(ii) *The intersection of the closed exteriors of the circles of curvature of C, and the intersection of the closed exteriors of the minimal circumscribed circles to C.*

REFERENCE

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HOMOTOPY GROUPS OF ONE-DIMENSIONAL SPACES

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In this paper we prove the following theorem:

If S is a one-dimensional separable metric space, then $\pi_k(S)=0$ *for all* $k>1$.

Actually it is proved that a much broader class of spaces than spheres have the property that mappings of these spaces into one-dimensional spaces are homotopic to constant maps. This class of spaces includes, for example, projective spaces and Lens spaces.

LEMMA 1.[2] *Let X be a compact metric space whose one-dimensional integral singular homology group is a torsion group. Then for any finite covering G of order one by arcwise-connected open sets, G does not contain a simple loop.*

PROOF. By a simple loop we mean a simple chain such that the first and last sets are the same. Let K be the nerve of G. Since K is one-dimensional, a simple loop in G implies a nonbounding one cycle in K. Hence it suffices to show that $H_1(K)=0$.

Let $\phi: X\rightarrow K$ be a canonical map. For each vertex v in K we choose a point $\psi(v)$ in the element of G corresponding to v. For each edge σ with vertices v_1 and v_2 we extend ψ on $\{v_1, v_2\}$ to a mapping of σ into the union of the two elements of G corresponding to v_1 and v_2. This is possible, since these two elements of G are arcwise connected and

Received by the editors August 22, 1956 and, in revised form, September 28, 1956.

[1] Supported by contract AF 18(600)-1571.

[2] The method of proof used below, which extended this lemma from manifolds M with $H_1(M)=0$ to compact spaces X with $H_1(X)$ a torsion group, was suggested by the referee.

must have a non-null intersection by the definition of the nerve of a covering. This defines a map $\psi: K \to X$, and it is easy to check that the map $\phi\psi: K \to K$ is star related with the identity so that $\phi\psi$ is homotopic to the identity map. It follows from this that $\psi_*: H_1(K) \to H_1(X)$ is an isomorphism into. Since $H_1(X)$ is a torsion group, so is $H_1(K)$. Since K is a graph, $H_1(K)$ is free abelian, so that $H_1(K) = 0$. This proves the lemma.

LEMMA 2. *If X is also a locally connected continuum, $Y = f(X)$ is one-dimensional and f is monotone, then Y is a dendrite.*

PROOF. A dendrite is a locally connected continuum which does not contain a simple closed curve. We suppose that Y contains a simple closed curve Γ and obtain a contradiction.

There exists a positive number ϵ such that from any covering of Γ by open sets of diameter less than ϵ one can extract a simple loop of open sets (which may not cover Γ). There exists a covering $\mathcal{U}$ of Y of order one by open sets of diameter less than ϵ. The set of all components of members of $\mathcal{U}$ has a finite subset $\mathcal{V}$ which is a covering of Y of order one by connected open sets of diameter less than ϵ. It follows that $\mathcal{V}$ contains a simple loop $V_1, \cdots, V_{n-1}, V_1$.

The covering $G = \{f^{-1}(V) \mid V \in \mathcal{V}\}$ is a covering of X of order one by connected open sets, and it contains a simple loop. By Lemma 1 this is not possible and Lemma 2 is proved.

THEOREM. *Let X be a locally connected continuum whose one-dimensional integral singular homology group is a torsion group. Let S be a one-dimensional separable metric space. Then any map $f: X \to S$ is homotopic to a constant map.*

PROOF. Let $f = gh$ be the monotone-light factorization of f, and let $Y = h(X)$. Since g is light, Y must be one-dimensional [**3**, p. 91]. By Lemma 2, Y is a dendrite and one of the authors [**2**] has shown that a dendrite is contractible. Hence h is homotopic to a constant and so is f. This proves the theorem.

COROLLARY. *If S is a one-dimensional separable metric space, then $\pi_k(S) = 0$ for $k > 1$.*

REMARK. If an arcwise connected one-dimensional separable metric space X has $\pi_1(X)$ a free finitely-generated group, then all singular homology groups $H_k(X, Z) = 0$, $k > 1$. This follows because X is aspherical and an aspherical space with the same fundamental group is obtained by taking a finite number of circles with one common point. Since the homology groups of this model are trivial in dimensions higher than one, the same is true of X (see [1, p. 481]).

REMARK. For $k=2$ this corollary can be proved without the theorem since a monotone image of the 2-sphere is a cactoid [4] which is either a dendrite or contains a 2-sphere. This also shows that if $\pi_2(S)\neq 0$, then there exists a light map of S^2 into S. However, there exists a space S for which $\pi_2(S)\neq 0$ such that any light map of S^2 into S is inessential.

PROBLEM.[3] Is every monotone image of S^k, $k>1$, simply connected? For $k=2$ the answer is "yes" by the theorem of R. L. Moore [4], but we do not know the answer for $k=3$.

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[3] *Added in proof*. This problem has been settled in the negative by R. H. Bing for $k=3$.