1}^n (F_k - F_\alpha) = \prod_{\alpha=k+1}^s (F_n - F_\alpha) \prod_{\alpha=s+1}^n (F_k - F_\alpha),$$

which reduces to (2) for $s=n$. Clearly in (3) we may assume $k=1$, $n \geq s$ without any loss in generality.

We wish to point out in this note that the formula (3) is a special case of the Newton interpolation formula (see for example [2, p. 10]). If $f(x)$ is a polynomial of degree less than s , then in the notation of [2],

$$(4) \quad f(x) = f(x_1) + [x_1x_2](x - x_1) + [x_1x_2x_3](x - x_1)(x - x_2) + \cdots \\ + [x_1x_2 \cdots x_s](x - x_1)(x - x_2) \cdots (x - x_{s-1}),$$

where

$$[x_1x_2] = \frac{f(x_1) - f(x_2)}{x_1 - x_2}, \quad [x_1x_2x_3] = \frac{[x_1x_2] - [x_2x_3]}{x_1 - x_3}, \quad \dots$$

In particular if we take

$$f(x) = (x - x_2)(x - x_3) \cdots (x - x_s),$$

then it is easy to verify that

Received by the editors September 5, 1953.

$$(5) \quad [x_1 x_2 \cdots x_r] = (x_1 - x_{r+1}) \cdots (x_1 - x_s) \quad (2 \leq r \leq s).$$

Substituting from (5), it is evident that (4) becomes

$$(6) \quad \prod_{\alpha=2}^s (x - x_\alpha) = \sum_{m=1}^s \prod_{\alpha=m+1}^s (x_1 - x_\alpha) \prod_{\alpha=1}^{m-1} (x - x_\alpha).$$

If we put $x = F_n$, $x_\alpha = F_\alpha$, (6) becomes

$$\prod_{\alpha=2}^s (F_n - F_\alpha) = \sum_{m=1}^s \prod_{\alpha=m+1}^s (F_1 - F_\alpha) \prod_{\alpha=1}^{m-1} (F_n - F_\alpha),$$

which proves (3).

REFERENCES

1. A. W. Goodman, *On some determinants related to p -valent functions*, Trans. Amer. Math. Soc. vol. 63 (1948) pp. 175–192.
2. N. E. Nörlund, *Vorlesungen über differenzenrechnung*, Berlin, 1924.
3. E. G. Rodeja F., *Note on a lemma of A. W. Goodman*, Proc. Amer. Math. Soc. vol. 2 (1951) pp. 314–317.

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