ERRATUM TO
ASYMPTOTIC AND CONVERGENT FACTORIAL SERIES
IN THE SOLUTION OF LINEAR ORDINARY
DIFFERENTIAL EQUATIONS

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An error in the proof of the entitled article was kindly pointed out by Professor H. L. Turrittin; its correction introduces a limitation on the argument of \( \omega \) in Theorem 1. To outline the correction in terms consistent with those of the article, let \( v_{\nu}(z) \) correspond to a segment on the Puiseux diagram for (1) extending from \((\beta_1, m(\beta_1))\) to \((\beta_1+N, m(\beta_1+N))\)—so that \( s_0 = (m(\beta_1+N) - m(\beta_1))/N \)— and call

\[
\tag{11}
\hat{p}_{\beta_1-m(\beta_1)} h^N + \cdots + \hat{p}_{\beta_1+N-m(\beta_1+N)} h^0 = 0
\]

the characteristic equation. Thus (11) has one root \( h_\nu \) which relates to the considered particular solution. Substitutions (4) and (5) convert (1) to a differential equation for \( v_{\nu}(z) \) in which the analogue of (11) has roots \((h_1-h_\nu), \ldots, (h_N-h_\nu)\) if \( h_1, \ldots, h_N \) are the roots of (11). For definiteness, let \( m \) of (4) be the least positive integer making \((m s_0)\) an integer or 0. Then the \( a_{\nu,\mu} \)'s of (3) and later equations are defined as the product of a gamma function and the solution of a difference equation of Poincaré type (reference of footnote 6, chap. XVII), and the characteristic equation for this difference equation has \((h_1-h_\nu)^{-1/(m s_0+1)}, \ldots, (h_N-h_\nu)^{-1/(m s_0+1)}\) as its roots. The \( a_{\nu,\mu} \)'s are a particular solution for which \( \lim_{\nu \to \infty} (a_{\nu,\mu+m(s_0+1)}/a_{\nu,\mu}) \) is one of \((h_1-h_\nu)^{-1}, \ldots, (h_N-h_\nu)^{-1}\), if it exists—we conjecture that this limit is one value among those of \((h_1-h_\nu)^{-1}, \ldots, (h_N-h_\nu)^{-1}\) having the largest modulus. If \( \alpha \) represents the argument of this limit we have, as a final result, that the convergence in Theorem 1 necessitates the additional restriction on arg \( \omega \):

\[
| \arg \omega - (\alpha + M \pi)/m(s_0 + 1) | < \pi/2m \quad s_0 + 1 \quad (\text{where } M = \text{odd integer}).
\]