From this it follows readily that the consistency of Z_C' is provable in Z_C. (Cf. Theorem IV, p. 260, of [2].)

Bibliography

1. Hao Wang, *The irreducibility of impredicative principles*, Math. Ann. vol. 125 (1952) pp. 56–66.

2. ———, *Truth definitions and consistency proofs*, Trans. Amer. Math. Soc. vol. 73 (1952) pp. 243–275.

3. R. McNaughton, Review of [1], J. Symbolic Logic vol. 18 (1953) pp. 265–266.

4. L. Henkin, *Completeness in the theory of types*, J. Symbolic Logic vol. 15 (1950) pp. 81–91.

5. B. Rosser and Hao Wang, *Non-standard models for formal logics*, J. Symbolic Logic vol. 15 (1950) pp. 113–129.

6. A. Tarski, *Der Wahrheitsbegriff in den formalisierten Sprachen*, Studia Philosophica vol. 1 (1936) pp. 251–405.

7. W. V. Quine, *Mathematical logic*, New York, 1940.

8. A. Mostowski, *On models of axiomatic systems*, Fund. Math. vol. 39 (1952) pp. 133–158.

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AN EXTENSION THEOREM FOR SOLUTIONS OF $d\omega = \Omega$

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Let U and V be open sets in E_n such that $\overline{V} \subset U$ and U is connected and homologically trivial, i.e., all homology groups of U beyond the zero-dimensional case vanish. Let Ω be an exterior differential form of degree p on E_n with infinitely differentiable coefficients whose exterior derivative vanishes: $d\Omega = 0$. The well known converse to the Lemma of Poincaré asserts that $\Omega = d\omega$ where ω is an infinitely differentiable $p-1$ form on E_n. Let us suppose however that we are merely given a $p-1$ form α on U such that $d\alpha = \Omega$ on U. The question immediately arises as to whether it is possible to prolong α to all of E_n. The example $U = \{(x, y) \mid x > 0\}$, $\alpha = (xdy - ydx)/(x^2 + y^2)$, $\Omega = 0$ shows us that the answer is negative. Nevertheless, *there exists a $p-1$ form β on E_n such that $\beta = \alpha$ on V and $d\beta = \Omega$ on E_n.*

To prove this, we shall take for granted the existence of an infinitely differentiable function f on E_n such that $f = 1$ on V and $f = 0$ outside of a closed subset of U. We have the form ω on E_n such that $d\omega = \Omega$ on E_n and the form α on U such that $d\alpha = \Omega$ on U. Thus $d(\alpha - \omega) = 0$ on U and so it follows from the hypotheses on U and what is essentially de Rham's second theorem that $\alpha - \omega = d\lambda$ on U,

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where λ is an infinitely differentiable $p-2$ form on U.[1] One sets $\mu=f\lambda$ on U, $\mu=0$ outside U so that μ is a $p-2$ form on E_n such that $d\mu=d(f\lambda)=d\lambda=\alpha-\omega$ on V. The form $\beta=\omega+d\mu$ solves the problem, for $d\beta=d\omega=\Omega$ on E_n and $\beta=\omega+(\alpha-\omega)=\alpha$ on V.

If any doubt remains about the case $p=1$, it is quickly settled when one notes that $\alpha-\omega$ is a function and so $d(\alpha-\omega)=0$ on U implies that $\alpha-\omega=c$, a constant, on U. Now $\beta=\omega+c$ is the solution.

This result and method of proof can be extended to multiply connected regions; we give a single instance:

Let Ω be a p-form on E_n such that $d\Omega=0$ and suppose that α is a $(p-1)$-form on $r>0$ $(r^2=x_1^2+\cdots+x_n^2)$ such that $d\alpha=\Omega$ on $r>0$. Finally, suppose that

$$\lim_{\epsilon\to 0}\int_{r=\epsilon}\alpha = 0$$

in the case $p=n$. Then given any $\epsilon>0$, there exists a $(p-1)$-form β on E_n such that $d\beta=\Omega$ on E_n and $\beta=\alpha$ on $r>\epsilon$.

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[1] This form of de Rham's theorem is given in the paper of A. Weil, *Sur les théorèmes de de Rham*, Comment. Math. Helv. vol. 26 (1952). It is pointed out on p. 138 that in case U is convex, then an elementary proof is possible. Such a proof is given for the case in which U is a cell on p. 94 of the second edition of W. V. D. Hodge, *Harmonic integrals*.