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NOTE ON A PAPER OF CIVIN AND YOOD

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It is possible to obtain stronger results than those of [1] in a small amount of space through the use of a different method. We follow the notation of [1] throughout.

Theorem 1. *Let B be a complex commutative Banach algebra with countable space $\mathfrak{M}$ of maximal regular ideals. Let A be a separating subalgebra. Then* (i) *if A is not contained in any maximal regular ideal it is determining. Otherwise* (ii) *there exists a* (*unique*) *maximal regular ideal M of B with $\hat{A}$ dense in $\hat{M}$.*

Proof. If $x \in A$ then $\mathrm{Cl}[\hat{x}(\mathfrak{M})] = \hat{x}(\mathfrak{M}) \cup \{0\}$ is a countable compact set. Then by a theorem of Lavrent'ev [2], polynomials in z are uniformly dense on $\mathrm{Cl}[\hat{x}(\mathfrak{M})]$ in all continuous complex valued functions on $\mathrm{Cl}[\hat{x}(\mathfrak{M})]$ and in particular uniformly approximate $\bar{z}$ on $\mathrm{Cl}[\hat{x}(\mathfrak{M})]$, i.e. polynomials in $\hat{x}$ uniformly approximate $\hat{x}^-$ on $\mathfrak{M}$. Then, $\mathrm{Cl}[\hat{A}]$ being a uniformly closed, separating, self adjoint algebra of continuous, complex valued functions vanishing at ∞ on a locally compact space $\mathfrak{M}$, equals $\mathcal{C}(\mathfrak{M})$ in case (i) or at worst some unique $\mathrm{Cl}[\widehat{\mathfrak{M}}]$ in case (ii) by the Stone-Weierstrass approximation theorem [3].

Theorem 3.3 and extensions of Theorem 3.5 and Corollary 3.6 of [1] eliminating the hypothesis of regularity and the necessity of limiting the discussion to maximal subalgebras follow as simple corollaries, e.g.

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Corollary 2. *The only closed separating subalgebras of a complex commutative B^*-algebra with a countable space of maximal regular ideals are these maximal regular ideals themselves.*

The method of proof of Theorem 1 and the Lavrent'ev Theorem will also yield the following more general version:

Theorem 3. *If A is a separating subalgebra of a complex commutative Banach algebra B such that all $x \in A$ have spectrum plus $\{0\}$ nowhere dense, nonseparating sets of complex numbers, then A is determining if it is not contained in a maximal regular ideal, and otherwise $\hat{A}$ is dense in $\hat{M}$ for some unique maximal regular ideal M of B.*

A number of related results are readily derivable from this theorem.

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