ON AN AXIOM OF BOURBAKI

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It is the purpose of this paper to show that Axiom A2 in Bourbaki's axiomatic system for set theory can be replaced by the weaker statement that every term x defines a set $\{x\}$ of which x is the only element. All references in the paper are to [1], and the terminology and notation of [1] are used.

We place ourselves into a theory with specific signs $=$ and $\in$, in which S1 through S8 are schemas, and A1, A4 and the statement

$$(1) \qquad (\forall x)\ \mathrm{Coll}_y\ (y = x)$$

are axioms. The theory may contain other specific signs, schemas and axioms.

Axiom (1) allows us to define a set $\{x\} = E_y\ (y = x)$. Criterion C51 (p. 65) then can be proved. It follows that there is a set ϕ such that $(\forall x)(x \notin \phi)$ is true. Sets $\{\phi\}$ and, using A4, $\mathfrak{P}(\{\phi\})$ can then be constructed. It follows that $\phi \in \mathfrak{P}(\{\phi\})$, $\{\phi\} \in \mathfrak{P}(\{\phi\})$, and $\phi \neq \{\phi\}$.

Let now R in S8 (p. 64) be the relation

$$(x = u \text{ and } y = \phi) \text{ or } (x = v \text{ and } y = \{\phi\}).$$

The relation

$$(\forall y)(\exists X)(\forall x)(R \Rightarrow (x \in X))$$

is true, as we may put $X = \{u\}$ for $y = \phi$ and $X = \{v\}$ for $y \neq \phi$. Then

$$\mathrm{Coll}_x\ ((\exists y)((y \in \mathfrak{P}(\{\phi\})) \text{ and } R))$$

is true, by S8 and C30 (p. 37). Since

$$((\exists y)((y \in \mathfrak{P}(\{\phi\})) \text{ and } R)) \Leftrightarrow (x = u \text{ or } x = v),$$

we have $\mathrm{Coll}_x\ (x = u \text{ or } x = v)$, and hence, using C27 (p. 36),

$$(\forall u)(\forall v)\ \mathrm{Coll}_x\ (x = u \text{ or } x = v).$$

This is a restatement of A2.

REFERENCE

1. N. Bourbaki, *Théorie des ensembles*, Chap. I and II., Actualités Scientifiques et Industrielles, no. 1212, Paris, Hermann, 1954.

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