MORE ON THE CONTINUITY OF THE REAL ROOTS OF AN ALGEBRAIC EQUATION

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Melvin Henriksen and I published [1] an incomplete restoration of the following theorem announced by Hewitt [2]:

THEOREM. Let \( C(X, R) \) be the ring of all continuous real-valued functions on a completely regular space \( X \); let \( M \) be a maximal ideal in \( C(X, R) \). The residue field \( C(X, R)/M = C_m \) is real closed.

Hewitt’s proof is defective only in showing a root for every polynomial of odd degree in \( C_m \); we used other results of [2] and the Tietze extension theorem, i.e., we proved the theorem for normal \( X \). This note recovers the whole theorem.

PROOF OF THEOREM. After Hewitt’s work [2], it remains to show that every polynomial \( P(x, w) = w^{2n+1} + \sum_{k=0}^{2n} a_k(x)w^k, a_k \in C(X, R) \), has a root in \( C_m \). If \( f \in C(X, R) \), let \( Z(f) = \{ x \in X | f(x) = 0 \} \), \( Z(M) = [Z(f) | f \in M] \). Decompose the real part of the root of \( P \) into continuous single-valued functions, \( \phi_1, \cdots, \phi_{2n+1} \), as in [1]. Let \( R_i = \{ x \in X | P(x, \phi_i(x)) = 0 \} \). Since the \( R_i \) cover \( X \) and \( Z(M) \) has the finite intersection property, some \( R_{i*} \) meets every element of \( Z(M) \). Then by [2, Theorem 36], \( R_{i*} \in Z(M) \); that is, \( P(x, \phi_{i*}(x)) \equiv 0 \pmod{M} \).

REFERENCES


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