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CORRECTION TO "UNIQUENESS OF THE PROJECTIVE PLANE WITH 57 POINTS"

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Professor Gunter Pickert has pointed out to me that there is an error in my paper [Proc. Amer. Math. Soc. vol. 4 (1953) pp. 912-916]. The equation on page 915 which reads $12 + 3s + 2t = 24$ should read $12 + 3s + 2t + u = 24$. This invalidates my conclusion $u = 0$ from which I deduce $U = 0$, which is necessary for the rest of the paper. I give here a new proof that $U = 0$.

We must show that in a 57 point plane a line containing points ACC is not possible, or that $U = 0$. The proof is by showing that if there is a line ACC we reach a contradiction. We may take ACC as $A_1C_4C_6$ by numbering points appropriately. We now reletter by the following rule:

$$\begin{array}{cccccccccc}
 A_1 & A_2 & A_3 & A_4 & B_1 & B_2 & B_3 & C_4 & C_6 & \\
 A_1 & B_1 & B_5 & A_0 & B_4 & B_2 & A_2 & B_3 & B_6 &
 \end{array}$$

and obtain the following configuration:

$$\begin{array}{cccc}
 A_0 & B_1 & B_2 & B_3 \\
 A_0 & B_4 & B_5 & B_6 \\
 A_0 & A_1 & A_2 & \\
 A_1 & B_1 & B_4 & \\
 A_1 & B_2 & B_5 & \\
 A_1 & B_3 & B_6 & \\
 A_2 & B_1 & B_5 & \\
 A_2 & B_2 & B_6 & \\
 A_2 & B_3 & B_4 &
 \end{array}$$

The remaining points in these lines must be filled in according to the following pattern:

	A_0	B_1	B_2	B_3	C_1	C_2	C_3	C_4		
	A_0	B_4	B_5	B_6	C_5	C_6	C_7	C_8	A	3
$L-$	A_0	A_1	A_2	D_1	D_2	D_3	D_4	D_5	B	6
	A_1	B_1	B_4	E_1	F_1	F_2	F_3	F_4	C	8
	A_1	B_2	B_5	E_2	F_5	F_6	F_7	F_8	D	5
	A_1	B_3	B_6	E_3	F_9	F_{10}	F_{11}	F_{12}	E	3
	A_2	B_1	B_5	E_3	F_{13}	F_{14}	F_{15}	F_{16}	F	24
$L-$	A_2	B_2	B_6	E_1	F_{17}	F_{18}	F_{19}	F_{20}	G	8
	A_2	B_3	B_4	E_2	F_{21}	F_{22}	F_{23}	F_{24}		57

The points E_1, E_2, E_3 provide the necessary remaining intersection points for these lines. All points as given here are necessarily distinct and there will be 8 remaining points which will be designated by the letter G .

Remaining lines are of the following patterns:

$P-$	A_0	E	E	E	G	G	G	G	
$Q-$	A_0	E	E	F	F	G	G	G	Remaining A_0
$R-$	A_0	E	F	F	F	F	G	G	$P + Q + R + S = 5.$
$S-$	A_0	F	F	F	F	F	F	G	
$N-$	A	C	C	F	F	F	G	G	Remaining A 's $N = 8.$
$U-$	B	B	D	E	G	G	G	G	Remaining BB 's
$V-$	B	B	D	F	F	G	G	G	$U + V = 3.$
$W-$	B	C	D	E	F	F	G	G	Remaining B 's
$X-$	B	C	D	F	F	F	F	G	$W + X = 24.$
$Y-$	C	C	D	E	E	E	G	G	Remaining CC 's
$Z-$	C	C	D	E	E	F	F	G	$Y + Z + T = 8.$
$T-$	C	C	D	E	F	F	F	F	

Counting BE 's we find $12 + 2U + W = 18$ whence $W = 6 - 2U = 2V$, and $X = 24 - W = 24 - 2V$. Counting CE 's we find $W + 6Y + 4Z + 2T = 24$ and combining with $Y + Z + T = 8$ and $W = 2V$ we obtain $Z = 4 - V - 2Y$, $T = 4 + V + Y$. Now count EE 's $3P + Q + 3Y + Z = 3$.

Hence if $P=1$ then $Y=Z=0$ whence $V=4$ in conflict with $U+V=3$. Hence we must have $P=0$.

Let us count occurrences of a point C on the lines:

Total occurrences: $3 + w + x + y + z + t = 8$ (1 on L , 2 on N 's)

$$CB: \quad 3 + w + x = 6,$$

$$CE: \quad w + 3y + 2z + t = 3.$$

Combining these we find $w+2y+z=1$. Hence $y=0$ and since there is no C on any Y we must have $Y=0$.

Let us now count occurrences of an F on lines. It is on 1 L and 1 N .

$$\text{Total:} \quad 2 + q + r + s + v + w + x + z + t = 8,$$

$$FE: \quad 1 + 2q + r + w + 2z + t = 3,$$

$$FC: \quad 2 + w + x + 2z + 2t = 8.$$

Now if $q=1$, from the second equation we have $r=w=z=t=0$, whence from the third equation $x=6$. But $q=1, x=6$ conflicts with the first equation. Hence $q=0$ and so $Q=0$.

We use two more counts on the number of lines:

$$A_0E: \quad 3P + 2Q + R = 3$$

$$GG: \quad 6P + 3Q + R + 8 + 6U + 3V + W + Y = 28.$$

Since $P=Q=0$, the first of these yields $R=3$. Putting this in the second and also $W=2V, Y=0$, we find

$$6U + 5V = 17.$$

Since $U+V=3$, we have $U=2, V=1$. Now with the relations above, we can determine the number of lines in each category:

P	Q	R	S	N	U	V	W	X	Y	Z	T
0	0	3	2	8	2	1	2	22	0	3	5.

We reach our final contradiction by showing that there is no point which can be the intersection point of the two lines of category U .

$$U - B B D E G G G G.$$

The remaining BB pairs in U and V are B_1B_6, B_2B_4 , and B_3B_5 and so the U intersection is not a B . Now consider occurrences of a D on lines. It is on 1 L .

$$\text{Total:} \quad 1 + u + v + w + x + z + t = 8.$$

$$DF: \quad 2v + 2w + 4x + 2z + 4t = 24.$$

Combining, $x+t=5+u$. But if $u=2$, then $x+t=7$ and these values conflict with the first equation. Hence we cannot have $u=2$ and so the two U lines do not intersect in a D . Now count occurrences of an E .

$$EA_0: \quad r = 1,$$

$$EG: \quad 2r + 4u + 2w + z = 8.$$

Here we cannot have $u=2$ and the two U 's do not intersect in an E .

For occurrences of a G we note that $n=2$.

$$GG: \quad r + n + 3u + 2v + w = 7.$$

This is not possible with $n=2$, $u=2$. Hence the two U lines do not intersect in a G . Since there is no point which can be the intersection of the two U 's, we have reached a final contradiction.

Professor Pickert further inquired the reason for the assertion on page 916 that a quadrilateral which did not generate a 7 point subplane must generate the whole 57 point plane. This depends on a result which appears to be well known, though not explicitly given in the literature. I give here a refinement due to Professor R. Bruck.

LEMMA. If a plane with $n+1$ points on a line has a proper subplane with $m+1$ points on a line, then $n \geq m^2$. If $n \neq m^2$, then $n \geq m^2 + m$.

PROOF. Let L be a line of the subplane π and P a point on L but not in π . Joining P to the m^2 point of π not on L we have m^2 further lines through P . These must be distinct since otherwise P would be the intersection of two lines of π , and hence a point of π . This gives m^2+1 lines through P and so $n \geq m^2$. If there are any more lines through P , then there is a line K through P containing no point of π . Hence the intersections of K with the m^2+m+1 lines of π are all distinct and so $n \geq m^2 + m$.

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