

where the  $L_i$  are nonsingular square matrices and where  $L_i$  is the transpose of  $L_{m-i}$ . Moreover,

$$\begin{pmatrix} & & & L_0 \\ 0 & & & \\ & \cdot & & \\ & & \cdot & \\ L_m & & & 0 \end{pmatrix}$$

is the matrix of the symmetric bilinear function over  $E_\infty^{2k}$  relative to  $\xi_\infty$ . By Lemma 1 we have  $\tau(E) = \tau_{\xi_\infty}(E_\infty)$ . This concludes the proof of our theorem.

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## THE PERIPHERAL CHARACTER OF CENTRAL ELEMENTS OF A LATTICE<sup>1</sup>

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A lattice being a Hausdorff space together with a pair of continuous lattice operations ( $\wedge$  and  $\vee$ ) the content of this note is best exhibited by quoting a corollary to our theorem: *If a compact connected lattice is (topologically) situated in Euclidean  $n$ -space then its center is contained in its boundary.* Thus, far from being "centrally located," the central elements are "peripheral."

The above is a consequence (see [3, p. 273]) of the

**THEOREM.** *If  $L$  is a compact connected lattice, if  $R$  is an  $(n, G)$ -rim [3] for  $L$  and if (i)  $a$  is central [1, p. 27] or if (ii)  $L$  is modular and  $a$  is complemented then  $a \in R$ .*

**PROOF.** The procedure is to introduce an appropriate multiplication into  $L$  so that  $L$  is a semigroup, to show that  $L$  is not simple (in the semigroup sense [3]) and that  $a$  is a left unit. Since  $L$  is compact it has a zero and unit, 0 and 1, as is well-known. Indeed, the set  $\bigcap \{x \vee L \mid x \in L\}$  is easily seen to consist of exactly one element, namely 1. If  $a = 1$  then the hypotheses of Theorem 1 of [3] are fulfilled using the multiplication  $(x, y) \rightarrow x \wedge y$  so that 1 being a unit for the multiplication,  $1 \in R$ . If  $a \neq 1$  let  $x \cdot y = (a' \wedge x) \vee y$ ,  $a'$  being a

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complement of  $a$ , see §9, Chapter II of [1]. Under (i) or (ii) the multiplication is associative (loc. cit.) and is clearly continuous. It is immediate that  $a \cdot y = y$  since  $a' \wedge a = 0$ , hence  $a$  is a left unit. It is clear that  $x \cdot 1 = 1$  so that  $L \cdot 1 \cdot L = 1 \cdot L$ . If  $1 \cdot L = L$  then 1 is a left unit for this multiplication by Theorem 1 of [2]. But  $1 \cdot a = 1 \neq a$ .

Hence  $L \cdot 1 \cdot L \neq L$  and it remains only to show that  $x \cdot L = L$  implies  $x \cdot R = R$ , in order to verify the hypotheses of Theorem 1 of [3]. But  $x \cdot L = \{(a' \wedge x) \vee y \mid y \in L\}$  so that  $x \cdot L = L$  gives  $a' \wedge x = 0$  and hence  $x \cdot R = \{(a' \wedge x) \vee y \mid y \in R\} = \{y \mid y \in R\} = R$ .

It is known (unpublished) that the center of any compact lattice is totally disconnected. L. W. Anderson (Tulane dissertation, 1956) has shown that a compact connected lattice which is (topologically) situated in the plane is distributive. In this case we see that all complemented elements are boundary—elements. It would be interesting to know if the Theorem remains valid upon the deletion (in (ii)) of the stipulation that  $L$  be modular. The hypothesis of modularity was used only in obtaining the associativity of the multiplication.

REMARK. If one desires to forego the commutativity of addition and multiplication then a modular lattice may be regarded (in many ways) as a ring-like system. For  $a \in L$  (modular!) let  $x + y = (x \wedge a) \vee y$  and  $x \cdot y = (x \vee a) \wedge y$ . It is clear that addition and multiplication are idempotent and associative and that multiplication distributes over addition: indeed,  $(x + y)z = yz = xz + yz$ . Moreover  $x + a = a = xa$  for all  $x \in L$ . If  $L$  has a zero or unit or if  $a$  has a complement then these elements play special roles, easily determined. Presumably the lattice structure can be recovered from such ring-like systems.

#### REFERENCES

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