COROLLARY 2. *If* $\phi: H^k(A) \to H^k(\overline{A})$ *is an isomorphism for* $k \leqq n$ *and univalent for* $k = n+1$, *and* $\phi: H(H^+(X, A)) \to H(H^+(X, \overline{A}))$ *is an isomorphism, then* $\phi: H^k(\Gamma A) \to H^k(\Gamma \overline{A})$ *is also an isomorphism for* $k \leqq n$ *and univalent for* $k = n+1$.

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ONE-PARAMETER TRANSFORMATION GROUPS IN THE PLANE

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Very little is known about the action of a one-parameter group R on two-space except when all orbits are circles, in which case the action is completely known [1]. In a forthcoming paper, A. Beck proves that any closed set can act as the set of fixed points for R. Hence, a very general description appears to be hopeless. However, here, we are able to prove the following result.

THEOREM. *Let* E *be the plane and* R *the real line acting on* E *as a group of transformations without fixed points* (*i.e., no point is left fixed by all of* R). *If* E/R *is Hausdorff, then* E *is fibred as a direct product of* R *and a cross sectioning line. Thus,* R *is equivalent to a group of translations.*

PROOF. Let $x \in E$. Since x is not fixed under R, there is a closed interval $[-a, a] = T$ about 0 in R, and an arc $C \subset E$, $x \in C$ but not an end point of C, such that $T^2(C)$ is a compact neighborhood of x and the mapping $(t, c) \to t(c)$ is one to one from $T^2 \times C \to T^2(C)$. That is, C is a local cross section to the local orbits of T^2 [1]. We shall show that C is a local cross section for the orbits of R.

Suppose, on the contrary, that for some $z \in C$, there is an $r > a$ such that $r(z) \in T(C)$. Let b be the greatest lower bound of such numbers. Then $b(z) \in -a(C)$, for if not, say $b(z) = t(c)$, $t \in T$, $c \in C$, and $t > -a$, then there is a t', $-a < t' < 0$, such that $t + t' > -a$. Hence $(b+t')(z) = (t+t')(c) \in T(C)$. But $t' < 0$ so that $b + t' < b$. By the choice of b, this implies $b + t' < a$. Since this implies $b + t'' < a$ for

Presented to the Society, April 19, 1957; received by the editors December 21, 1956.

$t' \leqq t < 0$, $b = a$. That is, for some t, $0 < t < a$, $(b+t)(z) \in T(c)$. But this contradicts the fact that $T^2(C)$ is homeomorphic to $T^2 \times C$. Thus $b > a$, and $b(z) \in -a(C)$.

Suppose $b(z) = -a(z)$. Then $[-a, b](z) = R(z)$ is a circle bounding a pre-compact region A by the Jordan curve theorem. Hence, if $z \in A$, $R(z) \subset A$ since orbits cannot intersect. Then, for any $r \in R$, $r(A^-) \subset A^-$. Thus r has a fixed point. For each n, let x_n be a fixed point for $1/2^n$. Let y be a limit point of the x_n's. Thus $R(y) = y$, contradicting our hypothesis that R acts without fixed points.

Now we may assume $b(z) \neq -a(z)$. Let $[b(z), -a(z)] = -a[(a+b)(z), z]$ denote the arc of $-a(C)$ joining $b(z)$ to $-a(z)$. Then $[-a, b](z) \cup [b(z), -a(z)]$ is a simple closed curve in E, and thus divides E into two parts A and B one of which is pre-compact. Moreover, if $R_+ = (0, \infty)$, $R_- = (-\infty, 0)$, $R_+(b(z))$ is contained in one, say it is A, while $R_-(-a(z))$ is contained in the other, for $R_+(b(z))$ cannot leave A except by crossing $[b(z), -a(z)]$ since an orbit cannot cross itself. But $R_+(b(z))$ cannot leave $T(C)$ except by crossing $a(C)$. A similar argument establishes that $R_-(-a(z)) \subset B$. Moreover, if $z \in A$, and $r > 0$, then $r(z) \in A$ since again $R(z)$ cannot cross $[-a, b](z)$, and cannot leave $T(C)$ through $-a(C)$. Hence, for $r > 0$, $r(A^-) \subset A^-$. But A^- is a closed two-cell since it is bounded by a simple closed curve. Thus r has a fixed point. Again find $x_n \in A^-$ such that $1/2^n(x_n) = x_n$. If y is a limit point of x_n, then $R(y) = y$, a contradiction.

This proves that C meets $R(x)$ in exactly one point for each $x \in T(C)$. Thus, C is a local cross section for all of R. Since we can find a local cross section for each point of E, and since E/R is Hausdorff, E is a fibre bundle over E/R with fibre R and base space a connected one-dimensional manifold. Since E is a principal fibre bundle over E/R, and R is the line, there is a cross section L of E/R in E such that the natural mapping $E \times R \to R(E)$ is a homeomorphism onto E [2]. Clearly L must be a line.

Bibliography

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