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A NOTE ON THE STONE-WEIERSTRASS THEOREM FOR QUATERNIONS[1]

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A result of M. H. Stone [1, p. 466], which is nicely presented by N. Dunford [2, p. 23], is as follows: Let A be a closed subalgebra of the B-algebra $C(X)$ of all continuous real-valued functions on the compact Hausdorff space X. Then $A = C(X)$ if and only if A distinguishes between every pair of distinct points of X, i.e., for every pair $x_1 \neq x_2$ of points in X, there is an f in X such that $f(x_1) \neq f(x_2)$.

If one substitutes the word complex for the word real in the above statement, it becomes false. A well known counter example is obtained by letting X be the set of complex numbers z such that $|z| \leq 1$ and letting A be the subalgebra of functions which are analytic in the interior of X.

The purpose of this note is to show that if the word quaternion is substituted for the word real in the above statement, it remains valid. To be specific, let A be a set of continuous quaternion-valued functions which satisfy the following conditions:

1. A is complete.
2. Given a quaternion q, the function $f(x) \equiv q$ is in A.
3. If f and g are in A, then fg and $f+g$ are in A.

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If A contains all continuous quaternion-valued functions, it obviously distinguishes between points. Letting A distinguish between points, consider two arbitrary distinct points x_1 and x_2. Choose an element of A which takes a different value at x_2 than at x_1. Multiply this function by an appropriate quaternion to obtain a function f such that real part $[f(x_1)] \neq$ real part $[f(x_2)]$. But the real part of f is $[f-ifi-jfj-kfk]/4$ which is an element of A. Therefore, A contains real valued functions which distinguish between points.

Since A is complete, and is closed under multiplication, addition and subtraction, it follows that the set of all real-valued functions in A is also complete and closed under these arithmetic operations. The Stone-Weierstrass Theorem implies that A contains all continuous real-valued functions on X. Therefore, A contains all continuous quaternion-valued functions on X.

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