2. A. D. Wallace, *A fixed point theorem*, Bull. Amer. Math. Soc. vol. 51 (1945) pp. 413–416.

3. L. E. Ward, Jr., *Partially ordered topological spaces*, Proc. Amer. Math. Soc. vol. 5 (1954) pp. 144–161.

4. G. T. Whyburn, *Analytic topology*, New York, 1942.

THE UNIVERSITY OF NEVADA

CORRECTION TO "UNIQUENESS OF THE PROJECTIVE PLANE WITH 57 POINTS"

MARSHALL HALL, JR.

Professor Gunter Pickert has pointed out to me that there is an error in my paper [Proc. Amer. Math. Soc. vol. 4 (1953) pp. 912–916]. The equation on page 915 which reads $12+3s+2t=24$ should read $12+3s+2t+u=24$. This invalidates my conclusion $u=0$ from which I deduce $U=0$, which is necessary for the rest of the paper. I give here a new proof that $U=0$.

We must show that in a 57 point plane a line containing points ACC is not possible, or that $U=0$. The proof is by showing that if there is a line ACC we reach a contradiction. We may take ACC as $A_1C_4C_6$ by numbering points appropriately. We now reletter by the following rule:

$$\begin{matrix} A_1 & A_2 & A_3 & A_4 & B_1 & B_2 & B_3 & C_4 & C_6 \\ A_1 & B_1 & B_5 & A_0 & B_4 & B_2 & A_2 & B_3 & B_6 \end{matrix}$$

and obtain the following configuration:

$$\begin{matrix} A_0 & B_1 & B_2 & B_3 \\ A_0 & B_4 & B_5 & B_6 \\ A_0 & A_1 & A_2 & \\ A_1 & B_1 & B_4 & \\ A_1 & B_2 & B_5 & \\ A_1 & B_3 & B_6 & \\ A_2 & B_1 & B_5 & \\ A_2 & B_2 & B_6 & \\ A_2 & B_3 & B_4 & \end{matrix}$$

The remaining points in these lines must be filled in according to the following pattern:

	A_0	B_1	B_2	B_3	C_1	C_2	C_3	C_4		
	A_0	B_4	B_5	B_6	C_5	C_6	C_7	C_8	A	3
$L-$	A_0	A_1	A_2	D_1	D_2	D_3	D_4	D_5	B	6
	A_1	B_1	B_4	E_1	F_1	F_2	F_3	F_4	C	8
	A_1	B_2	B_5	E_2	F_5	F_6	F_7	F_8	D	5
	A_1	B_3	B_6	E_3	F_9	F_{10}	F_{11}	F_{12}	E	3
	A_2	B_1	B_5	E_3	F_{13}	F_{14}	F_{15}	F_{16}	F	24
$L-$	A_2	B_2	B_6	E_1	F_{17}	F_{18}	F_{19}	F_{20}	G	8
	A_2	B_3	B_4	E_2	F_{21}	F_{22}	F_{23}	F_{24}		57

The points E_1, E_2, E_3 provide the necessary remaining intersection points for these lines. All points as given here are necessarily distinct and there will be 8 remaining points which will be designated by the letter G.

Remaining lines are of the following patterns:

$P-$	A_0	E	E	E	G	G	G	G	
$Q-$	A_0	E	E	F	F	G	G	G	Remaining A_0
$R-$	A_0	E	F	F	F	F	G	G	$P+Q+R+S=5$.
$S-$	A_0	F	F	F	F	F	F	G	
$N-$	A	C	C	F	F	F	G	G	Remaining A's $N=8$.
$U-$	B	B	D	E	G	G	G	G	Remaining BB's
$V-$	B	B	D	F	F	G	G	G	$U+V=3$.
$W-$	B	C	D	E	F	F	G	G	Remaining B's
$X-$	B	C	D	F	F	F	F	G	$W+X=24$.
$Y-$	C	C	D	E	E	E	G	G	
$Z-$	C	C	D	E	E	F	F	G	Remaining CC's
$T-$	C	C	D	E	F	F	F	F	$Y+Z+T=8$.

Counting BE's we find $12+2U+W=18$ whence $W=6-2U=2V$, and $X=24-W=24-2V$. Counting CE's we find $W+6Y+4Z+2T=24$ and combining with $Y+Z+T=8$ and $W=2V$ we obtain $Z=4-V-2Y$, $T=4+V+Y$. Now count EE's $3P+Q+3Y+Z=3$.

Hence if $P=1$ then $Y=Z=0$ whence $V=4$ in conflict with $U+V=3$. Hence we must have $P=0$.

Let us count occurrences of a point C on the lines:

$$\begin{array}{lll} \text{Total occurences:} & 3+w+x+y+z+t = 8 & (1 \text{ on } L,\ 2 \text{ on } N\text{'s}) \\ CB: & 3+w+x = 6, & \\ CE: & w+3y+2z+t = 3. & \end{array}$$

Combining these we find $w+2y+z=1$. Hence $y=0$ and since there is no C on any Y we must have $Y=0$.

Let us now count occurrences of an F on lines. It is on 1 L and 1 N.

$$\begin{array}{ll} \text{Total:} & 2+q+r+s+v+w+x+z+t = 8, \\ FE: & 1+2q+r+w+2z+t = 3, \\ FC: & 2+w+x+2z+2t = 8. \end{array}$$

Now if $q=1$, from the second equation we have $r=w=z=t=0$, whence from the third equation $x=6$. But $q=1$, $x=6$ conflicts with the first equation. Hence $q=0$ and so $Q=0$.

We use two more counts on the number of lines:

$$\begin{array}{ll} A_0E: & 3P+2Q+R = 3 \\ GG: & 6P+3Q+R+8+6U+3V+W+Y = 28. \end{array}$$

Since $P=Q=0$, the first of these yields $R=3$. Putting this in the second and also $W=2V$, $Y=0$, we find

$$6U+5V = 17.$$

Since $U+V=3$, we have $U=2$, $V=1$. Now with the relations above, we can determine the number of lines in each category:

P	Q	R	S	N	U	V	W	X	Y	Z	T
0	0	3	2	8	2	1	2	22	0	3	5.

We reach our final contradiction by showing that there is no point which can be the intersection point of the two lines of category U.

$$U- \quad B\ B\ D\ E\ G\ G\ G\ G.$$

The remaining BB pairs in U and V are B_1B_6, B_2B_4, and B_3B_5 and so the U intersection is not a B. Now consider occurrences of a D on lines. It is on 1 L.

$$\begin{array}{ll} \text{Total:} & 1+u+v+w+x+z+t = 8. \\ DF: & 2v+2w+4x+2z+4t = 24. \end{array}$$

Combining, $x+t=5+u$. But if $u=2$, then $x+t=7$ and these values conflict with the first equation. Hence we cannot have $u=2$ and so the two U lines do not intersect in a D. Now count occurrences of an E.

$$EA_0: \qquad r = 1,$$

$$EG: \qquad 2r + 4u + 2w + z = 8.$$

Here we cannot have $u=2$ and the two U's do not intersect in an E.

For occurrences of a G we note that $n=2$.

$$GG: \qquad r + n + 3u + 2v + w = 7.$$

This is not possible with $n=2$, $u=2$. Hence the two U lines do not intersect in a G. Since there is no point which can be the intersection of the two U's, we have reached a final contradiction.

Professor Pickert further inquired the reason for the assertion on page 916 that a quadrilateral which did not generate a 7 point subplane must generate the whole 57 point plane. This depends on a result which appears to be well known, though not explicitly given in the literature. I give here a refinement due to Professor R. Bruck.

LEMMA. *If a plane with $n+1$ points on a line has a proper subplane with $m+1$ points on a line, then $n \geqq m^2$. If $n \neq m^2$, then $n \geqq m^2+m$.*

PROOF. Let L be a line of the subplane π and P a point on L but not in π. Joining P to the m^2 point of π not on L we have m^2 further lines through P. These must be distinct since otherwise P would be the intersection of two lines of π, and hence a point of π. This gives m^2+1 lines through P and so $n \geqq m^2$. If there are any more lines through P, then there is a line K through P containing no point of π. Hence the intersections of K with the m^2+m+1 lines of π are all distinct and so $n \geqq m^2+m$.

OHIO STATE UNIVERSITY