ON ESSENTIAL FIXED POINTS

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The purpose of this note is to furnish an affirmative answer to a question posed at the Summer Institute on Set Theoretic Topology held at the University of Wisconsin in 1955. Let $X^X$ denote the space of continuous functions of $X$ into $X$ topologized by the compact open topology. A fixed point $p$ of a map $f \in X^X$ is called essential if for each neighborhood $U$ of $p$ there is a neighborhood $N$ of $f$ such that if $g \in N$, then $g$ has a fixed point in $U$.

**Theorem.** If $X$ is a compact Hausdorff space which has the fixed point property, then there is an $f \in X^X$ such that each fixed point of $f$ is essential.

**Proof.** Let $x_0$ be any element of $X$, and consider the map $f \in X^X$ where $f(X) = x_0$. Let $U$ be any neighborhood of $x_0$. Then $N = \{g : g(X) \subseteq U\}$ is a neighborhood of $f$ with the property that each $g \in N$ has a fixed point in $U$. Therefore $x_0$ is an essential fixed point of $f$. Since $x_0$ is the only fixed point of $f$, $f$ is the required map.

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Received by the editors June 16, 1958.