NOTE ON SOME FORMULAS OF RODEJA F.

L. CARLITZ

Generalizing some identities proved by A. W. Goodman [1], Rodeja F. [3] proved the formula

\[
G_n \prod_{a=1}^{m-1} (F_a - F_a) \sum_{m=k}^{n} \frac{G_n \prod_{a=1}^{m-1} (F_a - F_a)}{G_k \prod_{a=1}^{k-1} (F_k - F_a) \prod_{a=k+1}^{m} (F_k - F_a)} = \delta_n^k \quad (n \geq k \geq 1),
\]

where as usual a vacuous product is defined equal to 1; it is remarked that the factor \(G_n/G_k\) may be omitted without any loss in generality. It is easily verified that (1) is equivalent to

\[
\sum_{m=k}^{n} \prod_{a=1}^{m-1} (F_a - F_a) \prod_{a=m+1}^{n} (F_k - F_a) = \delta_n^k \prod_{a=1, a \neq k}^{n} (F_k - F_a).
\]

The author also states the somewhat more general formula

\[
\sum_{m=k}^{s} \prod_{a=k}^{m-1} (F_a - F_a) \prod_{a=m+1}^{n} (F_k - F_a) = \prod_{a=k+1}^{s} (F_a - F_a) \prod_{a=k+1}^{n} (F_k - F_a),
\]

which reduces to (2) for \(s=n\). Clearly in (3) we may assume \(k=1\), \(n \geq s\) without any loss in generality.

We wish to point out in this note that the formula (3) is a special case of the Newton interpolation formula (see for example [2, p. 10]). If \(f(x)\) is a polynomial of degree less than \(s\), then in the notation of [2],

\[
f(x) = f(x_1) + [x_1x_2](x - x_1) + [x_1x_2x_3](x - x_1)(x - x_2) + \cdots + [x_1x_2 \cdots x_s](x - x_1)(x - x_2) \cdots (x - x_{s-1}),
\]

where

\[
[x_1x_2] = \frac{f(x_1) - f(x_2)}{x_1 - x_2}, \quad [x_1x_2x_3] = \frac{[x_1x_2] - [x_2x_3]}{x_1 - x_3}, \quad \ldots.
\]

In particular if we take

\[
f(x) = (x - x_2)(x - x_3) \cdots (x - x_s),
\]

then it is easy to verify that

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(5) \[ [x_1 x_2 \cdots x_r] = (x_1 - x_{r+1}) \cdots (x_1 - x_s) \] (2 \leq r \leq s).

Substituting from (5), it is evident that (4) becomes

(6) \[ \prod_{a=2}^{s} (x - x_a) = \sum_{m=1}^{s} \prod_{a=m+1}^{s} (x_1 - x_a) \prod_{a=1}^{m-1} (x - x_a). \]

If we put \( x = F_n, x_a = F_a, \) (6) becomes

\[ \prod_{a=2}^{n} (F_n - F_a) = \sum_{m=1}^{n} \prod_{a=m+1}^{n} (F_1 - F_a) \prod_{a=1}^{m-1} (F_n - F_a), \]

which proves (3).

REFERENCES


Duke University