ON AN AXIOM OF BOURBAKI

OSWALD WYLER

It is the purpose of this paper to show that Axiom A2 in Bourbaki's axiomatic system for set theory can be replaced by the weaker statement that every term x defines a set $\{x\}$ of which x is the only element. All references in the paper are to [1], and the terminology and notation of [1] are used.

We place ourselves into a theory with specific signs $=$ and $\in$, in which S1 through S8 are schemas, and A1, A4 and the statement

$$(\forall x)\ \mathrm{Coll}_y\ (y = x) \tag{1}$$

are axioms. The theory may contain other specific signs, schemas and axioms.

Axiom (1) allows us to define a set $\{x\} = E_y\ (y=x)$. Criterion C51 (p. 65) then can be proved. It follows that there is a set ϕ such that $(\forall x)(x \notin \phi)$ is true. Sets $\{\phi\}$ and, using A4, $\mathfrak{P}(\{\phi\})$ can then be constructed. It follows that $\phi \in \mathfrak{P}(\{\phi\})$, $\{\phi\} \in \mathfrak{P}(\{\phi\})$, and $\phi \neq \{\phi\}$.

Let now R in S8 (p. 64) be the relation

$$(x = u \text{ and } y = \phi) \ \text{ or } \ (x = v \text{ and } y = \{\phi\}).$$

The relation

$$(\forall y)(\exists X)(\forall x)(R \Rightarrow (x \in X))$$

is true, as we may put $X = \{u\}$ for $y = \phi$ and $X = \{v\}$ for $y \neq \phi$. Then

$$\mathrm{Coll}_x\ ((\exists y)((y \in \mathfrak{P}(\{\phi\})) \text{ and } R))$$

is true, by S8 and C30 (p. 37). Since

$$((\exists y)((y \in \mathfrak{P}(\{\phi\})) \text{ and } R)) \Leftrightarrow (x = u \text{ or } x = v),$$

we have $\mathrm{Coll}_x\ (x = u \text{ or } x = v)$, and hence, using C27 (p. 36),

$$(\forall u)(\forall v)\ \mathrm{Coll}_x\ (x = u \text{ or } x = v).$$

This is a restatement of A2.

Reference

1. N. Bourbaki, *Théorie des ensembles*, Chap. I and II., Actualités Scientifiques et Industrielles, no. 1212, Paris, Hermann, 1954.

University of New Mexico

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