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A NOTE ON INVARIANT SUBRINGS

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The problem of invariant subrings has been studied in detail for certain types of rings satisfying the descending chain condition on one-sided ideals.

We prove here two theorems concerning invariant subrings of a ring R without assuming the descending chain condition.

THEOREM 1. *Let R be a ring with identity 1. If S is a subring of R with identity 1, and S has a representation as the complete matrix ring of order $n \geqq 2$ over a ring with identity, then S cannot be a proper invariant subring of R.*

PROOF. Let $E = \{e_{ij}\}$ be a set of n^2 matrix units contained in S, and let B be the centralizer of E relative to S. Then $S = B_n$. Let A be the centralizer of E relative to R. Then $R = A_n$, and $B \leqq A$. If t is an arbitrary element of A, we obtain (following Hattori [1]) that $e_{ii}(1+e_{ij}t)e_{ji}(1-e_{ij}t)e_{ii} = te_{ii}$ belongs to S for arbitrary $i \neq j$. Hence $t = \sum_{i=1}^{n} te_{ii}$ belongs to S, and $A \leqq S$. But then $A = B$, and $S = R$.

THEOREM 2. *Let R be a ring with identity 1, and not of characteristic 2. Assume that R has a representation as the complete matrix ring of order $n \geqq 2$ over a ring with identity. Let S be an invariant sub-sfield of R, with identity 1. Then S is a subfield of the center of R.*

PROOF. Let $E = \{e_{ij}\}$ be a set of n^2 matrix units contained in R, and let A be the centralizer of E relative to R, so that $R = A_n$. We note first that for every noncentral element x contained in R, there exists a square-nilpotent element $p \neq 0$ ($p^2 = 0$) such that $xp \neq px$. (For, if x commutes with every square-nilpotent element, then x commutes in

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particular with the elements e_{ij} and $e_{ii}ye_{jj}$ for $i \neq j$, and y arbitrary in R. This implies that x belongs to the center of R.) Now, suppose S is an invariant sub-sfield of R, and $1 \in S$. If S contains a noncentral element x, then R contains a square-nilpotent element $p \neq 0$ such that $xp \neq px$. Let $t = 2 + p$. Then t^{-1} and $(t-1)^{-1}$ belong to R. Using Hua's identity [2]

$$t = [x^{-1} - (t-1)^{-1}x^{-1}(t-1)][t^{-1}x^{-1}t - (t-1)^{-1}x^{-1}(t-1)]^{-1}$$

and the hypotheses on S, we conclude that t belongs to S. But then p belongs to S. This is impossible since p can have no inverse relative to the identity 1.

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