MORE ON THE CONTINUITY OF THE REAL ROOTS OF AN ALGEBRAIC EQUATION

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Melvin Henriksen and I published [1] an incomplete restoration of the following theorem announced by Hewitt [2]:

THEOREM. *Let $C(X, R)$ be the ring of all continuous real-valued functions on a completely regular space X; let M be a maximal ideal in $C(X, R)$. The residue field $C(X, R)/M = C_M$ is real closed.*

Hewitt's proof is defective only in showing a root for every polynomial of odd degree in C_M; we used other results of [2] and the Tietze extension theorem, i.e., we proved the theorem for normal X. This note recovers the whole theorem.

PROOF OF THEOREM. After Hewitt's work [2], it remains to show that every polynomial $P(x, w) = w^{2n+1} + \sum_{k=0}^{2n} a_k(x)w^k$, $a_k \in C(X, R)$, has a root in C_M. If $f \in C(X, R)$, let $Z(f) = [x \in X \mid f(x) = 0]$, $Z(M) = [Z(f) \mid f \in M]$. Decompose the real part of the root of P into continuous single-valued functions, $\phi_1, \cdots, \phi_{2n+1}$, as in [1]. Let $R_i = [x \in X \mid P(x, \phi_i(x)) = 0]$. Since the R_i cover X and $Z(M)$ has the finite intersection property, some R_{i*} meets every element of $Z(M)$. Then by [2, Theorem 36], $R_{i*} \in Z(M)$; that is, $P(x, \phi_{i*}(x)) \equiv 0 \pmod{M}$.

REFERENCES

1. M. Henriksen and J. R. Isbell, *On the continuity of the real roots of an algebraic equation*, Proc. Amer. Math. Soc. vol. 4 (1953) pp. 431–434.

2. E. Hewitt, *Rings of real-valued continuous functions*. I, Trans. Amer. Math. Soc. vol. 64 (1948) pp. 45–99.

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