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REMARK ON AUTOMORPHISMS OF GROUPS

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Let G be a group with center C. Let α be an automorphism of G and n an integer such that α^n is an inner automorphism. Thus there is a g in G such that $\alpha^n(x)=gxg^{-1}$ for all x in G. Applying α to both sides of this equation we have that $\alpha^n(\alpha(x))=\alpha(g)\alpha(x)\alpha(g)^{-1}$ for all x in G. Since every element in G can be written as $\alpha(x)$ for some x in G, it follows that g and $\alpha(g)$ induce the same inner automorphism of G. Thus $g^{-1}\alpha(g)=c$ where c is in C. Now if y is in C, then $(gy)^{-1}\alpha(gy)=g^{-1}y^{-1}gc\alpha(y)=cy^{-1}\alpha(y)$. Thus as x runs through all x in G which induce the inner automorphism α^n, the elements of the form $x^{-1}\alpha(x)$ run through the entire coset cC_α in C/C_α, where C_α is the subgroup of C consisting of all elements of the form $y^{-1}\alpha(y)$ (y in C). This element of C/C_α depends on n and will be denoted by $o(\alpha, n)$.

THEOREM. *If all the fixed points of α are in the center of G, then $\alpha^{n^2}=1$. Further $\alpha^n=1$ if and only if $o(\alpha, n)=(1)$.*

PROOF. Let g in G induce the inner automorphism α^n. Then by the previous remarks we have that $g^{-1}\alpha(g)=c$ where c is in C. Thus the abelian subgroup of G generated by C and g is stable under α. Since $\alpha^n(g)=g$, it follows that $\prod_{i=0}^{n-1}\alpha^i(g)$ is a fixed point of α and is thus in C. On the other hand, since $\alpha(g)=gc$, we have that $\prod_{i=0}^{n-1}\alpha^i(g)=g^nd$ for some d in C. Therefore g^n is in C which means that $\alpha^{n^2}=1$.

It is clear that if $\alpha^n=1$, then $o(\alpha, n)=(1)$. Suppose $o(\alpha, n)=(1)$. Then by our introductory remarks, we can choose a g in G such that g induces the inner automorphism α^n and $g^{-1}\alpha(g)=1$. Thus g is a fixed point of α. Consequently g is in the center of G, which means that $\alpha^n=1$.

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It should be observed that if $o(\alpha, n) = cC_\alpha$, then c has the property that $\prod_{i=0}^{n-1} \alpha^i(c) = 1$. Thus $o(\alpha, n)$ is actually an element of the cohomology group $H^3(Z_n, C)$, where Z_n is the integers mod n and a generator of Z_n operates on C as α does. It is easily seen that $o(\alpha, n)$ is the "obstruction" in the sense of Eilenberg and MacLane of the Q-kernel $Z_n \to A(G)/I(G)$ given by $m + Z \to \alpha^m I(G)$, where $A(G)$ and $I(G)$ are the automorphism and inner automorphism groups of G respectively [1]. Thus the above theorem gives another interpretation of the "obstruction" in this special case.

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