R. L. MOORE'S AXIOM 1' AND METRIZATION

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Let \( S \) be a Hausdorff space for which there exists a simple sequence \( G_1, G_2, \ldots \) of open coverings such that (1) for each \( n \), \( G_n \supseteq G_{n+1} \), and (2) if \( H \) and \( K \) are nonintersecting closed subsets of \( S \) one of which is compact, then for some \( n \) no element of \( G_n \) intersects both \( H \) and \( K \). At the 1957 Summer Meeting of the Society the question arose in connection with Mr. Armentrout's paper, *A study of certain plane-like spaces without the use of arcs*, as to whether or not \( S \) when satisfying certain rather complicated axioms was metric. I remarked that there did exist such nonmetric spaces. This observation was incorrect.

**Theorem.** The space \( S \) is metric.

**Proof.** Let \( p \) be a point of an open set \( R \). There exists a natural number \( n \) such that if \( g, h \in G_n \), \( p \in g \), and \( g \cdot h \neq 0 \), then \( g + h \subseteq R \). For suppose, on the contrary, that for each natural number \( n \), there exist \( g_n, h_n \in G_n \), \( p \in g_n \), \( g_n \cdot h_n \neq 0 \) and \( (g_n + h_n) \cdot (S - R) \neq 0 \); let \( p_n \) be a point of \( g_n \cdot h_n \). Obviously \( p_1, p_2, \ldots \) converges to \( p \). Let \( H = R \cdot (p + p_1 + p_2 + \cdots) \) and let \( K = S - R \). Both \( H \) and \( K \) are closed and \( H \) is compact. Furthermore, for each \( n \) some element of \( G_n \) intersects both \( H \) and \( K \). This is a contradiction.

It now follows from Moore's metrization theorem [1] that \( S \) is metric.

**References**


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1 A National Science Foundation Senior Postdoctoral Fellow.
2 Cf., Moore's Axiom 1' in [2, p. 324].
3 Abstract number 797, Bull. Amer. Math. Soc. vol. 63 (1957) p. 403