This proves $xy \in A$ and similarly $yx \in A$. Since I_f is obviously closed under addition, it is a two sided ideal. An application of Lemma 3 proves that A is commutative.

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A NOTE ON VALUED LINEAR SPACES

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Banaschewski [1] has given a simple and elegant proof of Hahn's embedding theorem for ordered abelian groups. His method can be used to prove the author's generalization of Hahn's theorem [2, p. 11]. In this note we make use of Banaschewski's method to prove a special case of the author's theorem (which is also a generalization of Hahn's theorem) that has been proven by Gravett [3].

Let (L, Δ, d) be a valued linear space [3]. That is, L is a vector space over a division ring K, Δ is a linearly ordered set with minimum element θ, and d is a mapping of L onto Δ such that for all $x, y \in L$, $d(x) = \theta$ if and only if $x = 0$, $d(x) = d(kx)$ for all $0 \neq k \in K$, and $d(x+y) \leqq \text{Max } [d(x), d(y)]$. For each $\delta \in \Delta$, let $C^\delta = \{x \in L: d(x) \leqq \delta\}$ and let $C_\delta = \{x \in L: d(x) < \delta\}$. Let W be the vector space of all mappings f of Δ into the join of the spaces C^δ/C_δ for which $f(\delta) \in C^\delta/C_\delta$ and $R_f = \{\delta \in \Delta: f(\delta) \neq C_\delta\}$ is an inversely well ordered set. W is a subspace of the unrestricted direct sum V of the C^δ/C_δ. W is also a valued linear space (W, Δ, d'), with $d'(f)$ the largest $\delta \in R(f)$.

Received by the editors February 1, 1958.

THEOREM. *There exists an isomorphism $x\rightarrow\bar{x}$ of L into W. Moreover, if $d(x)=\alpha$, then $\bar{x}(\alpha)=C_\alpha+x$ and $\bar{x}(\delta)=C_\delta$ for all $\alpha<\delta\in\Delta$. Thus this isomorphism is value-preserving.*

PROOF. Let $\mathfrak{S}$ be the set of all subspaces of L. There exists [1, Lemma 4, p. 431] a mapping π of $\mathfrak{S}$ into $\mathfrak{S}$ such that for all $C, D\in\mathfrak{S}$, $C\oplus\pi(C)=L$ and if $C\subseteq D$, then $\pi(C)\supseteq\pi(D)$. For $x\in L$ and $\delta\in\Delta$, let $\bar{x}(\delta)=C_\delta+x_\delta$, where $L=C^\delta\oplus\pi(C^\delta)$ and x_δ is the component of x in C^δ. Then $\bar{x}\in V$ and it follows easily that the mapping $x\rightarrow\bar{x}$ is an isomorphism of L into V. If $\alpha=d(x)<\delta$, then $x\in C^\alpha\subseteq C_\delta$, hence $\bar{x}(\alpha)=C_\alpha+x$ and $\bar{x}(\delta)=C_\delta$. To complete the proof it suffices to show that $R_{\bar{x}}$ is an inversely well ordered set (for each $0\neq x$ in L). Let Γ be a nonempty subset of $R_{\bar{x}}$, and let C be the join of the C^γ for all γ in Γ. $L=\pi(C)\oplus C$, and $x=y+z$ where $y\in\pi(C)$ and $z\in C$. $d(z)$ is the greatest element in Γ. For if $d(z)<\gamma\in\Gamma$, then $\bar{z}(\gamma)=C_\gamma$, and since $\pi(C^\gamma)\supseteq\pi(C)$, $\bar{y}(\gamma)=C_\gamma$. But then $C_\gamma=\bar{y}(\gamma)+\bar{z}(\gamma)=\bar{x}(\gamma)\neq C_\gamma$.

If, as in [2], Δ is partially ordered and d is multiple valued, then practically the same proof gives the embedding theorem [2, p. 11].

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