

ON THE DIRICHLET PROBLEM FOR LINEAR DIFFERENTIAL EQUATIONS

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In nonlinear elasticity, it is of interest to know for what real constants λ and μ the system of "linear elasticity equations"

$$(1) \quad \sum_{k=1}^n \left[(\lambda + \mu) \frac{\partial^2 u_k}{\partial x_k \partial x_k} + \mu \frac{\partial^2 u_i}{\partial x_k^2} \right] = 0, \quad (i = 1, \dots, n > 1)$$

admits a unique solution to the Dirichlet problem for sufficiently smooth regions and boundary data. This question also appears to be of some interest in the general theory of partial differential equations.

The system (1) is elliptic if and only if $(\lambda + 2\mu)\mu \neq 0$, strongly elliptic if and only if $(\lambda + 2\mu)\mu > 0$. When it is strongly elliptic, existence and uniqueness of solutions to the Dirichlet problem for a rather general class of regions and boundary data can be inferred from results of Browder [1] and Morrey [2]. It is noted in [3] that uniqueness fails when $(\lambda + 2\mu)\mu = 0$. For the elliptic systems obtained from (1) by setting $n = 2$, $\lambda = -3\mu \neq 0$ and replacing u_2 by $-u_2$ or u_2 by $2^{1/2}u_2$, Biçadze [4] has shown that uniqueness fails. To settle the question of uniqueness, it thus remains to dispose of the case $(\lambda + 2\mu)\mu < 0$, which we do in the following

THEOREM. *When $(\lambda + 2\mu)\mu < 0$, the system (1) will admit nonzero solutions which vanish on the boundary of some region R of given diameter and are analytic in the closure of R .*

PROOF. It is easily verified by direct substitution in (1) that

$$u_1 = (n - 1)\mu x_1^2 - (\lambda + 2\mu) \sum_{k=2}^n x_k^2 - \epsilon\mu, \quad u_2 = \dots = u_n = 0,$$

where ϵ is any positive constant, is a solution of (1) which is analytic everywhere. When $(\lambda + 2\mu)\mu < 0$, the surface $u_1 = 0$ is an ellipsoid and one can choose ϵ so that the region R interior to it has any prescribed diameter.

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ON THE IDENTITIES OF CERTAIN ALGEBRAS

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Introduction. One basic problem in the study of finite algebras is concerned with the existence of a finite basis for the identities of the algebra, i.e., a finite set of identities from which *all* the identities of the algebra are logical consequences. That the identities of a finite algebra need not have a finite basis (in the above sense) has already been observed by Lyndon [3]. This leads naturally to the following question: are there certain classes of finite algebras the identities of which possess a finite basis? We shall answer this question in the affirmative. Indeed, the main result of this paper is the following

THEOREM. *A functionally strictly complete algebra which contains more than one element has a finite basis for its identities.*

1. Preliminary concepts. In this section, we shall review some basic concepts and definitions, all of which are to be found in Foster [1; 2].

Let $\mathfrak{A} = (A, \rho, \dots)$ be a universal algebra with primitive operations ρ, \dots . Let $A = \{ \dots, x, \dots \}$.

A (k -ary) \mathfrak{A} -function $f(x_1, \dots, x_k)$ is a composition, via the primitive operations, of indeterminate symbols x_1, \dots, x_k over the set A together with a (possibly empty) set of constants¹ ($= \text{fixed} \in A$).

An \mathfrak{A} -function is called *strict* if it involves no constants.

In an obvious way each \mathfrak{A} -function $f(x_1, \dots, x_k)$ represents (or has associated with it) a mapping of the set A^k into A , where of course different \mathfrak{A} -functions need not represent different such mappings. If \mathfrak{A} -functions $f(x, \dots)$ and $g(x, \dots)$ represent the same mapping we

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¹ In more uniform terminology a constant may also be defined as a "0-ary" function.