NOTE ON A PAPER OF CIVIN AND YOOD

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It is possible to obtain stronger results than those of [1] in a small amount of space through the use of a different method. We follow the notation of [1] throughout.

Theorem 1. Let $B$ be a complex commutative Banach algebra with countable space $\mathcal{M}$ of maximal regular ideals. Let $A$ be a separating subalgebra. Then (i) if $A$ is not contained in any maximal regular ideal it is determining. Otherwise (ii) there exists a (unique) maximal regular ideal $M$ of $B$ with $\hat{A}$ dense in $M$.

Proof. If $x \in A$ then $\text{Cl}[\hat{x} (\mathcal{M})] = \hat{x}(\mathcal{M}) \cup \{0\}$ is a countable compact set. Then by a theorem of Lavrent'ev [2], polynomials in $z$ are uniformly dense on $\text{Cl}[\hat{x} (\mathcal{M})]$ in all continuous complex valued functions on $\text{Cl}[\hat{x} (\mathcal{M})]$ and in particular uniformly approximate $\hat{z}$ on $\text{Cl}[\hat{x} (\mathcal{M})]$, i.e. polynomials in $\hat{x}$ uniformly approximate $\hat{x}$ on $\mathcal{M}$. Then, $\text{Cl}[\hat{A}]$ being a uniformly closed, separating, self adjoint algebra of continuous, complex valued functions vanishing at $\infty$ on a locally compact space $\mathcal{M}$, equals $\mathcal{C}(\mathcal{M})$ in case (i) or at worst some unique $\text{Cl}[\hat{M}]$ in case (ii) by the Stone-Weierstrass approximation theorem [3].

Theorem 3.3 and extensions of Theorem 3.5 and Corollary 3.6 of [1] eliminating the hypothesis of regularity and the necessity of limiting the discussion to maximal subalgebras follow as simple corollaries, e.g.

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Corollary 2. The only closed separating subalgebras of a complex commutative $B^*$-algebra with a countable space of maximal regular ideals are these maximal regular ideals themselves.

The method of proof of Theorem 1 and the Lavrent'ev Theorem will also yield the following more general version:

Theorem 3. If $A$ is a separating subalgebra of a complex commutative Banach algebra $B$ such that all $x \in A$ have spectrum plus $\{0\}$ nowhere dense, nonseparating sets of complex numbers, then $A$ is determining if it is not contained in a maximal regular ideal, and otherwise $\hat{A}$ is dense in $\hat{M}$ for some unique maximal regular ideal $M$ of $B$.

A number of related results are readily derivable from this theorem.

References


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