

ERRATUM TO
 ASYMPTOTIC AND CONVERGENT FACTORIAL SERIES
 IN THE SOLUTION OF LINEAR ORDINARY
 DIFFERENTIAL EQUATIONS

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An error in the proof of the entitled article was kindly pointed out by Professor H. L. Turrittin; its correction introduces a limitation on the argument of ω in Theorem 1. To outline the correction in terms consistent with those of the article, let $v_\nu(z)$ correspond to a segment on the Puiseux diagram for (1) extending from $(\beta_1, m(\beta_1))$ to $(\beta_1 + N, m(\beta_1 + N))$ —so that $s_0 = (m(\beta_1 + N) - m(\beta_1))/N$ —and call

$$(11) \quad p_{\beta_1, -m(\beta_1)} h^N + \cdots + p_{\beta_1 + N, -m(\beta_1 + N)} (h)^0 = 0$$

the characteristic equation. Thus (11) has one root h_ν which relates to the considered particular solution. Substitutions (4) and (5) convert (1) to a differential equation for $v_\nu(z)$ in which the analogue of (11) has roots $(h_1 - h_\nu), \dots, (h_N - h_\nu)$ if h_1, \dots, h_N are the roots of (11). For definiteness, let m of (4) be the least positive integer making (ms_0) an integer or 0. Then the $a_{\nu, \mu}$'s of (3) and later equations are defined as the product of a gamma function and the solution of a difference equation of Poincaré type (reference of footnote 6, chap. XVII), and the characteristic equation for this difference equation has $(h_1 - h_\nu)^{-1/m(s_0+1)}, \dots, (h_N - h_\nu)^{-1/m(s_0+1)}$ as its roots. The $a_{\nu, \mu}$'s are a particular solution for which $\lim_{\mu \rightarrow \infty} (a_{\nu, \mu+m(s_0+1)}/a_{\nu, \mu})$ is one of $(h_1 - h_\nu)^{-1}, \dots, (h_N - h_\nu)^{-1}$, if it exists—we conjecture that this limit is one value among those of $(h_1 - h_\nu)^{-1}, \dots, (h_N - h_\nu)^{-1}$ having the largest modulus. If α represents the argument of this limit we have, as a final result, that the convergence in Theorem 1 necessitates the additional restriction on $\arg \omega$:

$$\left| \arg \omega - (\alpha + M\pi)/m(s_0 + 1) \right| < \pi/2m \left| s_0 + 1 \right|$$

(where $M = \text{odd integer}$).