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ON A CLASS OF UNIVERSAL ORDERED SETS

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An ordered set B is said to be $\aleph_\alpha$-universal if and only if every ordered set of power $\aleph_\alpha$ is similar to a subset of B. Let U_{ω_α} be the lexicographically ordered set of all sequences of 0's and 1's of type ω_α; and let H_α be the subset of U_{ω_α} consisting of all sequences $\{x_\xi\}_{\xi<\omega_\alpha}$ for which there is some $\xi_0<\omega_\alpha$ such that $x_{\xi_0}=1$ and, for $\xi>\xi_0$, $x_\xi=0$.

H_0, being countable, dense, and without first or last element, is similar to the set of rationals in their natural order, and therefore, is $\aleph_0$-universal. Sierpiński [2] has shown (as a direct consequence of his theorem that $H_{\alpha+1}$ is an $\eta_{\alpha+1}$-set) that, for any α, $H_{\alpha+1}$ is $\aleph_{\alpha+1}$-universal. Gillman [1] has given a demonstration that, for any limit ordinal α, H_α is $\aleph_\alpha$-universal. The purpose of this note is to give a very simple proof of these results, which does not depend on the ordinal α.

Theorem. *H_α is $\aleph_\alpha$-universal.*

Proof. Let A be an ordered set of power $\aleph_\alpha$. Fix some well-ordering $\{a_\beta\}_{\beta<\omega_\alpha}$ of A. Let $\prec$ denote the order in A. Define a function ϕ from A into H_α in the following way. Let a_τ be an element of A, and $\beta<\omega_\alpha$. Then the βth component $\phi_\beta(a_\tau)$ of $\phi(a_\tau)$ is defined by:

$$\phi_\beta(a_\tau) = \begin{cases} 1 & \text{if } \beta \leqq \tau \quad \text{and} \quad a_\beta \preceq a_\tau, \\ 0 & \text{otherwise.} \end{cases}$$

Now, let a_τ and a_σ be any elements of A, with $a_\tau \prec a_\sigma$. Clearly, if $\beta\leqq\sigma$, $\phi_\beta(a_\sigma)\geqq\phi_\beta(a_\tau)$. But, $\phi_\sigma(a_\sigma)=1$ and $\phi_\sigma(a_\tau)=0$. Hence, $\phi(a_\tau)$ pre-

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cedes $\phi(a_\sigma)$ in H_α. Thus, ϕ is a one-one order-preserving mapping of A into H_α. Q.E.D.

Note that H_0 is $\aleph_0$-universal and of power $\aleph_0$. Since $\overline{H}_{\alpha+1}=2^{\aleph_\alpha}$, $H_{\alpha+1}$ is of power $\aleph_{\alpha+1}$ if and only if $2^{\aleph_\alpha}=\aleph_{\alpha+1}$. Finally, for limit ordinals α, $\overline{H}_\alpha = \sum_{\beta<\alpha} 2^{\aleph_\beta}$, and, therefore, H_α is of power $\aleph_\alpha$ if and only if, for every $\beta<\alpha$, $2^{\aleph_\beta} \leqq \aleph_\alpha$ (and, hence, if $2^{\aleph_\beta}=\aleph_{\beta+1}$ for all $\beta<\alpha$). For $\alpha>0$, it seems to be an open problem to prove, without additional cardinality assumptions, the existence of an $\aleph_\alpha$-universal ordered set of power $\aleph_\alpha$.

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