ful counting shows that B is empty.) The seven-person game, $k=2$, is well known.

Added in proof. The 15 solutions mentioned at the end of §3 are main simple, with $x_7=0$. The projective plane games and their solutions V_{k+1} were previously described in an unpublished paper of Moses Richardson.

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CERTAIN TYPES OF HOMOGENEOUS CONTINUA

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According to the usual definition of homogeneity, a point set M is said to be homogeneous if for any two points x and y of M there is a homeomorphism of M onto itself carrying x into y. Some more general types of homogeneity previously defined in [2] will be studied in this paper, and it will be shown that there is a certain type of homogeneity such that every decomposable compact metric continuum possessing it is a simple closed curve. For bounded plane continua possessing the usual type of homogeneity, this problem has been only partially solved.[1] *Added in proof.* At the Summer Meeting in Laramie, September, 1954, Bing and Jones each presented an example of a decomposable homogeneous bounded plane continuum which is different from a simple closed curve [Bull. Amer. Math. Soc. Abstracts 60-6-766 and 60-6-770].

THEOREM 1. *If every proper subcontinuum of the compact metric continuum M is nearly homogeneous, then M is hereditarily indecomposable.*[2]

PROOF. Since every subcontinuum of M satisfies the hypothesis of this theorem, it will be sufficient to show that M is indecomposable.

Suppose that M is decomposable. Then there is some proper sub-

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[1] For these results, see [3, Theorem 2], [2, Theorem 8], and other references cited in [2].

[2] An example satisfying the hypothesis of this theorem has been described by Bing [1].

continuum H of M such that the closure of $M-H$ does not contain H. Let x be a point of $M-H$, and let K be a subcontinuum of M irreducible from x to H. Since $K-K\cdot H$ is a subset of $M-H$ and K is the closure of $K-K\cdot H$, it follows that K does not contain H. Let y be a point of $H-H\cdot K$, and let H' be a subcontinuum of H irreducible from y to K. Since K is nearly homogeneous and is irreducible between some two points, it follows from [2, Theorem 4] that K is indecomposable. Hence some composant of K intersects both H' and $K-K\cdot H'$, and this composant contains a continuum K' irreducible from H' to some point z of $K-K\cdot H'$. Then $H'+K'$ is a proper subcontinuum of M and is irreducible between y and z. Hence $H'+K'$ is nearly homogeneous, and by [2, Theorem 4], it is indecomposable. This involves a contradiction since both H' and K' are proper subcontinua of $H'+K'$. Hence M is indecomposable.

THEOREM 2. *If M is a decomposable compact metric continuum such that for every two nondegenerate proper subcontinua H and K of M there is a homeomorphism of M onto itself that carries H onto K, then M is a simple closed curve.*[3]

PROOF. Since M is decomposable, then some proper subcontinuum of M is not a continuum of condensation[4] of M. Hence no nondegenerate proper subcontinuum of M is a continuum of condensation of M. This implies that M is locally connected. Clearly M is not an arc, and since every nondegenerate locally connected compact continuum which is neither an arc nor a simple closed curve contains both an arc and a simple triod as proper subsets [5, p. 446], it follows that M is a simple closed curve.

THEOREM 3. *If $n>1$ and the plane continuum M is nearly n-homogeneous and is not locally connected, then M is indecomposable.*

A proof for the case in which M is bounded was indicated in [2, Theorem 9]. If M is unbounded, the method of inversion can be applied so that the proof is quite similar to the one for the bounded case.

THEOREM 4. *If $n>1$ and the unbounded continuum M is n-homogeneous and is a proper subset of the plane, then M is homeomorphic with a straight line.*

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[3] Bing [1, Theorem 15] has shown that the requirement that M be decomposable is necessary.

[4] A proper subcontinuum K of M is said to be a continuum of condensation of M if every point of K is a limit point of $M-K$.

Proof. The argument used in the proof of [2, Theorem 10] can be used to show that M is decomposable. Hence by Theorem 3, M is locally connected. By [2, Theorem 1], M is homogeneous. Mazurkiewicz [4, p. 146] has shown that every locally connected unbounded homogeneous proper subcontinuum of the plane is homeomorphic with a straight line.

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