and in G must separate G. But by Theorem I this set is 0-dimensional, a contradiction. Hence some element of G intersects each of D_1 and D_2, and thus some element of G' contains an arc h with endpoints in D_1 and D_2 respectively. But by an argument similar to that used in the proof of Theorem I we can exhibit an open subcollection U of G with U containing g and with $\overline{U}-U$ countable, a contradiction.

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A NOTE ON BASIC SETS OF HOMOGENEOUS HARMONIC POLYNOMIALS

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For any set of non-negative integers (b_j) such that $b_1 \leqq 1$ and $\sum_{j=1}^{k} b_j = n$, let

$$(1) \qquad H^n_{b_1 \cdots b_k} = \sum (-1)^{[a_1/2]} \frac{n!}{\prod_{j=1}^{k} a_j!} \cdot \frac{\left[\frac{a_1}{2}\right]!}{\prod_{j=2}^{k} \left(\frac{b_j - a_j}{2}\right)!} \prod_{j=1}^{k} x_j^{a_j}$$

where the summation is extended over all (a_j) such that,

(a) $a_j \equiv b_j \bmod 2, j = 1, 2, \cdots, k$,

(b) $\sum_{j=1}^{k} a_j = n$,

(c) $a_j \leqq b_j, j = 2, 3, \cdots, k$.

The polynomials (1) were shown by the authors to form a basic set of homogeneous harmonic polynomials in k variables [1].[1]

It is easily seen that the following differential recursion formulas hold for these polynomials:

Presented to the Society, November 26, 1954 under the title *A basic set of homogeneous polynomials in k variables*. II; received by the editors November 29, 1954.

[1] Numbers in brackets refer to bibliography at the end of the paper.

$$(2)\qquad \frac{\partial}{\partial x_1} H^n_{1b_2\cdots b_k} = nH^{n-1}_{0b_2\cdots b_k}, \qquad 1 + \sum_{j=2}^{k} b_j = n;$$

$$(3)\qquad \begin{aligned} &\frac{\partial}{\partial x_j} H^n_{b_1b_2\cdots b_j\cdots b_k} = nH^{n-1}_{b_1b_2\cdots(b_j-1)\cdots b_k}, \\ &\qquad\qquad j = 2, 3, \cdots, k,\ b_1 = 0, 1,\ \sum_{j=1}^{k} b_j = n; \end{aligned}$$

$$(4)\qquad \frac{\partial}{\partial x_1} H^n_{0b_2\cdots b_j\cdots b_k} = -n \sum_{j=2}^{k} H^{n-1}_{1b_2\cdots(b_j-2)\cdots b_k}, \qquad \sum_{j=2}^{k} b_j = n;$$

where $H^n_{c_1c_2\cdots c_j\cdots c_k}=0$ if $c_j<0$.

The relations (2) and (3) follow directly from (1) by differentiation. To prove (4) we note from (2) that

$$(5)\qquad \frac{\partial^2}{\partial x_1^2} H^{n+1}_{1b_2\cdots b_k} = (n+1)\frac{\partial}{\partial x_1} H^n_{0b_2\cdots b_k}$$

and from (3) that

$$(6)\qquad \frac{\partial^2}{\partial x_j^2} H^{n+1}_{1b_2\cdots b_j\cdots b_k} = (n+1)nH^{n-1}_{1b_2\cdots(b_j-2)\cdots b_k}, \qquad j = 2, \cdots, k.$$

Combining (5) and (6) and noting that $H^{n+1}_{1b_2\cdots b_k}$ is harmonic, we obtain (4).

For $k=2$ the basic sets (1) give the real and imaginary parts of $(x_1+ix_2)^n$. For $k=3$ they give a single formulation for the eight types first given by Protter [**2**] and later reduced by him to four types [**3**]. The differential recursion formulas generalize those given by Protter. The authors are indebted to Rosenbloom and Bers for calling their attention to these results of Protter.

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