SOME TRANSFORMATIONS OF WELL-POISED BASIC HYPERGEOMETRIC SERIES OF THE TYPE ${}_8\Phi_7$

R. P. AGARWAL

1. **Introduction.** Transformations of the basic series ${}_8\Phi_7$ have been studied by Bailey [2], Sears [3], and Slater [4]. Bailey [2] in 1936 gave two and three term relations connecting well-poised ${}_8\Phi_7$ series with special forms of second and third parameters in the numerator. Recently Sears [3] has given one such transformation connecting three well-poised ${}_8\Phi_7$ of the above type which, as will be shown later, is really equivalent to one of the transformations of Bailey. He has also given general transformations connecting ${}_8\Phi_7$ series without special forms of the parameters. These have been later studied and proved in other ways by Slater [4]. But a systematic study of these transformations has not yet been made as was done by Whipple [5] in the case of a well-poised ${}_7F_6$. The relations connecting three ${}_8\Phi_7$ series have appeared to be rather isolated results, so far.

In this paper, following Whipple, I classify the ${}_8\Phi_7$ series in different groups and study systematically the relations existing between the 192 allied series that occur in the classification. The fundamental three term relation due to Bailey [2] has been employed to obtain the various relations, of which some are believed to be new.

In the last two sections of this paper I have employed basic integrals of the Barnes type[1] to give extremely simple and elegant proofs of the two term relations and the fundamental three term relation of Bailey.

2. **Notation.** Following Bailey we write

$$\begin{aligned}\chi(a; b, c, d, e, f) &= \prod\left[\begin{matrix}aq/b,\ aq/c,\ aq/d,\ aq/e,\ aq/f,\ a^2q^2/bcdef\\ aq\end{matrix}\right]\\ &\quad\times {}_8\Phi_7\left[\begin{matrix}a,\ qa^{1/2},\ -qa^{1/2},\ b,\ c,\ d,\ e,\ f;\\ a^{1/2},\ -a^{1/2},\ aq/b,\ aq/c,\ aq/d,\ aq/e,\ aq/f\end{matrix}\ a^2q^2/bcdef\right],\end{aligned}\tag{2.1}$$

where

$$\prod\left[\begin{matrix}a,\ b,\ \cdots\\ c,\ d,\ \cdots\end{matrix}\right] \text{ denotes } \prod_{n=0}^{\infty}\frac{(1-aq^n)(1-bq^n)\cdots}{(1-cq^n)(1-dq^n)\cdots}.$$

Also,

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[1] See for instance Watson, Trans. Cambridge Philos. Soc. vol. 21 (1909) pp. 281–299, and Agarwal, *On integral analogues of certain transformations of well-poised basic hypergeometric series* (under publication in Quart. J. Math. Oxford Ser. A).

$$\mathfrak{p}\begin{bmatrix} a, \cdots; \\ b, \cdots; \end{bmatrix} \text{ denotes products of the type } \prod_{n=0}^{\infty} \frac{(1-q^{a+n})\cdots}{(1-q^{b+n})\cdots}$$

and

$$\omega(a; b, c, d, e, f)$$
$$= \mathfrak{p}\begin{bmatrix} 1+a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f; \\ 1+a, b, c, d, e, f; \end{bmatrix}$$
$$\times {}_8\Phi_7\begin{bmatrix} q^a, q^{1+a/2}, -q^{1+a/2}, q^b, q^c, q^d, q^e, q^f; & q^{2+2a-b-c-d-e-f} \\ q^{a/2}, -q^{a/2}, q^{1+a-b}, q^{1+a-c}, q^{1+a-d}, & \\ q^{1+a-e}, q^{1+a-f} & \end{bmatrix}.$$

3. In this section I give in a tabular form the numerator parameters of the well-poised ${}_8\Phi_7$ of the type (2.1) that occur in different relations in the following sections.

PARAMETERS OF THE 192 ALLIED χ-FUNCTIONS

$s = bcdef/a^2q$

Name of group	Numerator parameters	Convergence indicator $\lvert I\rvert < 1$	No. of permutations of each form
$G_p(0)$	$a; b, c, d, e, f$	q/s	1
	$bc/s; b, c, aq/de, aq/ef, aq/df$	qa/bc	10
	$aq/bs; q/s, aq/bc, aq/bd, aq/be, aq/bf$	b	5
$G_p(1)$ [$G_p(2)$ etc. are permutations]	$b^2/a; b, bc/a, bd/a, be/a, bf/a$	q/s	1
	$qb/as; q/s, q/c, q/d, q/e, q/f$	b	1
	$b^2c/sa; b, bc/a, aq/de, aq/ef, aq/df$	q/c	4
	$qb/sc; q/s, q/c, aq/cd, aq/ce, aq/cf$	bc/a	4
	$qb/cd; q/c, q/d, be/a, bf/a, aq/cd$	aq/ef	6
$G_n(0)$	$q/a; q/b, q/c, q/d, q/e, q/f$	s	1
	$qs/bc; q/b, q/c, ef/a, df/a, de/a$	bc/a	10
	$bs/a; bc/a, bd/a, be/a, bf/a, s$	q/b	5
$G_n(1)$ [$G_n(2)$ etc. are permutations]	$aq/b^2; q/b, aq/bc, aq/bd, aq/be, aq/bf$	s	1
	$as/b; s, c, d, e, f$	q/b	1
	$asq/b^2c; q/b, aq/bc, ef/a, fd/a, de/a$	c	4
	$sc/b; s, c, cd/a, ce/a, cf/a$	aq/bc	4
	$cd/b; c, d, aq/be, aq/bf, cd/a$	ef/a	6

The group G_n has been obtained from the respective group G_p by dividing q by the numerator parameters in order. It is to be understood that, in the expansions of $G_p(0)$ and $G_n(0)$, permutations of the letters b to f are allowed. In the expansions of $G_p(1)$ and $G_n(1)$ permutations of the letters c to f are allowed and so on. If b and c are permuted in $G_p(1)$, it becomes $G_p(2)$ and so on for the rest of the groups.

In the above table the number of permutations of the forms $G_p(0)$, $G_p(1)$, $G_n(0)$, and $G_n(1)$ only are mentioned in the last column. The number of permutations of each of the forms $G_p(r)$ and $G_n(r)$ $(r=2, 3, 4, 5)$ is sixteen, so that there are 192 allied series in all.

4. **Relations between two ${}_8\Phi_7$ series.** The fundamental two term relations of ${}_8\Phi_7$ series were given by Bailey [2], namely

$$\chi(a; b, c, d, e, f) = \chi(a^2q/def; b, c, aq/de, aq/df, aq/ef), \tag{4.1}$$

$$\begin{aligned} &\chi(a; b, c, d, e, f) \\ &\quad = \chi(a^3q^2/b^2cdef; aq/bc, aq/bd, aq/be, aq/bf, a^2q^2/bcdef). \end{aligned} \tag{4.2}$$

These two term relations express the equality between the sixteen series $G_p(0)$.

In (4.1) and (4.2) if we change a to q/a, b to q/b, and so on, we get the equality between the sixteen series of the group $G_n(0)$.

Also replacing a by b^2/a, c by bc/a, d by bd/a, e by be/a, and f by bf/a in (4.1) we get the equality between the first and the third series of the group $G_p(1)$. Similarly, by proper substitutions we can show that all the sixteen series of each of the groups $G_p(r)$ or $G_n(r)$ $(r=1, 2, 3, 4, 5)$ are equal among themselves.

5. **Three term relations between the allied series.** It can be easily shown that there are (apart from mere interchange of parameters) 110 different representations of a given ${}_8\Phi_7$ in terms of two other ${}_8\Phi_7$ series. Taking $G_p(0)$ to be the standard series $\chi(a; b, c, d, e, f)$ there are six typical transformations between three series, which we proceed to deduce systematically. Two of these relations, namely, a relation between $G_p(0)$, $G_p(1)$, and $G_n(2)$, and the relation between $G_p(0)$, $G_p(1)$, and $G_p(2)$ were given by Bailey [2].

The fundamental three term relation of Bailey between three series of the type $G_p(0)$, $G_n(2)$, and $G_p(1)$ is

$$\begin{aligned} &\prod (aq/def, def/a, bd/a, be/a, bf/a, q/c) \times \chi(a; b, c, d, e, f) \\ &\quad = \prod (aq/b, b/a, aq/ef, aq/df, aq/de, a^2q^2/bcdef) \\ &\qquad\qquad \times \chi(ef/c; e, f, aq/bc, aq/cd, ef/a) \\ &\qquad + (b/a) \prod (d, e, f, aq/bc, a^2q/bdef, bdef/a^2) \\ &\qquad\qquad \times \chi(b^2/a; b, bc/a, bd/a, be/a, bf/a). \end{aligned} \tag{5.1}$$

The relation between three G_p's (Bailey [2, 4.6]) can be obtained by eliminating $G_n(2)$ from (5.1) and the similar relation between $G_p(0)$, $G_n(2)$, and $G_p(3)$.

If we replace the $G_n(2)$ series by its equivalent series $\chi(bdef/aq;\, s, b, d, e, f)$, we get

$$\begin{aligned}(5.2)\qquad &\prod (aq/def,\, def/a,\, bd/a,\, be/a,\, bf/a,\, q/c)\times \chi(a;\, b, c, d, e, f)\\ &= \prod (aq/b,\, b/a,\, aq/ef,\, aq/df,\, aq/ed,\, a^2q^2/bcdef)\\ &\qquad \times \chi(bdef/aq;\, s, b, d, e, f)\\ &\quad + (b/a)\prod (d, e, f, aq/bc, a^2q/bdef, bdef/a^2)\\ &\qquad \times \chi(b^2/a;\, b, bc/a, bd/a, be/a, bf/a).\end{aligned}$$

If we interchange c and f in (5.2) we get Sears' result [**3**; 10.2].

Now, interchanging c and e in (5.2) and then eliminating $G_p(1)$ from the resulting transformation and (5.2) we get, after interchanging b and e in the final result, the relation

$$\begin{aligned}(5.3)\qquad &c\prod (de/a,\, df/a,\, ef/a,\, b/c,\, qc/b,\, bcdef/a^2,\, a^2q/bcdef)\\ &\qquad \times \chi(a;\, b, c, d, e, f)\\ &= \prod (aq/cf,\, aq/cd,\, aq/ce,\, b,\, a^2q^2/bcdef,\, a^2q/bdef,\, bdef/a^2)\\ &\qquad \times \chi(cdef/aq;\, s, c, d, e, f)\\ &\quad - (\text{the same expression with } c \text{ and } b \text{ interchanged}).\end{aligned}$$

This gives a relation between three series of the type $G_p(0)$, $G_n(1)$, and $G_n(2)$.

Now, eliminating $G_n(2)$ between (5.2) and (5.3) we get, after some simplification,[2]

$$\begin{aligned}(5.4)\qquad &\{S(def/a,\, bd/a,\, be/a,\, bf/a,\, cdef/a^2,\, c)\\ &\qquad + cS(de/a,\, ef/a,\, df/a,\, bcdef/a^2,\, b/c,\, b/a)\}\\ &\qquad \times \chi(a;\, b, c, d, e, f)\\ &= (b/a)\prod (aq/bc,\, aq/bf,\, aq/bd,\, aq/be,\, c, d, e, f,\, a^2q/bdef,\\ &\qquad bdef/a^2,\, a^2q/cdef,\, cdef/a^2)\\ &\qquad \times \chi(b^2/a;\, b, bc/a, bd/a, be/a, bf/a)\\ &\quad + \prod (aq/ce,\, aq/cd,\, aq/cf,\, aq/ed,\, aq/ef,\, aq/df,\, aq/b,\\ &\qquad b/a,\, a^2q/bdef,\, bdef/a^2,\, a^2q^2/bcdef)\\ &\qquad \times \chi(cdef/aq;\, S, c, d, e, f).\end{aligned}$$

This gives a relation between three series of the type $G_p(0)$, $G_p(1)$, and $G_n(1)$.

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[2] $S(a, b, c, \cdots)$ denotes $\prod(a, q/a, b, q/b, c, q/c, \cdots)$.

If in (5.4) we put $b^2/a=A$, $bc/a=C$, $bd/a=D$, $be/a=E$, and $bf/a=F$ it is transformed into a relation between three series of the type $G_p(0)$, $G_p(1)$, and $G_n(0)$, which can be used to get another relation between three series of the type $G_p(0)$, $G_n(1)$, and $G_n(0)$.

This completes the transformation theory of expressing $\chi(a; b, c, d, e, f)$ in terms of two other ${}_8\Phi_7$ series of the type given in the table in §3. Similar remarks follow for relations between a given series of any other group and two other ${}_8\Phi_7$ series.

6. **Integrals representing two term relations.** In a recent paper[8] I have shown that

$$(6.1)\quad {}_8\Phi_7\left[\begin{matrix} q^a, q^{1+a/2}, -q^{1+a/2}, q^b, q^c, q^d, q^e, q^f; \\ q^{a/2}, -q^{a/2}, q^{1+a-b}, q^{1+a-c}, q^{1+a-d}, \\ q^{1+a-e}, q^{1+a-f} \end{matrix} \; q^{2+2a-b-c-d-e-f}\right]$$
$$= \sin \pi(d+e+f-a-1)$$
$$\times \mathfrak{P}\left[\begin{matrix} d, e, f, 1+a, 1+a-d-e, 1+a-d-f, 1+a-e-f, \\ 1+a-b-c; \\ 1, d+e+f-a, 1+a-d-e-f, 1+a-b, 1+a-c, \\ 1+a-d, 1+a-e, 1+a-f; \end{matrix}\right]$$
$$\times \frac{1}{2\pi i}\int_C \mathfrak{P}\left[\begin{matrix} d+e+f-a+s, 1+a-b+s, 1+a-c+s, 1+s; \\ d+s, e+s, f+s, 1+a-b-c+s; \end{matrix}\right]$$
$$\times \frac{\pi q^s ds}{\sin \pi s \sin \pi(1+a-d-e-f-s)},$$

where C is a line parallel to $Rl(\omega s)=0$ ($\log q=-\omega$), with loops, if necessary, to ensure that the poles of cosec πs for which $s=n$ and those of cosec $\pi(1+a-d-e-f-s)$ for which $s=n+1+a-d-e-f$ ($n=0, 1, 2, \cdots$) lie only to the right of this line.

Now, consider the integral

$$(6.2)\quad \begin{aligned} &\theta(\alpha, \beta, \gamma, \delta; \rho, \sigma) \\ &= \frac{1}{2\pi i}\int_C \mathfrak{P}\left[\begin{matrix} 1+s, \rho+s, \sigma+s, 1+\alpha+\beta+\gamma+\delta-\rho-\sigma+s; \\ \alpha+s, \beta+s, \gamma+s, \delta+s; \end{matrix}\right] \\ &\times \frac{\pi q^s ds}{\sin \pi s \sin \pi(\sigma+\rho-\alpha-\beta-\gamma-\delta-s)}. \end{aligned}$$

Now, the left-hand side of (6.1) is symmetrical in b, c, d, e, and f. Hence, interchanging f and b and writing in the notation of (6.2) we get the transformation

[8] Agarwal, loc. cit.

$$\theta(d, e, f, 1+a-b-c; 1+a-b, 1+a-c) = \frac{\sin \pi(b+d+e-a)}{\sin \pi(d+e+f-a)}$$

$$\times \phi\begin{bmatrix} b, 1+a-b-d, 1+a-b-e, 1+a-c-f, \\ \qquad d+e+f-a, 1+a-d-e-f; \\ d+e+b-a, 1+a-b-d-e, f, 1+a-e-f, \\ \qquad 1+a-d-f, 1+a-b-c; \end{bmatrix} \tag{6.3}$$

$$\times \theta(d, e, b, 1+a-c-f; 1+a-f, 1+a-c).$$

Also, interchanging d with c and e with b in (6.1) and writing in the above notation we get

$$\theta(d, e, f, 1+a-b-c; 1+a-b, 1+a-c) = \frac{\sin \pi(b+c+f-a)}{\sin \pi(d+e+f-a)}$$

$$\times \phi\begin{bmatrix} b, c, 1+a-b-f, 1+a-c-f, d+e+f-a, \\ \qquad 1+a-d-e-f; \\ b+c+f-a, 1+a-b-c-f, d, e, 1+a-d-f, \\ \qquad 1+a-e-f; \end{bmatrix} \tag{6.4}$$

$$\times \theta(c, b, f, 1+a-d-e; 1+a-e, 1+a-d).$$

Putting $d=\alpha$, $e=\beta$, $f=\gamma$, $1+a-b-c=\delta$, $1+a-b=\rho$, and $1+a-c=\sigma$, i.e., $c=\rho-\delta$, $b=\sigma-\delta$, and $a=\rho+\sigma-\delta-1$, we get from (6.3) and (6.4)

$$\theta(\alpha, \beta, \gamma, \delta; \rho, \sigma) = \frac{\sin \pi(\alpha+\beta-\rho)}{\sin \pi(\alpha+\beta+\gamma+\delta-\rho-\sigma)}$$

$$\times \phi\begin{bmatrix} \sigma-\delta, \rho-\alpha, \sigma-\gamma, \rho-\beta, \alpha+\beta+\gamma+\delta-\rho-\sigma+1, \\ \qquad \rho+\sigma-\alpha-\beta-\gamma-\delta; \\ \alpha+\beta-\rho+1, \rho-\alpha-\beta, \gamma, \rho+\sigma-\delta-\alpha-\gamma, \\ \qquad \sigma+\rho-\delta-\beta-\gamma, \delta; \end{bmatrix} \tag{6.5}$$

$$\times \theta(\alpha, \beta, \sigma-\delta, \sigma-\gamma; \sigma+\rho-\delta-\gamma, \sigma)$$

$$= \frac{\sin \pi(\gamma-\delta)}{\sin \pi(\alpha+\beta+\gamma+\delta-\rho-\sigma)}$$

$$\times \phi\begin{bmatrix} \sigma-\delta, \rho-\delta, \rho-\gamma, \sigma-\gamma, \alpha+\beta+\gamma+\delta-\rho-\sigma+1, \\ \qquad \rho+\sigma-\alpha-\beta-\gamma-\delta; \\ \gamma-\delta+1, \delta-\gamma, \alpha, \beta, \rho+\sigma-\beta-\gamma-\delta, \\ \qquad \rho+\sigma-\alpha-\gamma-\delta; \end{bmatrix} \tag{6.6}$$

$$\times \theta(\rho-\delta, \sigma-\delta, \gamma, \rho+\sigma-\alpha-\beta-\delta; \rho+\sigma-\alpha-\delta, \rho+\sigma-\beta-\delta).$$

If in (6.6) we put $(\sigma-\alpha)$ for α, $(\sigma-\beta)$ for β, $(\sigma-\delta)$ for γ, $(\sigma-\gamma)$ for δ, σ for ρ, and $(2\sigma+\rho-\alpha-\beta-\gamma-\delta)$ for σ, the right-hand integral remains unchanged and hence we obtain the transformation

$$\theta(\alpha,\beta,\gamma,\delta;\rho,\sigma)=\frac{\sin\pi(\sigma-\rho)}{\sin\pi(\alpha+\beta+\gamma+\delta-\rho-\sigma)}$$

$$(6.7)\qquad \times\varphi\begin{bmatrix}\sigma+\rho-\alpha-\beta-\gamma-\delta,\ 1+\alpha+\beta+\gamma+\delta-\rho-\sigma, \\ \sigma-\alpha,\ \sigma-\beta,\ \sigma-\gamma,\ \sigma-\delta,\ \rho-\alpha,\ \rho-\beta,\ \rho-\gamma,\ \rho-\delta; \\ 1+\sigma-\rho,\ \rho-\sigma,\ \alpha,\ \beta,\ \gamma,\ \delta,\ \sigma+\rho-\alpha-\beta-\gamma, \\ \sigma+\rho-\alpha-\beta-\delta,\ \sigma+\rho-\beta-\gamma-\delta,\ \sigma+\rho-\alpha-\gamma-\delta;\end{bmatrix}$$

$$\times\theta(\sigma-\alpha,\sigma-\beta,\sigma-\gamma,\sigma-\delta;\sigma,2\sigma+\rho-\alpha-\beta-\gamma-\delta).$$

The relations (6.5) and (6.7) give the integrals representing the two term relations (4.1) and (4.2) respectively. We can employ (6.5), (6.6), and (6.7) to give us relations connecting four Saalschützian ${}_4\Phi_3$. The relation obtained from (6.6) was given by Sears [3, 11.1].

7. **Integral representing the fundamental three term relation** (5.1). We can write (6.1) in the form

$$\omega(a;b,c,d,e,f)=\sin\pi(d+e+f-a-1)$$

$$\times\varphi\begin{bmatrix}1+a-b-c,\ 1+a-d-e,\ 1+a-d-f,\ 1+a-e-f; \\ 1,\ b,\ c,\ d+e+f-a,\ 1+a-d-e-f;\end{bmatrix}$$

$$(7.1)\qquad \times\frac{1}{2\pi i}\int_C\varphi\begin{bmatrix}d+e+f-a+s,\ 1+a-b+s,\ 1+a-c+s,\ 1+s; \\ d+s,\ e+s,\ f+s,\ 1+a-b-c+s;\end{bmatrix}$$

$$\times\frac{\pi q^s ds}{\sin\pi s\sin\pi(1+a-d-e-f-s)}.$$

Hence, by analogy we have

$$\omega(2b-a;b,b+c-a,b+d-a,b+e-a,b+f-a)$$

$$=\sin\pi(b+d+e+f-2a-1)$$

$$(7.2)\times\varphi\begin{bmatrix}1-c,\ 1+a-d-e,\ 1+a-e-f,\ 1+a-d-f; \\ 1,\ b,\ b+c-a,\ b+d+e+f-2a,\ 1+2a-b-d-e-f;\end{bmatrix}$$

$$\times\frac{1}{2\pi i}\int_C\varphi\begin{bmatrix}b+d+e+f-2a+s,\ 1+b-a+s,\ 1+b-c+s,\ 1+s; \\ b+d-a+s,\ b+e-a+s,\ b+f-a+s,\ 1-c+s;\end{bmatrix}$$

$$\times\frac{\pi q^s ds}{\sin\pi s\sin\pi(1+2a-b-d-e-f-s)}.$$

Putting $s+b-a=t$ in (7.2), so that the products under the integral sign in it become the same as in (7.1), and combining with (7.1), we get after slight trigonometrical simplification that

$$
\begin{aligned}
&\wp\begin{bmatrix}1,\ b,\ c,\ d+e+f-a,\ 1+a-d-e-f; \\ 1+a-b-c,\ 1+a-d-e,\ 1+a-e-f,\ 1+a-d-f;\end{bmatrix} \\
&\quad\times\omega(a;\ b,\ c,\ d,\ e,\ f) \\
&\quad -q^{b-a}\wp\begin{bmatrix}1,\ b,\ b+c-a,\ b+d+e+f-2a,\ 1+2a-b-d-e-f; \\ 1-c,\ 1+a-d-e,\ 1+a-e-f,\ 1+a-d-f;\end{bmatrix} \\
&\quad\times\omega(2b-a;\ b,\ b+c-a,\ b+d-a,\ b+e-a,\ b+f-a) \\
&\quad=\sin\pi(a-b) \\
&\quad\times\frac{1}{2\pi i}\int_C \wp\begin{bmatrix}1+s,\ d+e+f-a+s,\ 1+a-b+s,\ 1+a-c+s; \\ d+s,\ e+s,\ f+s,\ 1+a-b-c+s;\end{bmatrix} \\
&\quad\times\frac{\pi q^s ds}{\sin\pi s\,\sin\pi(b-a-s)}.
\end{aligned}
\tag{7.3}
$$

Comparing the right-hand integral in (7.3) with (6.2) after making in it the substitutions $\rho=d+e+f-a$, $\sigma=1+a-c$, $\alpha=e$, $\beta=f$, $\gamma=1+a-b-c$, and $\delta=d$, we get from (6.1) the required transformation (5.1).

It may be remarked that, as shown in §5, the relation (5.1) can be used to find all the other relations between three series of the type ${}_8\Phi_7$ and so we can prove all the relations connecting three ${}_8\Phi_7$ by means of simple transformations and manipulations of integrals of the type (7.1).

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BEDFORD COLLEGE, LONDON